1. What is the place value of the "0" in the numeral 402,563 ?

2a. Write in words: 21,032,301,007

2b. Write the Hindu-Arabic numeral for the number:
"six billion, two hundred million, thirty-four thousand, nine hundred two".

3. What is the largest number that results from rearranging the digits in 1023649 ?

4. Write 73,021 in expanded form, and in expanded form using powers of ten.

5. Write $7 \cdot 10^3 + 5 \cdot 10^2$ in Hindu Arabic numerals.

6. Write 487 and 1999 using Roman numerals.

7. The Roman numeral MCDLXXXVIII would be written in Hindu-Arabic numerals as __.

8. 210 x 356 is closest to which of the following?
700 7000 70000 None

9. Write word problems for 13 – 9 illustrating:
a. take-away subtraction
b. part-whole (or missing addend) subtraction
c. comparison interpretation for subtraction

10. Indicate if the word problem is measurement division or partitive division.
a. Alice tied 15 sticks into 3 equal bundles. How many sticks were in each bundle?
b. 72 eggs is how many dozen?
c. 36 balls are packed into boxes of 6. How many boxes are there?
d. We drove 1280 miles through California in 5 days. What was our average distance per day?

11a. Does the equation, 2 ÷ 0 = 0 ÷ 2, follow from the commutative property?
Give a complete explanation.

11b. Explain why 0 ÷ 0 is undefined.

12. Fill in the arithmetic property that justifies each step:
a. $(5 \times 397) \times 2 = (397 \times 5) \times 2$ _____
$= 397 \times (5 \times 2)$ _____
b. $10 \times (19 + 8) = 10 \times (8 + 19)$ _____
c. $999 + 0 = 999$ _____

13. Draw rectangular arrays to illustrate $3 \times 5 = 5 \times 3$
(Hint: what must be done to the array for 3×5 to get the array for 5×3?)

14. Draw a rectangular array illustrating the distributive property for
$3 \times (4+5) = 3 \times 4 + 3 \times 5$

15. Illustrate mental math techniques to calculate the following. Write your answers in ways that clearly show the steps involved in mental calculation.

a.	167 + 19 + 33 + 6.3 + 81 + 3.7	b.	435 + 96	c.	355 – 97
d.	9 × 67	e.	48 × 25	f.	76 × 1001
g.	340 ÷ 5	h.	24 × 38 + 24 × 12	j.	4 × 63.2 × 25

16. Suppose you want to estimate 42x92. Which is a better estimate: 40x92 or 42x90?
Explain your choice without actually finding each product and then taking the difference.

17. Using a chip model, base ten blocks, or bundles, clearly explain all the steps for the standard addition, subtraction, and multiplication algorithms:

a. 1354	b. 2027	c. 235
+ 3678	– 537	× 3

18. Fill in the blanks so that the result is a correct multiplication calculation:

$$\begin{array}{r} \_\,8 \\ \times\, 2\,6 \\ \hline 4\,0\,\_ \\ \_\,3\,\_\,\_ \\ \hline \_\,\_\,\_\,\_ \end{array}$$

19. A student's paper shows this (incorrect) work on an assignment:

a.		b.	
72		47	
− 28		− 29	
56		28	

In each case, explain what the student's method might be. What answer would the student get if the same method were used to compute 82 – 54 ?

20. a. Explain the steps of the long division algorithm for whole numbers using the partitive model for the division of 1062 cents (10 dollars and 6 dimes and 2 pennies) between three people.
b. Illustrate the division, 1396 ÷ 6, using a measurement model.
21. Prove by counterexample, using only whole numbers, division is not associative.
22. A case holds 24 bottles of cider. How many cases will 7,400 cider bottles completely fill? How many bottles of cider will be left over?
Number of full cases _____
Number of jars left over _____
23. If this month is September, what month will it be 580 months from now?
24. Compute the following multiplication: 521 x 43. When you multiply the 2 by the 4, the product ends up in a particular position (place value). Explain why that is the correct place value. (Do not give an answer that essentially states that the algorithm says so.)
25. A student writes the following equations in order to solve 58 – 15:
58 – 10 = 48 – 5 = 43. This is incorrect because, for example, 58 – 10 does not equal 48 – 5. Use the student's strategy, but write correct equations showing the steps.
26. Illustrate the identity, $(a + b)(a + b) = a^2 + 2ab + b^2$, by a rectangular array.
Use the distributive, commutative and associative properties to justify the statements:

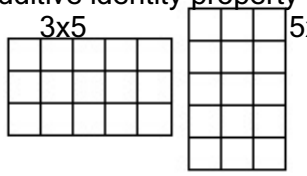
$(a + b)^2 = (a + b)(a + b)$ (Why?)
$= (a + b)a + (a + b)b$ (Why?)
$= (a^2 + ba) + (ab + b^2)$ (Why?)
$= a^2 + ba + ab + b^2$
$= a^2 + ab + ab + b^2$
$= a^2 + (ab + ab) + b^2$
$= a^2 + (1ab + 1ab) + b^2$
$= a^2 + (1+1)ab + b^2$
$= a^2 + 2ab + b^2$ because 1+ 1 = 2

