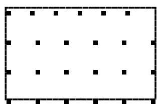
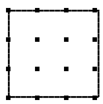
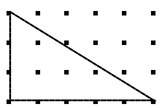
A SURFACE can be covered with ANY standard units which are themselves SURFACES– TWO-DIMENSIONAL ! We can use triangles, trapezoids, even circles; but we use squares because they are the most convenient.

1. 

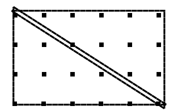
A rectangular region 3 units high, 5 units long, would require 3 rows of 5 squares each, or 3·5 square units, to cover. So we say the *AREA* is 15 square units.
Similarly, for any rectangle, we can demonstrate that
the **area** of any **rectangle** must be the length of the base times the height: **$A = b \cdot h$**

1½. 

A **square** is just a rectangle which has both sides, base and height, equal in length.
The *length* of the *side* of the square is usually referred to as *s*.
For the special rectangles which are square, the area formula is **$A = s \cdot s = s^2$**

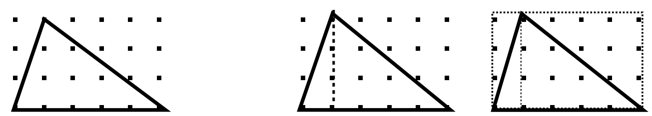
2. 

A TRIANGLE can be viewed as a half-rectangle ...
So the *area* of a ***triangle***
is ***half the area of the completed rectangle***

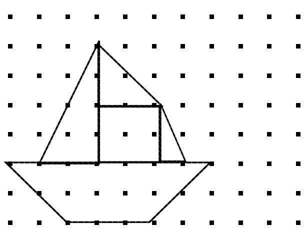


$A = \frac{1}{2} b \cdot h$

Even if the triangle is not a right triangle,
the triangle's area is still...
half of the area of a rectangle
with the same base and height.

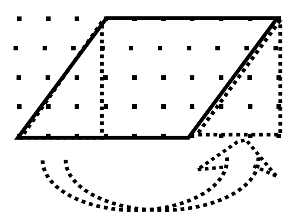


3. Areas of figures with straight sides can be found by breaking the figure into rectangles and triangles. (Try this one !)

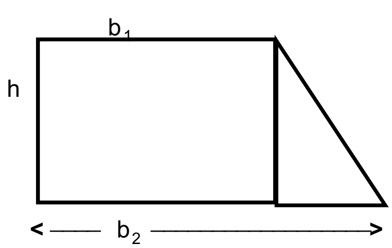


		Total area:	
2x4/2	=	4 sq u	
2x2/2	=	2 sq u	
2x2	=	4 sq u	
2x1/2	=	1 sq u	
2x7	=	14 sq u	
– 2·(2x2/2)	=	– 4 sq u	= 21 sq u

4. The PARALLELOGRAM can be divided into a rectangle and two triangles. Furthermore, because opposite sides are parallel, a right triangle can be "cut off" and translated to the opposite side, showing the area of a parallelogram is base width times height.



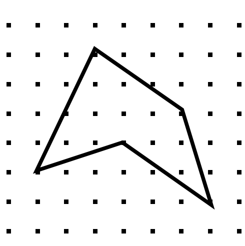
(Height is perpendicular to base!) **$A = b \cdot h$**

5. 

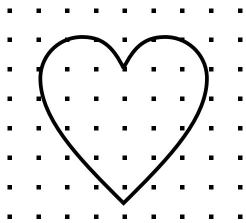
The two parallel sides of a TRAPEZOID are usually called its *bases*. Once again, by dissecting the trapezoid into triangles and a rectangle, we can determine a formula for the area– and discover a *general principle* of area.... The trapezoid's area can be seen as the product of *(average width)* · height !!

$$A = \frac{(b_1 + b_2)}{2} \cdot h$$

(average width) · height

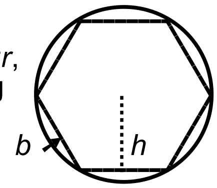
6. 

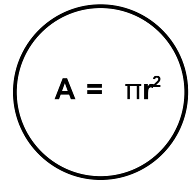
The area of ANY POLYGONAL REGION can be found exactly, by partitioning the region into rectangular and triangular regions... *especially easy* if the polygon joins lattice points. This is always simple if working *from the outside in*. Example:
Draw a rectangle that *just* encloses the figure. Partition the region inside the rectangle but outside the figure into rectangular and triangular regions whose base & height are measured along horizontal & vertical segments joining lattice points.

7. 

Not all figures' areas are easy to *exactly* determine. Area inside an IRREGULAR REGION can be ESTIMATED by superimposing a grid of square units over the region, then counting the number of square units in the region. Various methods exist for estimating the area of the region covered by less than a whole square. One method "averages" the partial squares by counting each partial square as a half square.
Notice this is an approximation, not exact.

8. AREA OF A CIRCLE:



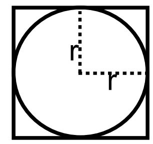


Given the circumference of a circle of diameter $2r$ is $C = 2\pi r$, we can even determine the area within a circle, by inscribing a regular polygon of n sides and computing the area within. The area for the figure shown is

$A_p = n\cdot(½)b\cdot\bullet h = (½)\,P\bullet h$

As n increases, $h \longrightarrow r$ and $nb = P \longrightarrow C$.

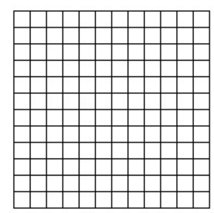
Then $A_p \longrightarrow (1/2)(2\pi r)(r) = \pi r^2$.



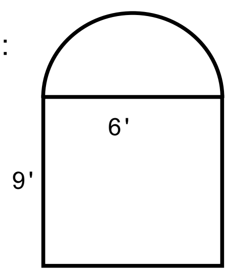
P.S. An argument such as given above is too sophisticated for some audiences. Note the illustration at right shows that the area of a circle is clearly less than $4r^2$.

Notice that all area formulas result in ***UNITS•UNITS, or*** ***SQUARE UNITS.***
A flat surface may be covered with any 2-dimensional figures, such as triangles or hexagons. So we could describe or reckon areas in terms of triangular or hexagonal units. However, square units are generally easier to work with and understand. But, in any case, **area is 2D** and generally involves the product of two linear dimensions—expressed in **units·units**, or **square units**.
Area is intrinsically TWO-DIMENSIONAL, and requires two-dimensional standard units to measure!

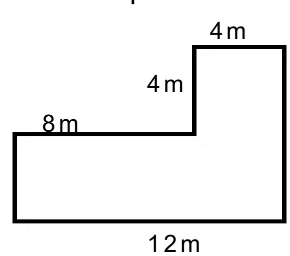
9. a. What is a square foot? How many square inches are in a square foot?



b. Find the area of this demilune window:



c. Assuming anything that looks like a right angle IS one, find the area of this L-shaped room:



d. Assuming any arcs are semi- or quarter-circle boundaries, find the area: