Math 310 Fall 2007Test #1 Statistics & Probability ANSWERS

SHOW WORK *as if making a solutions guide for students.* Use extra paper if necessary.

Don't forget labels, scales, & legends!

#1-3 The ages, in years, of ten members of the IHHAA are given below.

75 96 96 93 84 54 51 75 69 24

() 1. Classify the data in a stem-and-leaf diagram. so there are at least five classes.

Ages of ten members of the IHHAA*

2	4
3	
4	
5	1 4
6	9
7	5 5
8	4
9	3 6 6

Legend

5	1 4

represents ages 51 & 54 years of two members of IHHAA

* International Habitat for Humanity Alumni Assn.

() 2. Draw a histogram using classes which correspond to those in problem (1).

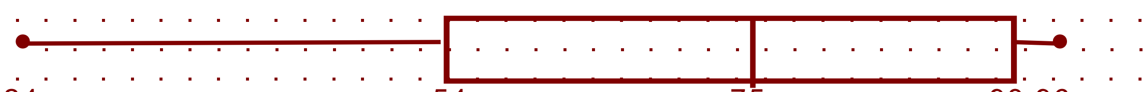
Ages of ten members of the IHHAA

() 3. Draw a box plot* for this data.

* Not a line plot

* Not a bar chart.

Ages, in years, of ten members of the IHHAA



WORK → 24 51 54 69 75 ^ 75 84 93 96 96

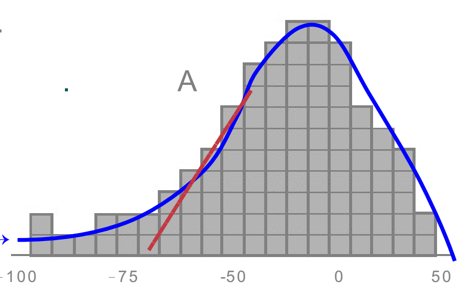
Min, Q_1, Q_2, Q_3, Max

IQR = 93 – 54 = 39

Test value for outlier = 1.5 (39) = 68.5 … No outliers.

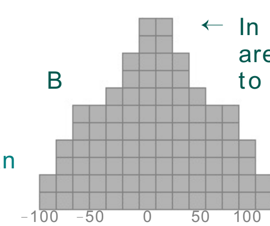
() 4. Choose (circle the letter of) the BEST completion of each statement.

For data A, shown at right, it appears:
the median is greater than the mean.
The data is skewed left (see the "Tail").
If data were symmetric (see the red line), then mean would = median.
When skewed left, the very low values in the tail "pull" the mean down,
but do not have the same effect on the median.
So the mean is below the median, thus the answer above..

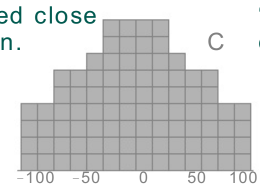


Of these three data distributions, B & C & D, the one with the greatest standard deviation is: D

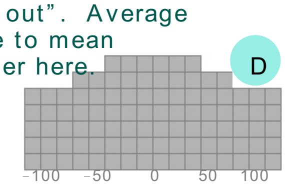
The average distance to the mean is greatest for distribution D.
This will also be true for the s.d.
the s.d. is greatest for distribution D.



← In B, many data are clustered close to the mean.



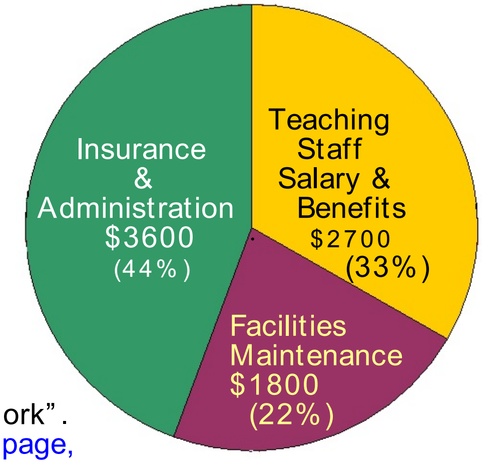
In D, the data are more "spread out". Average distance to mean is greater here.



