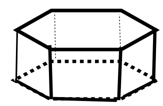
**Math 310 ♡ Self-test ♢ Geometry Basics (Chapter 9+ 10)**

This is NOT a comprehensive study guide. The following may be used to *TEST YOURSELF* ***after*** **review** of the assigned exercises and class work. *Memorization of these questions and answers is NOT advised.*

1. Find the word or phrase or number which *best* completes each statement.
a. The intersection of two distinct planes which
   i. are parallel is .... ii. are perpendicular is .... iii. form a dihedral angle is ....

b. The intersection of
   i. two parallel lines is .... ii. four concurrent lines is .... iii. two skew lines is ....

c. The number of planes passing through (i.e. containing)
   i. a line is .... ii. two distinct points is ....
   iii. two parallel lines is .... iv. three non-collinear points is ....
   v. a line and a separate point is .... vi. a pair of intersecting lines is ....



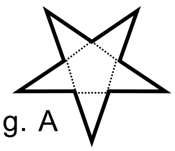
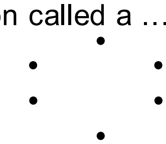


fig. A

d. A hexagonal hatbox most closely resembles the polyhedron...

e. A simple closed plane curve consisting of line segments is a ....

f. An equilateral five-pointed "star" (fig. A) can be folded up into a convex polyhedron called a ....



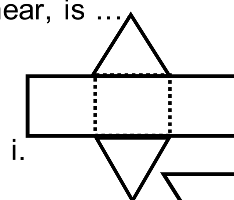
g. A quadrilateral with ALL SIDES EQUAL is called a ....
   A quadrilateral with ALL INTERIOR ANGLES EQUAL is called a ....

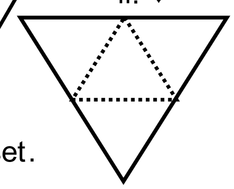
h. The number of segments connecting six points, no 3 of which are collinear, is ....

i. Coplanar lines which do not intersect are ....

j. Lines which cannot be contained in a plane are ....

k. The figures at right could be folded up into polyhedra called.... i. ii. ↓





l. The measure of each of the interior angles of a regular pentagon is ....

m. The pairs of angles formed by two intersecting lines are called....

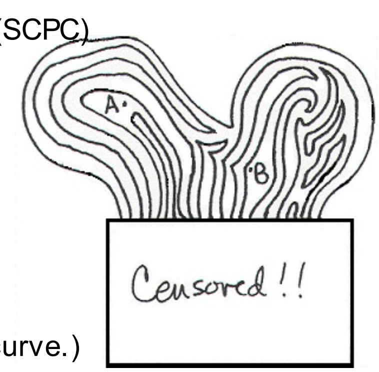
n. The intersection of a line and a plane can be a ______ or a ______ or the empty set.

o. Given points A and B, how many rays pass through the two, with one as an endpoint?

p. The set of all points in a plane a distance "r" from a given point "P" is a ....

q. The set of all points in space a distance "r" from a given point "P" is a ....

2. According to the Jordan Curve Theorem, a simple closed plane curve (SCPC) separates the plane– into __ regions, called ...?
   Given a point and a SCPC, can you easily determine whether the point is on the interior or exterior of the curve? How?




3. Given the curve shown* at right is a simple closed curve in the plane of the paper, are points A and B on the same side of the curve?
   *How do you know?*
   * (*Well, partially shown. The censor thought the curve too curvy and decided to cover part of it.*)(The censor's strip is not part of the curve.)

4. Name the five regular convex polyhedra (Platonic solids) & describe each one briefly (number of faces, type of faces, what forms a vertex of each). Explain why there are no regular convex polyhedra with octagonal faces.

5. Find the indicated sums of the measures of angles:
a. The sum of measures of the two acute angles in a right triangle.
b. The sum of the measures of interior angle and exterior angle at any vertex of a regular pentagon.
c. The sum of measures of a straight angle and two angles which are complementary to each other.
d. An angle measures $72°27'10''$. Find the sum of the measures of its complement & supplement.

