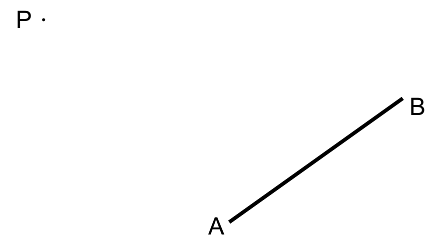
Don't just follow the "instructions". You may see how to make the following constructions on your own, and the methods you devise may differ from those given. Give yourself a chance to discover methods yourself. Then *read the "instructions".*

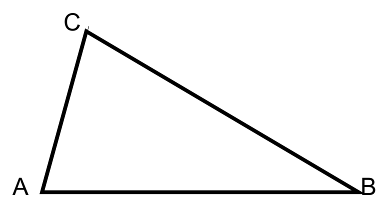
C1. COPY A SEGMENT. Given line segment $\overline{AB}$, construct a congruent copy at P.



1. Place compass point at A; open compass to just span $\overline{AB}$.
2. Place compass point at P (which we can think of as A'; draw a circle with center P (aka A').

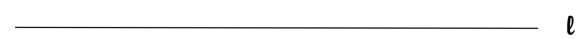
   (Important: don't change compass opening! This is one of the functions of the compass – it can measure!)
3. Select any point on the circle. Its distance from A' (= P) is the same as the length of segment AB. Use the straightedge to draw the segment A'B'.

C2. COPY A TRIANGLE. Given a triangle ABC, construct a congruent triangle A'B'C' with A'B' on ℓ .



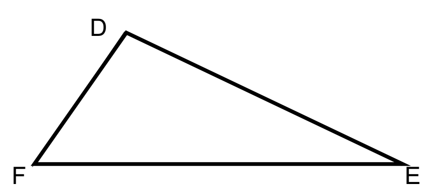
1. Place compass point at A; measure $\overline{AB}$. (✓: Draw arc through B).
2. Mark A' on ℓ Place compass point at A'; draw arc through ℓ; mark the intersection point B'.
3. Now locate C' as follows: use compass to measure the span of AC
   3a. Draw an arc of radius AC, centered at A'

   3b. Use a similar procedure to draw an arc of radius BC centered at B'.

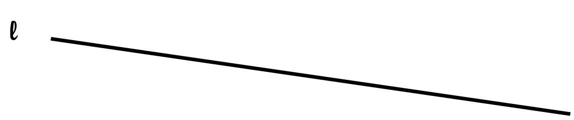


   3c. The point where these arcs intersect is C' ... just the right distance from A' & B'.
4. Draw the triangle, connecting A'B'C'.

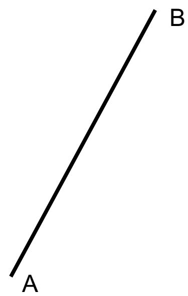
If the sides of one triangle are congruent to the respective sides of a second triangle, the entire triangles are congruent. ("SSS")



E1. Copy triangle DEF to line ℓ so that the longest side of the new triangle, D'E'F', lies on line ℓ



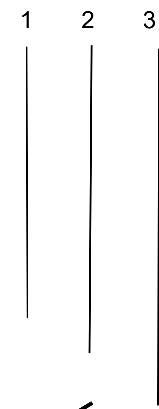
E2.



Given a line segment AB, construct an equilateral triangle with side AB.

How do I do that? What do I know? ...
The three sides of the triangle must be congruent.
One side is already specified.

1. Place compass point at B; draw arc through A.
2. Place compass point at A; draw arc through B.
3. The point where these arcs intersect is just the *right distance* from each. Call it point C, the third vertex of the triangle.
4. Draw the triangle!

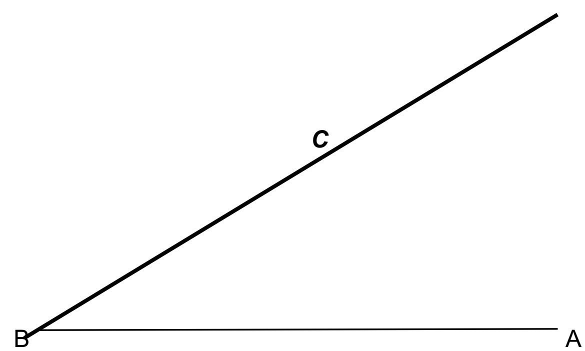


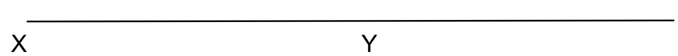
E3. **Construct a triangle whose sides are the lengths of the three segments at right:**
The theorem that guarantees all such triangles will be congruent is the ___________ .
Can this be done with ANY three lengths? (Try 2" & 2" and 5".)