() 5. Showing ALL your work, construct a pie chart illustrating the distribution of school expenditures.
In the ALUFSD Unified School District, the following yearly expenditures are made for each student:

Teaching staff salary & benefits:	*$2700*
School plant maintenance:	*$1800*
Insurance & Administration:	*$3600*
Total expenditure per student:	$8100

ALUFSD Unified School District yearly expenditures per student



Teaching salary etc. deserves 2700/8100, or
1/3 of the circle.
1/3 of 360° = 120° sector

School plant and maintenance should have
1800/8100 = 2/9 ...2/9 of 360° = 80° sector.

Ins. & admin. get 3600/8100 = 4/9 of the pie,
or a sector with 4/9 of 360° = 160° central angle.

Note: explanation shown above ↑ constitutes "showing your work".
Notice this does not mean scribbling computations all over the page, but making clear how the work is being done.

() 6. Showing your work, for the quiz scores in the frequency table at right:
a. calculate the **mean**. b. calculate the **standard deviation**.

quiz score	*frequency*
3	*1*
7	*1*
10	*3*

6a. Mean $= \dfrac{\Sigma(x)}{n}$

$$= \frac{3 \cdot 10 + 7 + 3}{5} = \frac{40}{5} = 8$$

NOTE: A mean lower than 3 or higher than 10 is clearly impossible! (Why?)

6b. Variance $= \dfrac{\Sigma(x - \text{mean})^2}{n}$

$$= \frac{3 \cdot (10-8)^2 + (7-8)^2 + (3-8)^2}{5} = \frac{3 \cdot 4 + 1 + 25}{5} = \frac{38}{5} = 7.6$$

Std Dev $= \sqrt{7.6}$

NOTE: Std dev should be more than 0 and less than 3.5. It is!

() 7. The 25 students in Miss Horne's 4th-grade averaged 60 on the state reading test; the remaining 15 4th-graders, in Mr. King's class, averaged 80. What is the mean for all the fourth graders?

mean $= \dfrac{\text{Total (of all tests) points}}{\text{\# of tests}}$

$= \dfrac{25 \cdot 60 + 15 \cdot 80}{25 + 15}$ ←Total points from Horne's class, plus total from King's class.
← # of tests from Horne's plus # from King's class.
Note Horne's 25 students have more "weight" in computing the mean.

$= \dfrac{1500 + 1200}{25 + 15}$

$= \dfrac{2700}{40}$

$= 67.5$

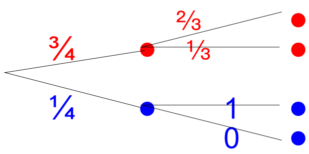
Notice this: The combined mean is 7.5 units away from 60, Horne's mean, and 12.5 units away from 80, King's mean. So the larger class (Horne's) has more influence on the mean than the smaller class (King's). In fact the ratio of these distances (7.5 to 12.5, or 3 to 5) is the inverse of the ratio of the classes' relative sizes (25 to 15, or 5 to 3)

() 8. Suppose Frank has earned 85, 92, and 86 on three tests, and he needs a 90% average for an A. What score must he earn on the fourth test to reach the "A" level?

His mean must be 90% for an A, so he needs $\dfrac{85 + 92 + 86 + x}{4} = 90$

$85 + 92 + 86 + x = 360$ $\quad X = 360 - (85 + 92 + 86) = 360 - 263 = 97$

() 1. A jar contains four marbles: three red, one blue. Two marbles are drawn from the jar.
a. What is the probability the marbles drawn are both red?
b. What is the probability the marbles drawn are both blue?
c. What is the probability the two different-color marbles are obtained?



●● ½
●● ¼
●● ¼
●● 0

a. P(●●) = P(● 1st) P(● 2nd | ● 1st) = (¾)(⅔) = ½

b. P(●●) = P(● 1st) P(● 2nd | ● 1st) = (¼)(0) = 0

c. P(two different) = P(●●) + P(●●) = (¾)(⅓) + (¼)(1) = ½

Notice the probabilities on the branches from a single point must always add up to 1.
Anything else would not make sense! (WHY?)
Tree diagrams lead to the distinct outcomes of an experiment; to find the probability of an event, we can simply add up the probabilities of the outcomes that make up that event.

PS: The events listed in parts a, b and c make up the entire sample space with no overlap (2 red or 2 blue, or 2 different)—therefore these probabilities (a, b and c) must all add up to 1.

