§7.1 #1 2 3 6 7 8 9 10 11 14 15 16 21 22 23 29

The solutions given are not the only method of solving each question.

1a. Based on experimental results, P(up) = 56/80 = 7/10 or 70%

1b. and P(down) = 24/80 = 3/10 or 30% ... or you could say P(down) = 1 – P(up) = 1 – .7 = .3

1c. If the experiment is repeated, we would most likely get similar (but not identical) results, because every tack-toss is a chance event, so we don't always, or even usually, get ideal (theoretical) results.

2a. NOT uniform, because P(Face) = 3/13 and P(Not Face) = 10/13.

2b. P(♥) = P(♦) = P(♣) = P(♠) = 1/4

2c. P(Black) = P(♠) + P(♣) = ½ and P(Red) = P(♥) + P(♦) = ½

2d. NOT uniform, because P(**K**) = 1/13 and P(even #) = P(2 or 4 or 6 or 8 or 10) = 5/13

3. Select the last digit of a [randomly selected] telephone number.

3a. SS = { 0, 1, 2, 3, 4, 5, 6, 7,8, 9 }

3b. Event the digit is < 5 = { 0, 1, 2, 3, 4 }

3c. Event the digit is odd = { 1, 3, 5, 7, 9 }

3d. Event digit is not 2 = { 0, 1, 3, 4, 5, 6, 7, 8, 9 }

3e. 5/10 & 5/10 & 9/10

6. Card is [selected at random] from an ordinary deck of 52. R = red, F = face, C = club, **Q**= queen

6a. P(R) = 26/52 = ½

6b. P(F) = 12/52 = 3/13 .

P(A or B) = P(A) + P(B) – P(A and B)
(The general addition "rule".)

6c. P(R or 10) = P(R) + P(10) – P(R and 10) .
= 26/52 + 4/52 – 2/52 = 28/52 = 7/13

6d. P(**Q**) = 4/52 or 1/13

6e. P(not **Q**) = 1 – P(**Q**) = 12/13

6f. P(F or C) = P(F) + P (C) – P(F&C) = 12/52 + 13/52 – 3/52 = 22/52

6g. P(FC) = P(**K**♣) + P(**Q**♣) + P(**J**♣) = 3(1/52) = 3/52

7. A drawer contains 6 B, 4 br and 2 G socks.

7a. P(br) = 4/12 = 1/3 .

7b. P(B or G) = P(B) + P(G)
= 6/12 + 2/12
= 8/12
= 2/3

7c. P(red) = 0/12

7d. P(not B) = 1 – P(B) = 1 – 6/12 = ½

P(A or B) = P(A) + P(B) – P(A and B)
P(A or B) = P(A) + P(B) – 0
(The addition "rule" when A & B are mutually exclusive ....
P(B AND G) = 0 because they cannot both occur together.
The sock cannot be both Black AND Green, since there are no such socks in the drawer.

8. A bag contains cards numbered 1 through n (EG: If n=15, #s are 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15)
If the probability of obtaining a card numbered ≤10 is .4, then what is n?

Some students answered n = 10.
These students did not take a second look at their answer. They should have asked themselves :
If there are 10 cards, numbered 1 2 3 4 5 6 7 8 9 10,
what is the probability the card drawn is numbered ≤ 10 ?

We ask ourselves, could n be 15? If there are 15 cards, then P (# ≤ 10) = 10/15 = 2/3 ≠ 4/10
Could n be 20? If there are 20 cards, then P (# ≤ 10) = 10/20 = 1/2 ≠ 4/10
What is n? If there are n cards, then P (# ≤ 10) = ?

9. Riena has 6 unmarked CDs, for E, M, F, A, C and S. She chooses one at random.

9a. P(E) = 1/6

9b. P(not M nor C) = P(not (M or C)) = 1 – P(M or C) = 1 – 2/6 = 4/6 or 2/3

10. The game of craps. A pair of fair dice is rolled.

10a. P(win on 1^{st} roll) = P(sum is 7 or sum is 11) = P (sum is 7) + P(sum is 11) = 6/36 + 2/36 = 8/36 = 2/9

10b. P(lose on 1^{st} roll) = P(sum is 2,3 or 12) = P(11 or 12 or 21 or 56 or 65) = 5/36

10c. P(neither win nor lose on 1^{st} roll) = 1 – P(win or lose on 1^{st} roll)
= 1 – (P(win on 1^{st} roll) + P(lose on 1^{st} roll))
= 1 – (8/36 + 5/36) = 23/36

10d. Which of these has the highest probability?
4 5 6 8 9 10
From the sample space, you can see that P(6) = P(8) = 5/36, and the remaining sums are less likely.

