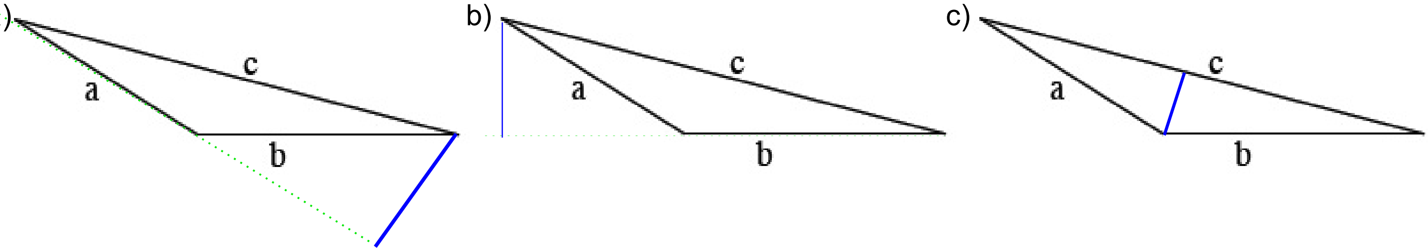
The solutions given are not the only method of solving each question.

§D. BASIC DEFINITIONS, LINES, ANGLES, TRIANGLES, POLYGONS

1. Draw the altitudes (heights) of the triangle if:
   a) side a is the base   b) side b is the base   c) side c is the base

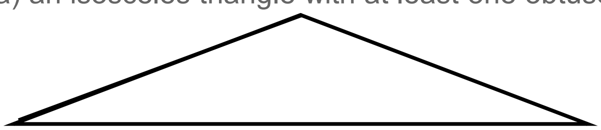
a) b) c)

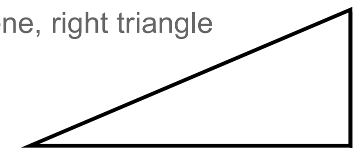


2. The perimeter of an isosceles triangle is 30 cm. Which of the following cannot be the length of the base?
   A. 1 cm   B. 5 cm   C. 9 cm   D. 15 cm

   Answer: D cannot be the base, as that would leave only a total of 15cm. length for the two remaining sides, and their total length must exceed the length of the longest side.
   That is: 30 cm perimeter – 15cm. for the base = 15 cm available for the two shorter sides. But the sum of the lengths of the two shorter sides must be more than the length of the longest side.

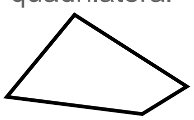
3. Use your straightedge to draw
   a) an isosceles triangle with at least one obtuse angle   b) a scalene, right triangle



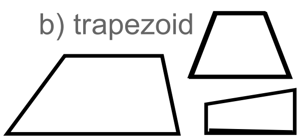


4. Define each figure given below fully and make a sketch of a typical example. (A definition may be based on any **previous** definition.) You may assume that "polygon" has been defined previously.

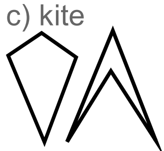
a) quadrilateral



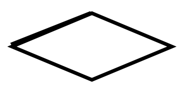
b) trapezoid



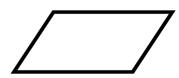
c) kite



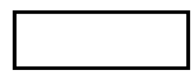
d) rhombus



e) parallelogram

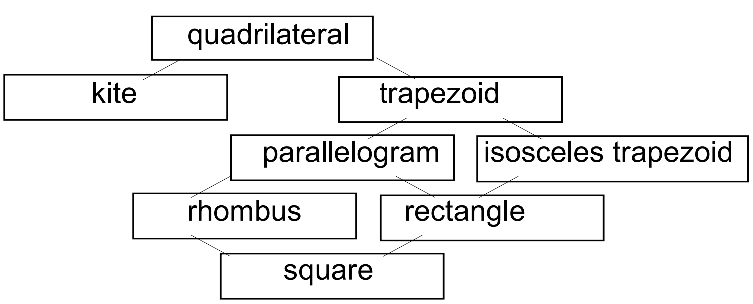


f) rectangle



a. A quadrilateral is a polygon with four sides.
b. A trapezoid is a quadrilateral with a pair of parallel sides.
c. A kite is a quadrilateral with two pairs of adjacent (consecutive) congruent sides.
d. A rhombus is a quadrilateral with all four sides congruent.
e. A parallelogram is a quadrilateral with two pairs of parallel sides.
f. A rectangle is a parallelogram with a right (90°) interior angle.

5. Put the following in the empty boxes below to show the relationship among the terms:
   isosceles trapezoid, parallelogram, quadrilateral, rectangle, rhombus, kite, trapezoid, square



(Showing that, for instance, a square is both a rhombus and a rectangle.... or that a rectangle is both a parallelogram and an isosceles trapezoid.)

