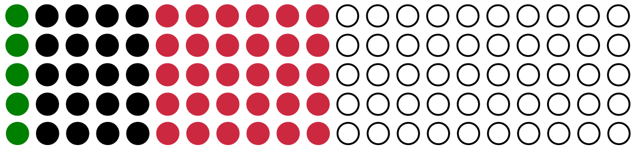
Simple PROBABILITY Questions for Winners - Answers with some explanation:



1a. $^{3}/_{10}$ ⇦ $^{30}/_{100}$ Since all 100 marbles are equally likely to be selected, & 30 are "winners" (●).

1b. $^{1}/_{4}$ ⇦ $^{25}/_{100}$ P(B or G) = P(●) + P(●) OR We can see there are 5+20 = 25 "winners"....

1c. $^{7}/_{10}$ ⇦ P(not R) = 1 − P(R) = 1 − $^{3}/_{10}$ (Using the result of #1.) Also, 70 marbles are not-●.

1d. 0 ⇦ P(multicolor) is 0 because it cannot happen. There are no multicolor marbles here.

2a. $^{1}/_{4}$ or 25% ⇦ $^{24}/_{96}$ ...since in 24 out of 96 occasions when we had these conditions, we got rain. Past history is our best predictor of what weather will happen now.

2b. $^{1}/_{2}$ The coin is *fair*. Therefore, every toss is equally likely to produce tails.

3a. 1 ⇦ All the cards are "winners" (face cards or aces).
P(F or A) = P(F) + P(A) = $^{3}/_{4}$ + $^{1}/_{4}$

A♥	A♦	A♣	A♠
K♥	K♦	K♣	K♠
Q♥	Q♦	Q♣	Q♠
J♥	J♦	J♣	J♠

3b. $^{1}/_{16}$ ⇦ P(A and D) = P(card selected is both an ace and a diamond) = P(A♦)

3c. $^{7}/_{16}$ ⇦ P(A or D) = P(A) + P(D) − P(AD) = P(A) + P(♦) − P(A♦) = $^{1}/_{4}$ + $^{1}/_{4}$ − $^{1}/_{16}$

3d. The "addition rules" (for mutually exclusive events in part a).

3e. F & A are mutually exclusive, which is why P(F or A) = P(F) + P(A)
— there is no overlap to worry about, so P(A&F) = 0.
A& D are NOT mutually exclusive. They can happen together. In fact: P(A&D) = $^{1}/_{16}$

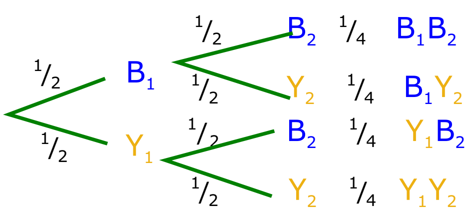
Additional Note on the independence of A & D:

A & D are independent. P(A) = $^{1}/_{4}$, regardless of what suit the card is in— so the chance of getting an ace is not affected by getting (or not getting) a diamond.

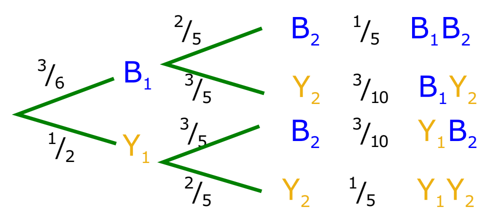
We usually prove two events, A & D, are independent by checking that P(AD) = P(A)•P(D).
In this case we see: $^{1}/_{16}$ = $^{1}/_{4}$•$^{1}/_{4}$ proving the point which is intuitively made above.

4. Each face of the die has an equal chance of turning up whenever the die is tossed.
Since half the faces are blue, the chance of turning up blue each time is ½.
$P(B_1B_2) = P(B_1)\cdot P(B_2)$
The multiplication rule for independent events supports this result.
But we can understand it best by modeling the experiment with a tree diagram.

#4



#5



5. $^{3}/_{6}$•$^{2}/_{5}$ = $^{1}/_{5}$

The probability "rule" that applies in this instance is the general rule for multiplication:

$$P(AB) = P(A)\cdot P(B|A).$$

And, once again, we see this most easily using a tree diagram to model the experiment.

6. In #5: The two events, getting a blue result first toss (B_1), and blue on the second toss (B_2), are independent because every toss of the die has the same possible results with the same chances. The die is unaffected by previous results.

In #6: The two events, getting a blue result on the first draw (B_1), and blue on the second draw (B_2), are DEPENDENT because obtaining a blue marble the first time reduces the proportion of blue marbles in the jar, thus reducing the probability of getting a blue on the next draw. As we wrote above, $P(B_2|B_1) = {}^2/_5$. By comparison, $P(B_2|\text{NOT } B_1) = {}^3/_5$. Thus the chance of B_2 is affected by B_1.

An additional argument would be appropriate in a mathematical probability course:
The defining condition for independence is that $P(AB) = P(A)\bullet P(B)$...
... so we could ask if it is true that $P(B_1B_2) = P(B_1)\bullet P(B_2)$.
The tree diagram shows that $P(B_1B_2) = {}^6/_{30} = {}^1/_5$ and that $P(B_2) = {}^1/_2$ (see #14).
We know $P(B_1) = {}^3/_6$. Does $P(B_1B_2) = P(B_1)\bullet P(B_2)$? Does ${}^1/_5 = {}^1/_2 \bullet {}^3/_6$? (NO!)

7a. $P(1H \,\&\, 2T) = P(HTT) + P(THT) + P(TTH) = 3 \bullet {}^1/_2 \bullet {}^1/_2 \bullet {}^1/_2 = {}^3/_8$

7b. P(at least one T) = P(TTT) + P(TTH) P(THT)+P(HTT) + P(THH) +P(HTH)+P(HHT) ... OR...
P(at least one T) = 1 – P(NO T) = 1 – P(HHH) = $1 - {}^1/_8 = {}^7/_8$

8. P(win) = P(3or4 on fair die toss) • P(T on fair coin toss) • P(red card from deck)
$= {}^2/_6 \bullet {}^1/_2 \bullet {}^1/_2 = {}^1/_3 \bullet {}^1/_2 \bullet {}^1/_2$

9. In order to answer this one, we must know what letters are in the English alphabet (ABCDEFGHIJKLMNOPQRSTUVWXYZ) and what letters are considered "vowels" (AEIOU) ... We will not consider "sometimes vowels" to be "vowels". Answer: 5:21

10a. P(W) = #W/#inclass = ${}^{30}/_{50} = {}^3/_5$

10b. The odds ARE in favor of selecting a woman, 30 to 20 (or 3 to 2).

10c. The odds against selecting a woman are 20 to 30 (or 2 to 3).

11a. If the odds in favor of "win" are 7 to 3: picture the outcomes as ☺☺☺☺☺☺☺ ☹☹☹
Then P(win) = P(☺) = ${}^7/_{10}$

11b. If the odds are against you, 5 to 4: ☹☹☹☹☹ ☺☺☺☺ ... P(win) = ${}^4/_9$

12. The class "favors" girls, 3 to 2: G G G B B

Boys make up two-fifths of the CLASS.
If a student is selected at random from the class, P(boy) = ${}^2/_5$
Odds in favor of selecting a boy are 2:3

13. Making the assumption that Jan does not select the same prize twice:

13a. $P(B_1A_2D_3) = P(B_1) \bullet P(A_2|B_1) \bullet P(D_3|B_1A_2) = {}^1/_5 \bullet {}^1/_4 \bullet {}^1/_3$ (B_1 means selects B first, A_2 means selects A second, et cetera.)

13b. $P(A_1D_2B_3) = P(B_1) \bullet P(A_2|B_1) \bullet P(D_3|B_1A_2) = {}^1/_5 \bullet {}^1/_4 \bullet {}^1/_3$

13c. P(selects all but C & E) = P(selects A,B,D, in any order)
= P(BAD) + P(ADB) + P(BDA) + P(ABD) + P(DAB) + P(DBA) = $6 \bullet {}^1/_5 \bullet {}^1/_4 \bullet {}^1/_3 = {}^1/_{10}$

Easier way:
P(does not select C or E) =
P(1^{st} selection is not C or E)•P(2^{nd} is not)•P(3^{rd} is not)= ${}^3/_5 \bullet {}^2/_4 \bullet {}^1/_3$

14. This is the probability that the second marble is blue:

$P(B_2) = P(B_1B_2) + P(Y_1B_2)$ (These probabilities can be found on the tree diagram.)
$= P(B_1)\bullet P(B_2|B_1) + P(Y_1)\bullet P(Y_2|B_1)$
$= {}^3/_6 \bullet {}^2/_5 + {}^3/_6 \bullet {}^3/_5$
$= {}^6/_{30} + {}^9/_{30} = {}^{15}/_{30} = {}^1/_2$.