27. Give a "teacher's solution" to each of these, using bar diagrams:
a. There are three times as many boys as girls. If there are 96 children, how many girls are there?
b. There are three children in a family. Ed is 20 pounds heavier than Fred who weighs twice as much as Ned. If the three children weigh 180 pounds altogether, how much does Ed weigh?
c. John and Wendy have a total of 1012 pennies. Wendy has 134 less than John. How many pennies does John have?
d. A bag of cookies contains two varieties, chocolate chip and oatmeal. There are four times as many chocolate chip as oatmeal cookies. If there are 36 more chocolate chip than oatmeal cookies, how many cookies are there altogether?

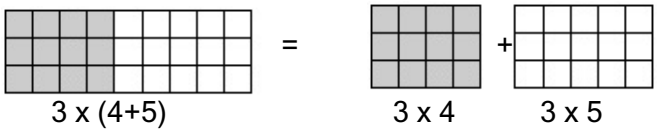
Answers

1. 0 is in the ten-thousands place.
2a. 21 billion, 32 million,3 hundred one thousand, seven.
2b. 6,200,034,902
3. 9643210 is the largest number using those digits.
4. $73,021 = 70000 + 3000 + 20 + 1 = 7x10^4 + 3x10^4 + 2x10 + 1x10^0$
5. 7500
6. CDLXXXVII and MCMXCIX

7. 1488
8. 70000 (2x35 = 70 so 200x350 would be 1000 times as great)
9a. Jan had 13 books checked out from the library, but he took back9 of them, so how many does he still have checked out? (TAKE-AWAY)
9b. Marie has 9 blue ribbons, but her goal is to have 13 of them. How many more does she need? (PART-WHOLE OR MISSING ADDEND)
9c. Jack has 13 stickers and Suz has only 9. How many more has Jack than Suz? (COMPARISON)
10a. Partitive division
10b. Measurement division
10c. Measurement division
10d. Partitive division
11a. Does 2 ÷ 0 = 0 ÷ 2, follow from the commutative property?
NO. There is no commutative property for division. Further, 2÷0 is undefined.
11b. 0÷0 is undefined because no single value can be assigned to this.
Eg. 6÷2 = 3 because 2x3 = 6 and 3 is the only number that will do this.
We could say 0÷0 = 652 because 0x652 = 0.... but then any number will work in place of 652, so there is not a unique multiplier.
12a. Commutative property of multiplication
Associative property of multiplication
12b. Commutative property of addition
12c. Additive identity property
13. 3x5 5x3

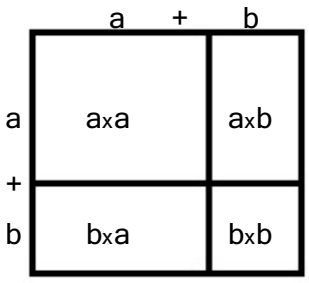


14.



15. a. 167 + 19 + 33 + 6.3 + 81 + 3.7 = 167+33 + 19+81 +6.3+3.7 = 310 compatible #s, comm & assoc. prop. of +
b. 435 + 96 = 431 + 100 = 531 compensation
c. 355 - 97 = 358 - 100 = 258 compensation
d. 9 × 67 = (10 - 1) x 67 = 670 - 67 = 603 distributive property, 9 = 10 - 1
e. 48 × 25 = 12 x 4 x 25 = 12 x 100 = 1200 compatible numbers, associative property of multiplication.
f. 76 × 1001 = 76x1000 + 76x1 place value, distributive property
g. 340 ÷ 5 = (340 ÷ 10) x 2 = 68
h. 24 × 38 + 24 × 12 = 24 x (38 + 12) = 24 x 50 = 240 x 5 = 240x10 ÷2 = 1200
j. 4 × 63.2 × 25 = 63.2 x 4 x 25 = 63.2 x 100 = 6320 compatible numbers, commutative prop. of mult.

