Solutions for Section G: Congruence

When you read these solutions, you should have a copy of the original sample questions from Section G, including the diagrams of the figures.

G1. $\triangle ABE \cong \triangle ACD$. Explanation: $\triangle ABC$ is iscosceles, so $\angle B \cong \angle C$, and $\overline{AB} \cong \overline{AC}$. Since $\overline{BE} \cong \overline{CD}$ by hypothesis, $\triangle ABE \cong \triangle ACD$ follows from SAS.

Another correct answer is $\triangle ABD \cong \triangle ACE$, which follows in a similar way from SAS.

G2. Since $\triangle ABC$ is iscosceles, $m\angle ABC = m\angle ACB$. By assumption, $\angle EBA$ is supplementary to $\angle ABC$ and $\angle DCA$ is supplementary to $\angle ACB$. Therefore, $m\angle EBA = 180 - m\angle ABC = 180 - m\angle ACB = m\angle DCA$. Therefore, $\angle EBA \cong \angle DCA$.

G3. From the fact that a rectangle has four congruent right angles and taking into account E, F, G, H are midpoints of the sides of rectangle $ABCD$, it follows from SAS that,

$$\triangle HAE \cong \triangle FBE \cong \triangle FCG \cong \triangle HDG$$

Since corresponding parts of congruent triangles are congruent, $\overline{HE} \cong \overline{FE} \cong \overline{FG} \cong \overline{GH}$. Therefore, quadrilateral $HEFG$ is a rhombus.

G4. From the assumptions of the problem, it follows from SSS that

$$\triangle KIT \cong \triangle KET$$

Therefore, $\angle ITS \cong \angle ETS$. Using this and the assumptions of the problem it follows that

$$\triangle ITS \cong \triangle ETS$$

Therefore, $\angle TSI \cong \angle TSE$. Since these two angles are also supplementary, they each have measure $90°$. Therefore, $\overline{KT} \perp \overline{IE}$.