6. Sketch each figure; find the number of faces, vertices & edges. Show Euler's formula holds.
   a. prism with a pentagonal base  b. pyramid with hexagonal base  c. octahedron

7. Given AB||CD and EF a transversal of the two: Select one angle, mark it "$\alpha$"; then find all angles congruent to the selected angle, and mark them "$\alpha$" also. How many did you find?

8. For a 70-sided convex polygon find the sum of the measures of:
   a. the interior angles  b. the exterior angles

9. **True-False** (where false, *how do you know?*):
   a. A cone is a type of polyhedron.
   b. A rectangle is a square.
   c. Every rectangle is a parallelogram.
   d. An equilateral triangle is a polygon.
   e. A pyramid is a type of prism.
   f. Every right triangle is acute.
   g. A triangle is an example of a simple closed curve.
   h. A figure-8 is an example of a simple closed curve.
   i. If two angles of a triangle are complementary, then the triangle is a right triangle.
   j. There is only one quadrilateral composed of segments of length 2cm, 4cm, 2cm, 4cm, in that order.
   k. There is only one triangle composed of segments of lengths 10 cm, 6 cm & 3 cm, in that order.

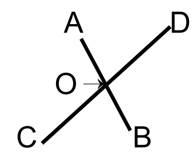
10. a. Is it possible for *a line & a plane* to intersect in: i. just 1 point? ii. exactly 2 points? iii. exactly 3 points? iv. infinitely many points? v. no points?
    b-d: Same questions for: b. *two distinct lines*  c. *two distinct planes*  d. *two distinct triangles*

11. Why are triangles discussed and used so much in geometry?

12. Can the line segments ________ ______________ __________________ form the sides of a triangle?

    How many DIFFERENT triangles can be formed using those 3 lengths?

13. Let O be the midpoint of intersecting segments $\overline{AB}$ and $\overline{CD}$.
    Prove △ACO and △BDO are congruent.




14. The sun is low in the sky, and casts a sixteen-foot shadow for a five-foot tall person. At the same time, the shadow of the redwood tree is 352 feet long. What is the height of the redwood tree?

15. Answer the questions of **parts D E F G H* & I #3-5 on the Practice Final** and #8 on p. 610.
    * In addition to constructions in part H, also *know these*: angle bisector, altitude, median.

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Answers (let's hope):

1. **a**-i. $\phi$ **a**-ii. a line **a**-iii. a line  Note: "$\phi$" means the empty set, which contains nothing.
   **b-**i. $\phi$ **b-**ii. a point **b-**iii. $\phi$
   **c-**i. infinite **c-**ii. same! (*essentially same as a line*) **c-**iii. one **c-**iv. one **c-**v. one **c-**vi. one.
   **d**. right hexagonal prism **e**. polygon **f**. pentagonal pyramid
   **g.** rhombus; rectangle **h**. 1+ 2+ 3+ 4+ 5 = 15(draw them!) **i**. parallel **j**. skew
   **k.** i. triangular based prism;
   **k**. ii. right triangular pyramid – or regular tetrahedron, since the faces are all equilateral triangles.
   **l**. 3(180°)/5 = 108° **m.** vertical angles **n.** a <u>point</u> or a <u>line</u> or $\phi$
   **o**. two: $\overrightarrow{AB}$ and $\overrightarrow{BA}$ **p**. *circle*. **q**. replace "plane" with "space", and the answer is *sphere*.

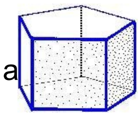
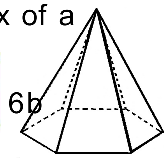
2. A SCPC divides the plane into three disjoint connected sets of points: the 'interior' points ('inside' the curve), the 'exterior' points ('outside' the curve), and the set of points which make up the curve. The points of the curve lie between the interior and exterior sets of points. Crossing the curve moves from interior to exterior or vice versa.

3. As to where A & B are, you should know! A is on the inside, & B is outside!... Also, A & B are on opposite sides of the curve; to get from A to B you must cross the curve an ___ number of times.