6. Circle T for True or F for False.
   a. T or F If AM = MB, then A, M, and B are collinear.
   b. T or F If two angles are congruent, then they are right angles.
   c. T or F The supplement of an obtuse angle is an obtuse angle.
   d. T or F A line and a point not on the line are coplanar.
   e. T or F Two distinct lines can have no more than one point in common.
   f. T or F Two skew lines determine one and only one plane.
   g. T or F If a plane contains one point of a line, then it must contain the entire line.

a. T True if we understand correctly that "AM" and "MB" are lines through points A and M, and through M and B, respectively.

b. F Congruent angles have the same measure, not necessarily 90° !

c. F Never! An obtuse angle exceeds 90°, and its supplement is always acute.

d. T True because a line and a point not on the line determine a plane.

e. T True, because two distinct points determine a line (only one line passes through the two points).

f. F Two skew lines are two lines that cannot be contained in a single plane.

g. F False. Consider a line perpendicular to the plane ("piercing" the plane, as a needle stuck through a piece of paper)– meets the plane at one point only. If a line and plane contain two distinct points, then the entire line lies in the plane.

§E. MISSING ANGLES

1a. Add the angle measurements and express your answer in degrees, minutes, and seconds: 18° 35' 29'' + 22° 55' 41''

These sum to: $40° \, 90' 70''$, which should be expressed as: $41° \, 31' 10''$.

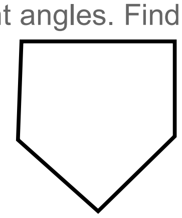
1b. If the measure of an angle is 22° 55' 41'', what is the measure of its supplement?

$$\begin{array}{r} 180° \\ -\,22° \, 55'41'' \end{array} = \begin{array}{r} 179° \, 59'60'' \\ -\,22° \, 55'41'' \end{array} = 157° \, 4'19''$$

Its complement?

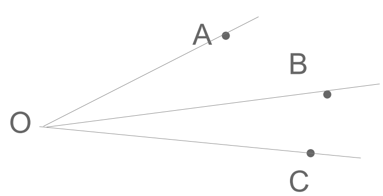
$90° - 22° \, 55'41''$, which is 90° less than the answer in the first part of #1b, above, thus: $67° \, 4'19''$

2. Home plate on a baseball field is a pentagonal region with three right angles and two congruent angles. Find the measures of each of these two congruent angles. Explain your reasoning.



The sum of the measures of all the interior angles in a pentagon must be $3 \cdot 180° = 540°$. **
Since three angles are right, 270° are used by those three, leaving only 270° for the remaining two congruent angles, whose measures, then, must each be half of 270°, or 135°.
(Look at the picture! It is nearly evident.) [We know this** by triangulation of the polygon.]

3. Given:



$m\angle AOB = x + 10°$
$m\angle BOC = 2x - 5°$
$m\angle AOC = 50°$

Is OB the angle bisector of AOC ?

Taking the sketch at face value:

If $(x + 10) + (2x - 5) = 50°$ then $x = 15°$, and thus:
$m\angle AOB = 15° + 10° = 25°$ and $m\angle BOC = 2 \cdot 15° - 5° = 25°$
Yes, each angle has half the measure of the angle AOC.

4. Each of the interior angles of a polygon has the same measure. The sum of the measures of the interior angles is 360°. Which of the following could be the polygon?
A. a rectangle B. a regular hexagon C. a regular pentagon D. an equilateral triangle

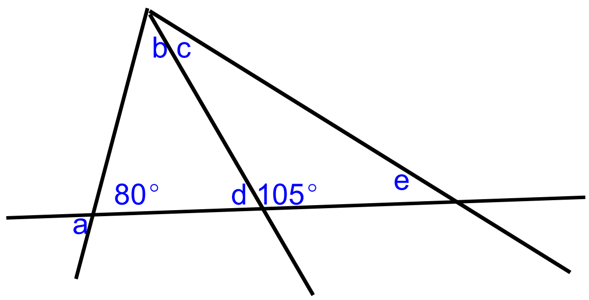
A. The polygon could be a rectangle, and only a rectangle. Since the sum of the measures of the interior angles is 360°, the polygon must be a quadrilateral, with four interior angles. If they are all congruent then each must measure 90°. A quadrilateral with four right angles is a rectangle.

5. If the exterior angles of a regular polygon are 22.5° each, how many sides does the polygon have?

The sum of all exterior angles of a convex polygon is 360°. Sixteen angles at 22.5° each totals 360°, so the regular polygon must have 16 sides.

Another approach: If the exterior angles measure 22.5°, then the interior angles must be the supplementary 157.5°. Knowing the measure of the interior angle of a regular n-gon is 180°(n–2)/n, we determine n = 16.

