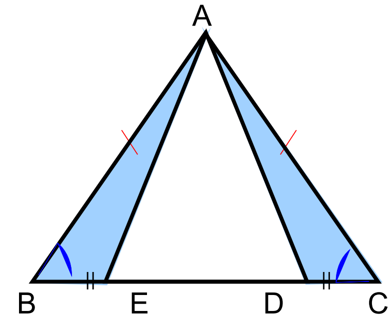
The solutions given are not the only method of solving each question.

§G. CONGRUENCE Added features of diagrams are shown in colors.

1. Triangle ABC below is equilateral. Congruent line segments are indicated. Identify one pair of congruent triangles and explain carefully why they are congruent.

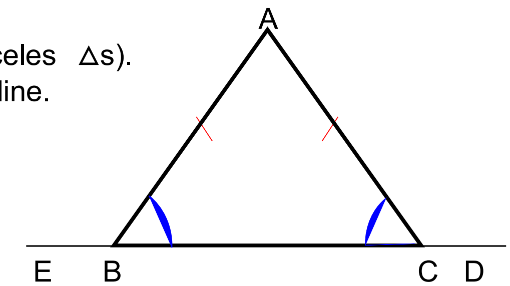


Note there are several pairs of congruent triangles in the figure sketched. Probably the easiest pair to prove congruent consists of $\triangle ABE$ and $\triangle ACD$.

$\triangle ABE \cong \triangle ACD$. Here's why:

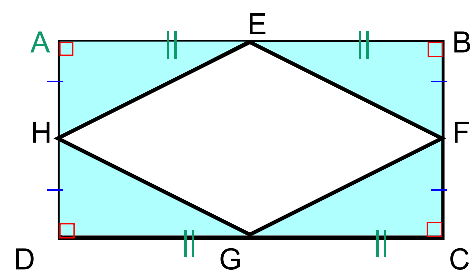
Because $\triangle ABC$ is equilateral, $\overline{AB} \cong \overline{AC}$ (marked \ ).
$\angle B \cong \angle C$ because $\triangle ABC$ is also isosceles ($\overline{AB} \cong \overline{AC}$).
That $\overline{BE} \cong \overline{CD}$, is given as part of the hypothesis.
By "SAS", $\triangle ABE \cong \triangle ACD$.

2. Prove that $\angle EBA$ is congruent to $\angle DCA$ [given $\overline{AB} \cong \overline{AC}$ in $\triangle ABC$, and E,B,C,D collinear] .



Since $\overline{AB} \cong \overline{AC}$, $\angle ABC \cong \angle ACB$ (previously proven property of isosceles $\triangle$s).
$\angle EBA$ & $\angle ABC$ are supplementary, since E,B,C and D lie on a straight line.
$\angle DBA$ & $\angle ACB$ are likewise supplementary
Since $\angle EBA$ and $\angle DBA$ are supplementary to congruent angles, their measures must be the same (details below).
(ie. $m(\angle EBA) = 180° - m(\angle ABC) = 180° - m(\angle ACB) = m(\angle ACD)$)

3. E, F, G, H are the midpoints of the sides of Rectangle ABCD. Prove that Quadrilateral EFGH is a rhombus.



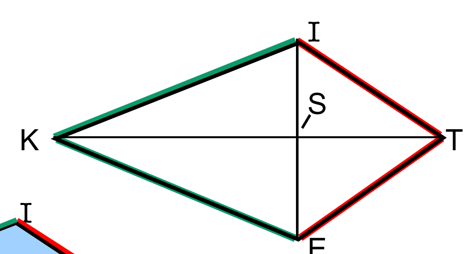
Since ABCD is a rectangle, all four angles, $\angle A$, $\angle B$, $\angle C$, $\angle D$, are right, and thus are congruent.

$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$ because opposite sides of any parallelogram are congruent.

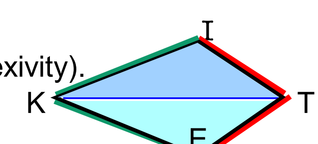
Since E is the midpoint of $\overline{AB}$, $\overline{EB} \cong \overline{AE}$.
Similarly, G is midpoint of $\overline{CD}$, so $\overline{DG} \cong \overline{CG}$— halves of $\overline{CD}$.
Furthermore, they are all congruent to each other, since they are all halves of two congruent sides.
That is, $\overline{EB} \cong \overline{AE} \cong \overline{DG} \cong \overline{CG}$.
By the same reasoning, $\overline{AH} \cong \overline{DH} \cong \overline{BF} \cong \overline{CF}$

Thus by "SAS", $\triangle AEH \cong \triangle BEF \cong \triangle CGF \cong \triangle DGH$
Since CPCTC, $\overline{EH} \cong \overline{GF} \cong \overline{GH} \cong \overline{EF}$.

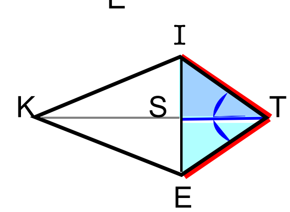
Thus, by definition, EFGH is a rhombus.



4. Given: Quadrilateral KITE with KI $\cong$ KE and IT $\cong$ ET
Prove KT $\perp$ IE.



$\overline{KI} \cong \overline{KE}$ and $\overline{TI} \cong \overline{TE}$ (given), and $\overline{KT} \cong \overline{KT}$ (reflexivity).
So, by "SSS", $\triangle KIT \cong \triangle KET$.
Since CPCTC, $\angle ITS \cong \angle ETS$



That, plus $\overline{TI} \cong \overline{TE}$ and $\overline{TS} \cong \overline{TS}$, guarantees, by "SAS", that $\triangle ITS \cong \triangle ETS$.
Since CPCTC, $\angle TSI \cong \angle TSE$; further, the sum of the measures of $\angle TSI$ and $\angle TSE$ is 180°, so each must measure 90°.
Since $\overline{KT}$ and $\overline{IE}$ meet at right angles, they are perpendicular. ■ (The end)