C3. COPY AN ANGLE

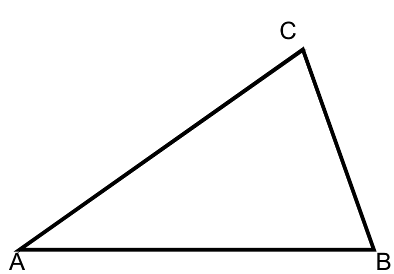
Given ∠*ABC*, construct a CONGRUENT ANGLE *with vertex* X, *through* Y.

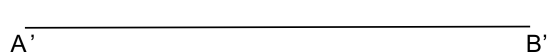
1. Place compass point at *B*; draw an arc with decent radius (NO itsy bitsies!),, intersecting *AB* at *D* and *BC* at *E*. (Mark *D* & *E*.)
2. Place compass point at *X*, and draw an arc matching the one in step 1. Mark *D′*. *(D' corresponds to D.)*
3. Placing the compass point at *D*, draw an arc through *E*; copy onto the new construction.
4. You have located E′. Draw the angle. *(Essentially you created a triangle including angle ABC, then copied it!)*





E4. Copy the angle at A (∠CAB) to A'. Then copy the angle at B (∠CBA) **to B'** (USE the GIVEN B' !). Extend the rays so they intersect. Mark the point where the rays intersect as C'.

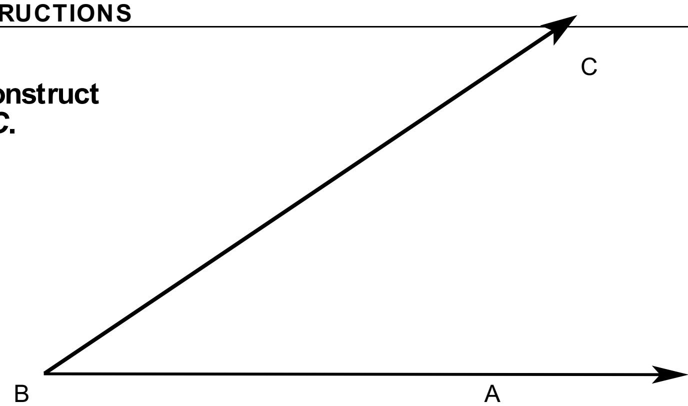




What can you say about the angles of the new triangle, A'B'C' ?

Is triangle A'B'C' congruent to triangle ABC?

C4. **BISECT an ANGLE** **Given angle $\angle ABC$, construct two angles with half the measure of $\angle ABC$.**



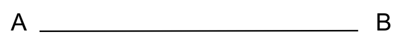
1. Place compass point at *B*; draw an arc of fair radius (NO squinchies!!!), intersecting *AB* at *D* and *BC* at *E,* as in C3.
2. Place compass point at *D*, and draw an arc through *E* toward *AB*.
3. Repeat with compass point at *E*, drawing arc through *D* toward *BC*.
4. The point where these arcs cross is on the bisecting line, as is B. Draw the line.

Why did this work? What two congruent triangles were created?

C5. **PERPENDICULAR BISECTOR**
Given a line segment *AB*, construct a perpendicular bisector of *AB*.

1. With compass point on *A*, draw arc through *B*, extending arc more than 60° both ways.
2. Repeat with compass point at *B*, drawing an arc through *A*.
3. The two points where these arcs cross are on the desired line

...because the points on the ⊥ *bisector are equidistant from A & B. (Why?)*



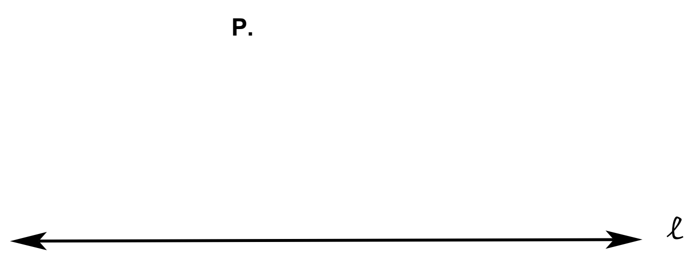
E5. **Given a line ℓ, construct a line perpendicular to ℓ.**

(This is just like C5, but less specific. Take advantage of the construction in C5! i.e. create a segment !)

C6. **PERPENDICULAR FROM A POINT**
Given a point *P* and a line ℓ not containing the point, construct a perpendicular to the line, through the point.

Hint: Compare this task to E5– where you made your own segment.
If we find two points on ℓ that are equidistant from P, we'll have a situation similar to C5.

☆ *(There is a very simple alternative method.)*



C6a. PARALLEL LINE

Given a line "ℓ" and a point *P* not on ℓ, construct a line through *P parallel to* ℓ.

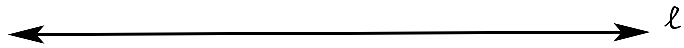
(Note there are many ways to accomplish this!)

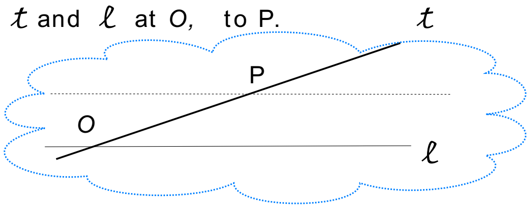
E.g. construct a line k $\perp$ to ℓ through P;

P.

then construct a $\perp$ to k at P. (the hard way)

Or: Draw a line t through P intersecting ℓ at point *O* ... then copy the ∠, formed by t and ℓ at *O*, to P.





Or... Find another way to do this .
(There is a very simple way!)

E6. Construct a parallelogram with given three points as vertices.
(What do you know about parallelograms?)
(There are multiple possible outcomes here.)

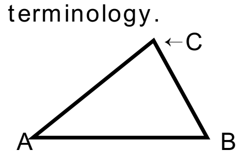
A •

B •

• C

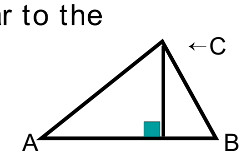
TERMS YOU MUST KNOW (Taking a break from construction here):

Before we move on to the constructions on the next page, let's make sure we know the terminology.
We often speak of the "base" of a triangle, viewing it as the side the triangle is "sitting on", in this case, AB. In fact, any side of the triangle can be the "base". Identify the other two bases of this triangle:

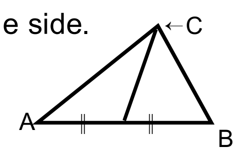


An **ALTITUDE** of a triangle is a segment from the vertex of a triangle, perpendicular to the opposite side, or "base". For each triangle, there are three altitudes.
Draw the missing altitudes.

Another exercise:
Draw a triangle on a sheet of thin notebook paper.
FOLD the paper to locate an altitude of the triangle.



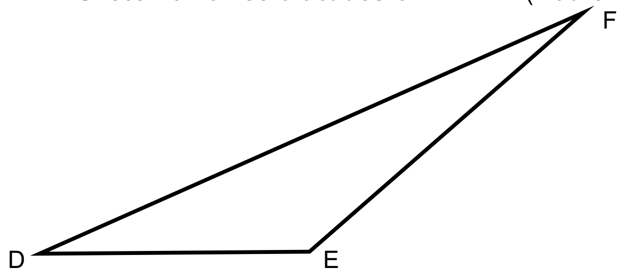
A **MEDIAN** of a triangle is a segment from one vertex to the midpoint of the opposite side.
One median, the median through C, is shown in the triangle at right.
Draw the other two medians.

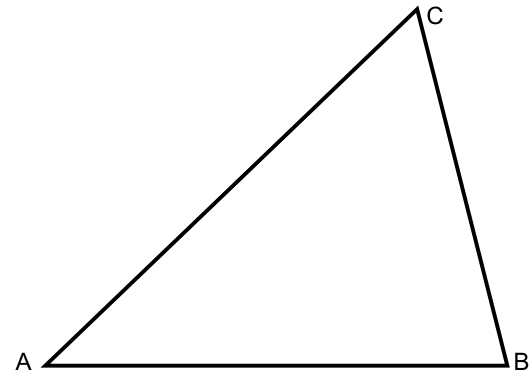


EC-5 MATH 310 GEOMETRY:

✓ **Yourself:** Using the skills developed earlier, you should be able to perform the following constructions.

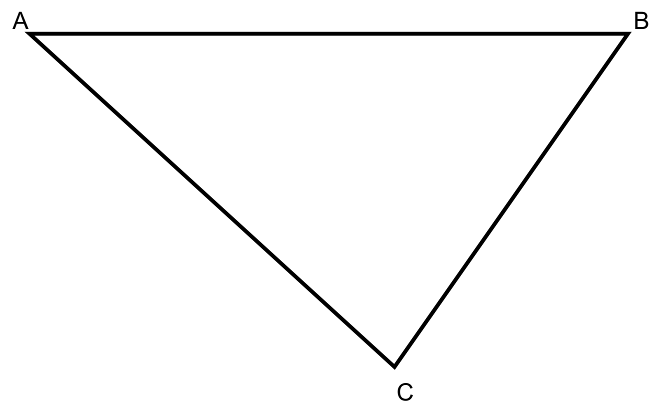
EH1. **SKETCH/CONSTRUCT ALTITUDES** **Given △ABC, construct at least one ALTITUDE of the triangle.** (*Hint: C6*)
Sketch all three altitudes of △DEF. (You should do this for both acute and obtuse triangles!)