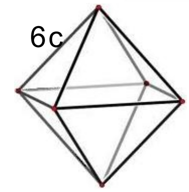
**Geo Basics Review Answers continued.**

4. Equilateral triangles make tetra-, octa- and icosa- ; squares make the cube, or hexa-; regular pentagons make the dodeca- hedron. For more information, see text or your notes. Regular octagons have interior angles of 135°; three of these cannot fit together to make the vertex of a convex polyhedron, as the total exceeds 360°.

5. **a**. 90° **b**. 180° **c**. 180°+ 90°= 270° **d**. 125°5′40″ [= 2(17°32′50″) + 90° ]

6a  6b 

6. Sketches, see right.
   **a**. 10,7,15; *EF:* 10+ 7 – 2= 15. **b**. 7,7,12; *EF:* 7+ 7– 2= 12 **c**. 8,6,12; *EF:* 8+ 6– 12= 2

6c 

7. See text; you should have found four angles congruent to each other (*and the remaining four would be supplementary to the ones you marked, & therefore congruent to each other*).

8. 12240°; 360°; (not asked, but the number of diagonals is 70•69/2 – 70 = 2415)

9. **a**. *F*, the base of a cone is not necessarily a polygonal region (e.g. base may be circular region).
   BTW, it *is true* that a pyramid is a type of cone; so every pyramid is a cone.
   **b**. *F*, a rectangle with two different-length sides is not a square.
   BTW: Can we say a square is a rectangle? Yes! Is a square a parallelogram? Yes!
   **e**. *F*, a pyramid has triangular "sides", a prism has parallelograms (which may be rectangles, etc.).
   **f**. *F*, NO right triangle is acute:
   a 90° angle violates the definition of acute triangle—i.e. all ∠s must measure < 90°.
   **h**. *F*, a figure-8 crosses itself.
   (Notice one of the consequences: unlike a *simple* closed plane curve, this curve separates the plane into *four* distinct connected sets of points.)
   **j**. *F*, there are infinitely many. Build one rectangular, then “squash” it down to a parallelogram.
   **k**. *F*, there are NONE, because the two shorter sides are too short to span across the longest side.
   (The lengths of the two shortest sides must sum to more than the longest side of a △.)

10. **a-**i. Yes, this is the typical situation– picture the line piercing the plane as a needle pierces paper.
    **a-**ii & iii. No, if two points of the line lie in the plane, the entire line must lie in the plane.
    **a-**iv. Yes, if line lies in the plane, all points of the line will be points of intersection (shared by both line & plane).
    **a-**v. Yes, if line lies in a plane parallel to the given plane, there will be no points of intersection.
    **b-**i. Yes; a typical scenario of two intersecting lines. Not parallel, not skew, just intersecting at a point.
    **b-**ii &iii &iv. No; if lines share two points, they are the same line (two points determine a line).
    **b-**v. Yes, if they are parallel, no points of intersection; if they are skew, ditto.
    **c-**i. NO **c-**ii. NO **c-**iii. NO **c-**iv.yes **c-**v. yes **d-**i. yes **d-**ii. yes **d-**iii. yes **d-**iv.yes **d-**v. yes

11. Triangles are very important in geometry, as once a minimal number of features are determined, such as the lengths of the three sides, the triangle is completely determined. For this reason, triangles are important in the real world, *not just in the study* of geometry; if we want an object to *keep its shape*, we *build* triangles *into* it! If we want to mark a location, we “triangulate” it!

12. There is only one triangle that can be constructed with those segments, in that any other triangle with the same length sides would be a congruent copy. P.S. There IS such a triangle, because the lengths of the two shortest sides together exceed the length of the longest side.

13. H:352′ = 5 : 16 So H = 5(352′)/16 = 110′. The redwood must be 110 feet tall.

14. $\overline{AO} \cong \overline{BO}$ since O is the midpoint of $\overline{AB}$.
    Similarly, $\overline{CO} \cong \overline{DO}$.
    ∠AOC ≅∠BOD (vertical angles) ∴ by SAS thm. △ACO ≅△BDO

15. Look on-line (Notices page!) for solutions to Practice Final questions. See text for #8.