16. 42x90. In each case you are rounding off by 2, which is multiplied by the other number. Would you rather have the error of rounding off (-2) multiplied by 42, or by 92? 40 x 92 is off by 2x92. 42x90 is off by 42x2.
17. These problems are in your class notes; we did a chip model for each of these in class.
18. The digits in bold are the ones to fill in the blanks: 68x26 = 408 + 1360 = 1768
19a. The student decided to subtract 8– 2 = 6 rather than 2– 8, requiring unbundling. He probably takes the smaller number from the larger, avoiding unbundling. By this method, 82– 54 would be 32.
19b. Forgot he "borrowed" or unbundled a ten, leaving 3–2 rather than 4 – 2. BTM 82 – 54 would be 38.
20. These divisions are illustrated on pages 3–4 and 3–5 of the class notes.
21. (24 ÷ 6) ÷2 = 4 ÷ 2 = 2. 24÷ (6÷2) = 24 ÷ 3 = 8. So (24 ÷ 6) ÷2 ≠ 24÷ (6÷2) [÷ is not associative]
22. 7400 = 24 x 308 + 8 So 7400 bottles will fill 308 cases of 24, with 8 bottles left over.
23. Every 12 months it will be September again. 580 = 12 x 48 + 4. 4 months after Sept.→ January!
24. Write the long multiplication on your paper– 521 x 43 The product of 2 x 4, 8, is in the hundreds column, which is where it belongs, because that is really 20 x 40, taking into account the place values!
25. 58 – 15 = (58 – 10) – 5 = 48 – 5
26. Definition of exponents. $w^2 = w\ w$
Distributive property (of x over +)
Distributive property (of x over +)
Associative property of +
Commutative property of multiplication
Associative property of +
Identity property of mult.
Distributive prop (of x over +)
because 1+ 1 = 2

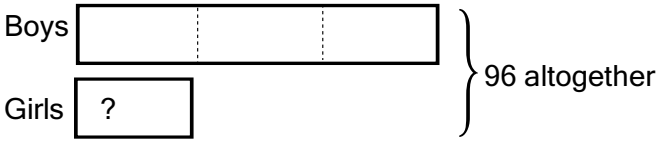


$(a + b)^2$

IS

$a^2 + 2ab + b^2$

27a. There are three times as many boys as girls. If there are 96 children, how many girls are there?



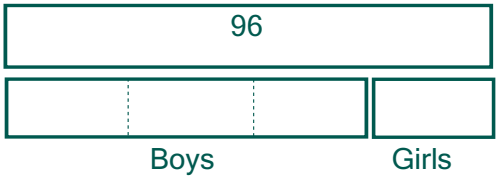
$4 \times ? = 96$

$? = 96 \div 4 = 24$

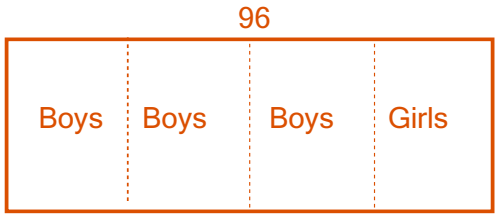
There are 24 girls.

Check: If 24 girls, then $3 \cdot 24 = 72$ boys.

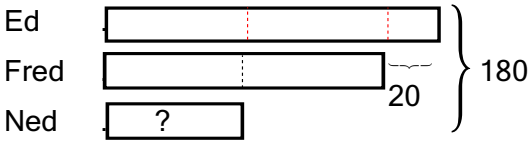
$24 + 72 = 96$ ✔



Two Alternate views....



27b. Ed is 20 pounds heavier than Fred who weighs twice as much as Ned. 180 pounds total all 3. Ed's weight?



180 [The red lines were not in the initial picture.]

− 20

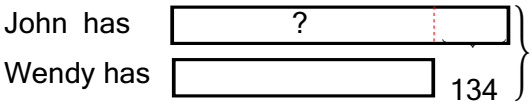
160 3 boys' total weight without Ed's extra 20 pounds

$160 \div 5 = 32$.

Ned weighs 32 pounds so Ed weighs 84 pounds

Check: Ned 32, Fred 64, Ed 84. $32+64+84 = 180$ ✔

27c. John and Wendy have a total of 1012 pennies. Wendy has 134 less than John. How many pennies has John?



1012

− 134

878 total pennies without John's "extra" 134.

$878 \div 2 = 439$ Wendy has 439 pennies.

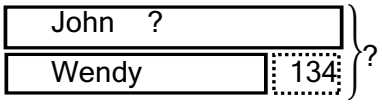
439

+ 134

573 John has 573 pennies.

Check:
573+439 = 1012,
573– 439 = 134 ✔

Alternate view:



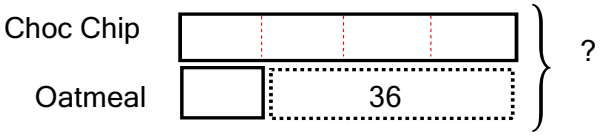
1012

+ 134

1146

If Wendy were to get 134 more pennies, then she and John would have equal amounts.
This would be $1146 \div 2 = 573$. John must have 573 pennies.

27d. four times as many choc chip as oatmeal cookies. 36 more choc chip than oatmeal cookies, how many cookies?



3 units = 36 cookies $36 \div 3 = 12$

1 unit = 12 cookies 12

5 units = 60 cookies $\times 5$

60

Check: $48 \div 12 = 4$, $48 - 12 = 36$, $48+12 = 60$ ✔