() 2. You roll a pair of fair dice, one green, the second one red.
Let "A" be the event the green die turns up 6 dots.
Let "B" be the event the red die turns up 3 dots.
Let "C" be the event the red die # is less than the green die #.

a. P(A) = P(6) = $^1/_6$ since "6" is one of the six equally likely outcomes for the green die.

P(B) = P(3) = $^1/_6$ since "3" is one of the six equally likely possibilities for the red die.

P(C) = $^{15}/_{36}$ = $^5/_{12}$

b. P(A or B) = P(A) + P(B) – P(A&B) = P(6) + P(3) – P(63) = $^1/_6$ + $^1/_6$ – $^1/_{36}$

c. P(A and B) = P(63) = $^1/_{36}$

d. Are A & B *independent events*? Yes
Are A & C *independent events*? No

"Independent" means each one has no effect on the other.
Intuitively, the outcomes on the green die and red die have no effect on each other.
This can also be determined as follows:

Events A & B are independent iff P(A & B) = P(A) P(B)	$^1/_{36}$ = ($^1/_6$)($^1/_6$) P(A & B) = P(A) P(B). A & B are independent	$^5/_{36}$ ≠ ($^1/_6$) ($^5/_{12}$) P(A & C) ≠ P(A) P(C) A & C are not independent

Are A & C *mutually exclusive events*? No.

A & C are not mutually exclusive– one does not preclude the other.
P (A and C) = P(green die is 6 and red die produces a lower number) = ($^1/_6$)($^5/_6$) = $^5/_{36}$
(Not 0)

Though these conditions are often confused,
independence and mutual exclusivity are nearly opposite in their meaning.

Independent events: the occurrence of one has NO EFFECT on the probability of the other.

Mutually exclusive events: one's occurrence virtually PRECLUDES the other.

() 3a. Out of the last 300 reservations made at a certain restaurant, only 282 showed up.
What is the probability a reservation made for this Friday night at 7:30 will be honored?

P(show up) = $\frac{282}{300} = \frac{47}{50}$ Based on the best information available— past history.

3b. A fair coin has been tossed twice; heads have turned up both times.
What is the probability the next toss of the coin will turn up tails?

P(tails) = $\frac{1}{2}$ Since the coin is fair. (Here we have better information than in part a; coin is fair.)

3c. Three marbles will be drawn from a jar containing six marbles, 4 black and 2 white.
What is the probability that the second marble taken out will be white?

P(second marble W) = $\frac{2}{6}$ (Intuitively: any one of the six marbles might be 2^{nd} , equally.)

(A tree diagram of all the outcomes and their probabilities will confirm the above observation.)

() 4. Find the probability of obtaining *exactly* 1 *head* in 3 tosses of a fair coin.

Draw a tree diagram and see that there are three ways for this to happen:

P(exactly 1 H in 3 tosses) = P(HTT or THT or TTH)

= P(HTT) + P(THT) + P(TTH)

= P(H)P(T)P(T) + P(T)P(H)P(T) + P(T) P(T)P(H)

= (½)(½)(½) + (½)(½)(½) + (½)(½)(½)

= $^1/_8$ + $^1/_8$ + $^1/_8$

= $^3/_8$

() 5a In a game in which the *odds against you* are **3:1**, what is the probability of winning?
☹ ☹ ☹ ☺

If the ODDS are against you, 3 to 1, it is as if there are 4 outcomes, and only 1 favors you.

So your chance of winning is 1/4.

5b. The probability of rain tomorrow is 10%. What are the odds against rain tomorrow?

P(rain) = 10% ....so P(not rain) = 90%

The odds are in fact against rain.... 90 to 10 (or 9 to 1)

() 6. A swimming class consists of 6 boys and 14 girls.
Of the boys, 4 have won a blue ribbon. Of the girls, 6 have won a blue ribbon.
Given that a blue-ribbon member of this class is selected, what is the probability she is a girl?

INTUITIVELY: If a blue-ribbon winner is selected, then we know the student selected is one of the 4 boys and 6 girls who have won a blue ribbon (one of the ten blue-ribbon winners). The probability she is a girl is 6 out of 10.

"By the book" (using the definition of conditional probability):

$$P(G \mid BR) = \frac{P(G \text{ and } BR)}{P(BR)} = \frac{6/20}{10/20} = \frac{6}{10}$$