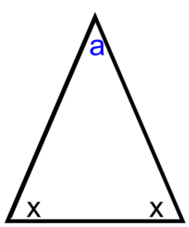
6. In the diagram, $m\angle b = m\angle c$, $m\angle a = 80°$, $m\angle d = 75°$. Find $m\angle e$.



First observe that the angle vertical to a measures 80°.
We know d measures 75°.
Therefore, b must measure 180° – (80° + 75°) = 25°.
Given c congruent to b, c must measure 25°
Since d measures 75°, its adjacent supplement measures 105°.
So e must measure 180° – (25° + 105°) = 50°

(We could have reduced the steps in the above exercise by knowing that the measure of an exterior angle of a triangle is the sum of the measures of the two extreme remote interior angles of the triangle.)

7. If the non-base angle of an isosceles triangle measures 32.33°, what are the measures of the other two angles?

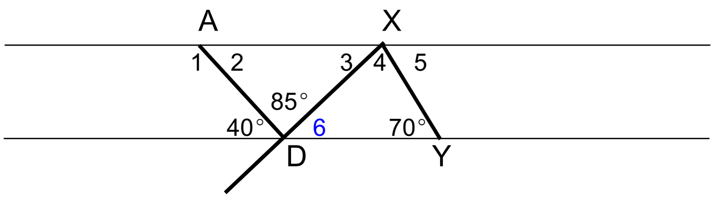


The sum of measures of interior angles of triangle is 180°. Angle a contains 32.33°, leaving a total of 180° – 32.33° = 147.67° for the remaining two angles. We know the angles at the base, opposite the two congruent sides of the isosceles triangle, have equal measures. Therefore each is half of 147.67°, or 73.835°.

The solutions given are not the only method of solving each question.

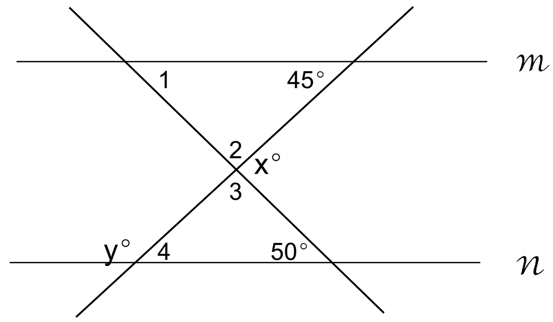
§F. PARALLEL LINES

1. If AX || DY, find the measure of each angle: a) $\angle 1$ b) $\angle 2$ c) $\angle 3$ d) $\angle 4$ e) $\angle 5$



a. 140° Supplement of 40°.
b. 40° Alternate interior to the given 40°.
c. 55° $180^\circ - (m\angle 2 + 85^\circ)$
d. 55° $180^\circ - (m\angle 3 + m\angle 5)$ or $180^\circ - (m\angle 6 + 70^\circ)$
e. 70°

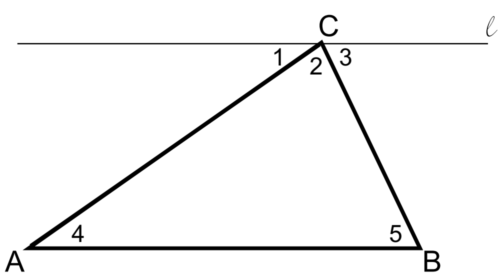
2. The lines m and n are parallel. Find the angles x and y.



$m\angle 4 = 45^\circ$ (Alternate interior angles...)
$m\angle 3 = 85^\circ$ ($180^\circ - (m\angle 4 + 50^\circ)$)
$x^\circ = 95^\circ$ (Supplement of $\angle 3$)
$y^\circ = 135^\circ$ (Supplement of $\angle 4$)

Other approaches might use $\angle 2$ & $\angle 1$:
$m\angle 2 = m\angle 3$
$m\angle 1 = 180^\circ - (m\angle 2 + 45^\circ)$

3. Explain why the sum of the measures of the three angles in any triangle is 180 degrees. You may use a picture and properties of parallel lines. Label your drawing and explain your argument using words and mathematical symbols.



We assume the existence of a line ℓ through vertex C that is parallel to the opposite side, AB, of the triangle.

Lines ℓ and the line through AB are parallel, and the line through AC constitutes a transversal of the two parallel lines. So $\angle 1$ and $\angle 4$ are alternate interior angles for a transversal of parallel lines, and thus must be congruent.
$\angle 1 \cong \angle 4$

Similarly, the line through CB is a transversal of the parallel lines, and so $\angle 3$ and $\angle 5$ are alternate interior angles for a transversal of parallel lines, and thus must be congruent.
$\angle 3 \cong \angle 5$

$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ because the three adjacent angles form a straight angle.

Conclusion: $m\angle 4 + m\angle 2 + m\angle 5 =$
$m\angle 1 + m\angle 2 + m\angle 3$ because $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 5$
$= 180^\circ$

Therefore the sum of the measures of the interior angles of triangle ABC must be 180°.