10e. P(sum = 1) = 0/36... no such outcomes, cannot happen.

10f. P(sum < 13) = 36/36 (The highest sum is 12, so every possibility in the SS is < 13.)

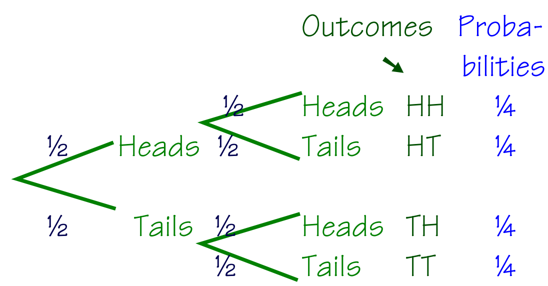
10g. P(sum = 7) = 6/36 = 1/6.
Out of 60 rolls, the best prediction is that 1/6 of them will have a sum = 7: 1/6 of 60 = 10

11. According to weather prediction, P(rain) = 30%. Thus P(not rain) = 1 – 30% = 70%
because these two events (rain and no rain) are complementary.

14. Does each player have an equal opportunity of winning?
a. NO. "I" wins, no matter what.
b. Yes. P(I win) = P(H) = ½ P(Uwin) = 1 – P(I win) = 1 – ½ = ½
c. Yes. P(I win) = P(1) = 1/6. P(U win) = P(6) = 1/6
d. Yes. P(I win) = P(Even) = P(2,4,6) = 3/6 or ½ P(U win) = P(1,3,5) = 3/6 = ½
e. NO. P(I win) = P(3,4,5,6) = 4/6 = 2/3, leaving only 1/3 for P(U win).
f. Yes. P(I win) = P(11) = 1/36. And P(U win) = P(66) = 1/36.
g. NO. P(I win) = P(sum = 3) = P(12 or 21) = 2/36
P(U win) = P(sum = 2) = P(11) = 1/36
h. NO. P (I win) = 15/36. P(U win) = 21/36

15. A fair coin is tossed twice.

TREE:



a. P(exactly one head) = P(HT) + P(TH) = ¼ + ¼ = ½
b. P(at least one head) = P(1 H) + P(2H) = ½ + ¼ = ¾
or P(at least one head) = P(HT or TH or HH) = ¼ + ¼ + ¼ = ¾
c. P(at most one head) = P(1 H) + P(0H) = ½ + ¼ = ¾

16. If a student is chosen at random, every one of the 350+320+310+400 = 1380 students has an equal chance of being selected. So P(Freshman) = 350/1380 = 35/138

21. If A and B are mutually exclusive, and P(A) = 0.3 and P(B) = 0.4, P(AUB) = P(A or B) = P(A) + P(B)

22. Class comprised of 35 men and 45 women— 20 business, 30 biology, 10 compsci & 20 math majors.
If a single student is chosen AT RANDOM [from the 80 students]....
a. P(female) = 45/80=9/16 b. P(compsci) = 10/80=1/8 c. P(not math) = 3/4 d. 7/8

23. A box contains 25% ● and 75 % ○
the number of black balls is doubled
Ball is now drawn at random from the box.

EG original situation could be ●○○○
& new situation would then be: ●●○○○
P(●) = 2/5 = 4/10 = 40%

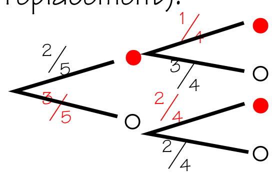
29. Look at the Probability tree for heads and tails in #15. The possibilities are BB, BG, GB, GG.

§7.2 # 1 3 6 10 24 31 35*

The solutions given are not the only method of solving each question.

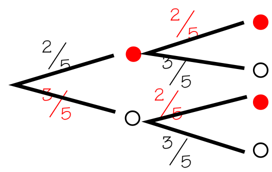
1. Box : [○○○ ●●] Two balls are drawn (without replacement).