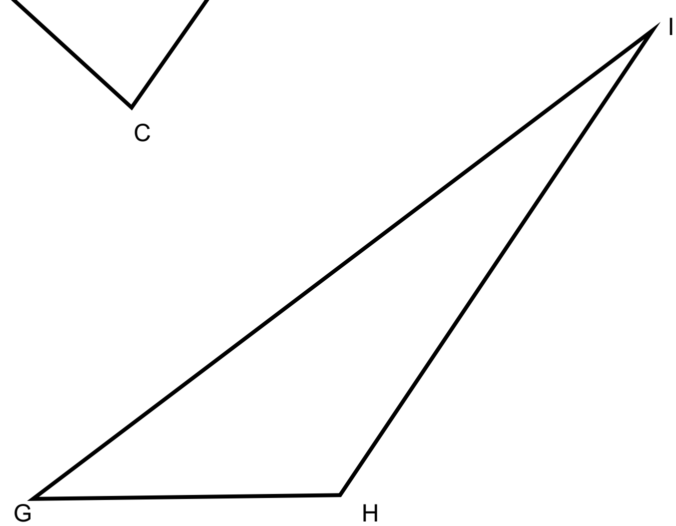


EH2. Given a triangle ABC, **SKETCH THE MEDIANS of the triangle**. *(Use a ruler!)*

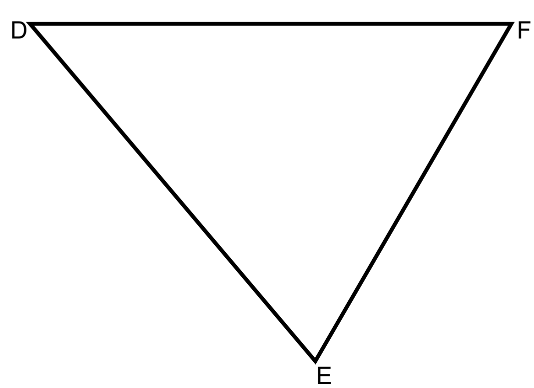
How would you construct a median?



EH3. Sketch
or Construct all three **ANGLE BISECTORS** of △GHI.

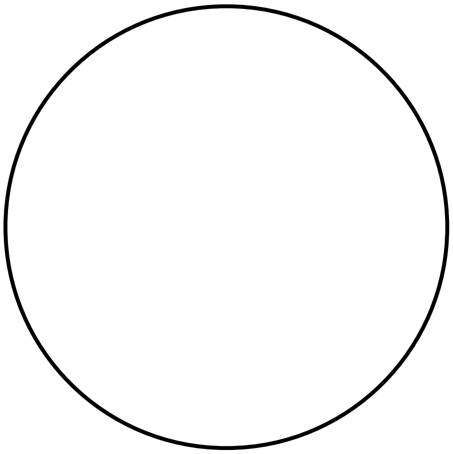


EH4. Construct all three **PERPENDICULAR BISECTORS** of the **SIDES** of △DEF.



The circle is the set of all points in the plane that are "r" distant from a fixed point "$\mathcal{O}$" (in the plane).
The distance "r" is the radius, and the point "$\mathcal{O}$" is called the center, of the circle.
Consider that all points of the circle are equally distant from the center.
The perpendicular bisector of any chord (*line segment joining any two points on the circle)* must pass through....

C7.



FIND THE CENTER OF A CIRCLE

Given a circle *C*, construct its center $\mathcal{O}$.

Hint: Draw any chord AB (or just select A & B on C).

What must be *the distance from $\mathcal{O}$ to A and B?*

$\mathcal{O}$ must lie on the $\perp$ bisector of AB! Construct that!

Then repeat, to "pinpoint" the center $\mathcal{O}$.

EH5. **Given three non-collinear points,**
construct a circle through the three points.
(Hint: find the circle's center.)

A •

B •　　　　• C

Question: Given a triangle ABC, would it be possible to find one circle that passes through all three vertices? (This circle is called the circumcircle, and triangle ABC is said to be inscribed in it.)

EH6. Consider a regular hexagon; three major diagonals section the interior into six equilateral regions.
What is the perimeter of the regular hexagon?
If inscribed in a circle, how does this perimeter relate to the radius of the circle?
This gives an easy method for constructing a regular hexagon.
Try it, starting with a circle.