1a. Tree diagram is at right.
P(two different colors) =
P (●○) + P(○●) =
$\frac{6}{20}$ + $\frac{6}{20}$



Outcome	Probability
●●	$(\frac{2}{5})(\frac{1}{4}) = \frac{2}{20}$
●○	$(\frac{2}{5})(\frac{3}{4}) = \frac{6}{20}$
○●	$(\frac{3}{5})(\frac{2}{4}) = \frac{6}{20}$
○○	$(\frac{3}{5})(\frac{2}{4}) = \frac{6}{20}$

1a. Tree diagram at right.
P(two different colors) =
P (●○) + P(○●) =
$\frac{6}{25}$ + $\frac{6}{25}$
Note this probability is less than in part a.
This makes sense, because ●● & ○○ are now more likely.



Outcome	Probability
●●	$(\frac{2}{5})(\frac{2}{5}) = \frac{4}{25}$
●○	$(\frac{2}{5})(\frac{3}{5}) = \frac{6}{25}$
○●	$(\frac{3}{5})(\frac{2}{5}) = \frac{6}{25}$
○○	$(\frac{3}{5})(\frac{3}{5}) = \frac{9}{25}$

3. Box: [A D M N O R] Three letters are drawn, one at a time,

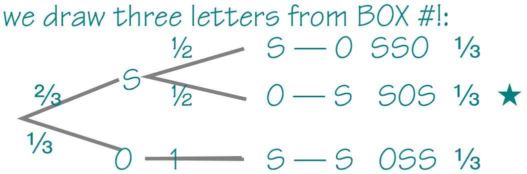
3a. with replacement: find P (D, A, N) (in that order!)
In order to "win" this game, the first letter MUST be D, the second MUST be A, the third MUST be N.
P(D)•P(A)•P(N) = $(\frac{1}{6})(\frac{1}{6})(\frac{1}{6}) = \frac{1}{216}$

3b. Without replacement, the numbers become: P(D)•P(A|D gone)•P(N|D & A gone) = $(\frac{1}{6})(\frac{1}{5})(\frac{1}{4}) = \frac{1}{120}$)

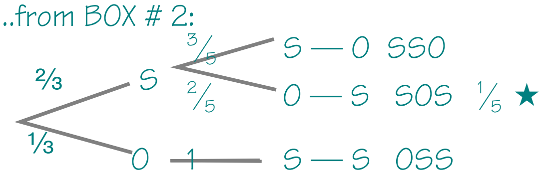
6. You must decide which box you will use: [S O S] BOX 1 Or [S O S S O S] BOX 2

6a. (drawing without replacement....)
Using the first box, with S O S, P("S,O,S" in that order) = (⅔)(½) 1 = ⅓
...Using the 2nd box, P("S,O,S" in that order) = $(\frac{4}{5})(\frac{2}{5})(\frac{3}{4}) = \frac{1}{5}$
Here are tree diagrams that show how that happens.

If we draw three letters from BOX #!:



...from BOX # 2:



6b. If the letters are being replaced between drawings, from either box, every time we draw out a letter, the probability of an S is $\frac{4}{6} = \frac{2}{3}$ and probability of obtaining an O is $\frac{2}{6} = \frac{1}{3}$.
So the chances of S-O-S are the same using either box.!

That is, on the probability trees, which are structured just like those above, "S" always has probability ⅔ and "O" always has probability ⅓. So there is no difference between drawing from BOX 1 with three letters, and drawing from BOX 2 with six letters, because the proportions of Ss and Os always stay the same.

10. Four coins (penny, nickel, dime and quarter) are tossed [assume they are fair].
What is the probability of obtaining at least three heads?

BEFORE you start multiplying and adding probabilities, you must consider what constitutes this experiment— FOUR tosses of fair coins.

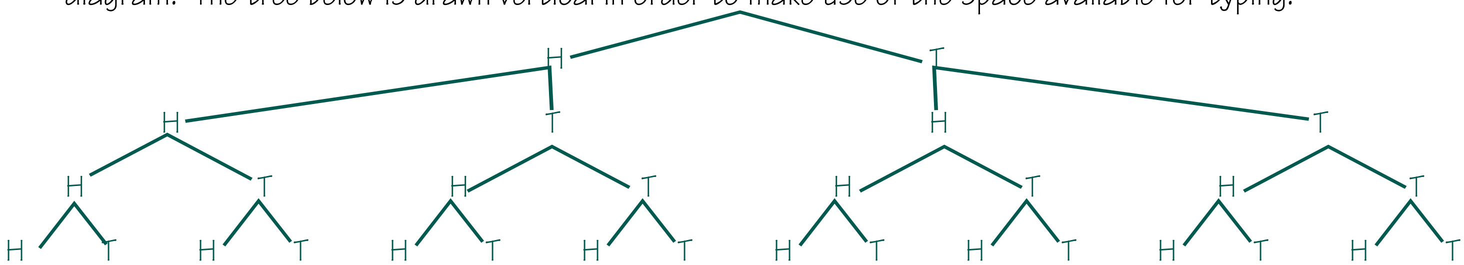
So you CANNOT just say P(HHH). "HHH" does not even represent an outcome for this experiment.
The Sample Space contains outcomes such as "HHHH" and "HHHT"

10 cont'd.

One way to approach this problem is to consider a few outcomes, then find how many there are. (keeping in mind that equally likely outcomes are desirable).

Since there are two outcomes for each coin toss, there should be 2x2x2x2 = 16 different outcomes.

Another approach, which would also demonstrate that there are 16 distinct outcomes, uses a tree diagram. The tree below is drawn vertical in order to make use of the space available for typing.



To complete this tree diagram, fill in the outcomes (HHHH, HHHT, etc.) and the probabilities (each branch splits ½ & ½). Then the answer to the question should be clear.

24. Ask yourself what is the probability of hitting water, IF water covers two-thirds of the earth. Then answer their question. Answer: 139600/(139600 + 57500).

31. Two socks are randomly drawn from a Drawer of Four Blue, four White, and four Gray socks .

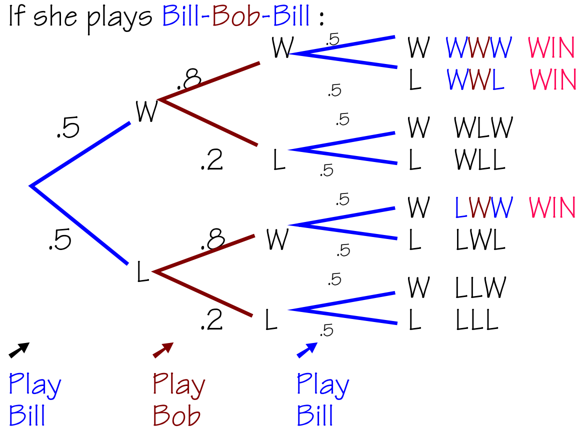
31a. A tree diagram will branch 3 ways (B,W,G) at each of 2 stages, with 9 distinct outcomes.
End results:
BB & WW & GG each have P= $(\frac{4}{12})(\frac{3}{11}) = \frac{1}{11}$ All others have P = $(\frac{4}{12})(\frac{4}{11}) = \frac{4}{33}$

31b. P(matching color) = P(BB or WW or GG) = $\frac{1}{11} + \frac{1}{11} + \frac{1}{11} = \frac{3}{11}$

31c. P(GG) = $\frac{1}{11}$

31d. If four socks are removed from the drawer, the chance of them all being different colors is:
$(\frac{12}{12})(\frac{8}{11})(\frac{4}{10})(\frac{0}{9})$ ← Notice that last factor is 0, because there is NO chance the fourth sock won't match a previous sock, so the whole product is zero.
Or consider: with only 3 colors, how can 4 socks all be different? If they cannot all be different, then at least two must match. Therefore the P is 1.

35. Draw the two trees! To WIN, she must defeat her opponent twice in a row:
If she plays Bill-Bob-Bill :



So, overall, if she plays Bill-Bob-Bill, she has three ways to WIN, and their probabilities are:

P(WWW) + P(WWL) + P(LWW)

.5(.8).5 + .5(.8).5 + .5(.8).5

So P(WIN) = .60

If she plays Bob-Bill-Bob, the probability of WIN (by winning 2 games in a row) is:
P(WIN) = P(WWW) + P(WWL) + P(LWW)
= .8(.5).8 + .8(.5).2 + .2(.5).8 = .48
(The tree diagram is similar to the one shown above, but the first and last probability splits are .8 & .2, and the middle split is .5 & .5.)

We compare these probabilities, and it is clear she should play Bill-Bob-Bill.
(Put the surer win in the middle.)