(5) 1. *Order* the following rational numbers (least to greatest) : $\dfrac{2}{3}$ $\dfrac{14}{25}$ $0.\overline{67}$ $0.6\overline{7}$

___ < ___ < ___ < ___

(5) 2. Calculate these:

a. $\dfrac{3}{8}\left(\dfrac{2}{3} \div \dfrac{1}{6} \right) - \dfrac{1}{3} \quad =$

b. $3\dfrac{3}{4} \times 1\dfrac{1}{5} \quad =$

(5) 3. Express the repeating decimal $0.\overline{67}$ as a fraction in lowest terms. Do work on backside of paper.

(5) 4. True or false:

a.   " **If** $\dfrac{a}{b}$ and $\dfrac{c}{d}$ **are both rational** then $\dfrac{ac}{bd}$ **must be rational also.** "

b. $3 + \sqrt{5}$ is irrational.

c. $\sqrt{5} \times \sqrt{20}$ is irrational.

d. $1.02002000200002\cdots$ is irrational.

(5) 5. Circle those fractions which have a terminating decimal form:

$\dfrac{7}{3^2}$ $\dfrac{2^{19}}{5^9}$ $\dfrac{3}{8^{11}}$ $\dfrac{60}{150}$

(5) 6. a.   Round 34.9782 to the nearest tenth.   b. Round 1952.788 to the nearest hundred.

c.   Write $7 \cdot 10^3 + 3 \cdot 10^0 + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}$ in standard decimal form.

(5) 7. When you multiply 52.03 by 3.12, you multiply the whole numbers $5203 \cdot 312$ (& get 1623336). Then you place a decimal point 4 spaces from the right side of 1623336. Write the fraction product which justifies that part of the process. That is, use fractions to show WHY we need to put that decimal point in that place.

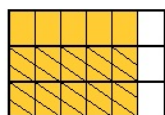(5) 8. Write a statement of the multiplication fact illustrated here.
(The first two diagrams just give background.)
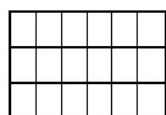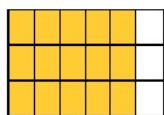
If this is a unit (1),   then this is . . .   & this represents ...

___ · ___ = ___

(5) 9. The legend on a map shows that $^3/_4$ of an inch on the map corresponds to 12 miles. *How far* do we need to drive if the distance on the map appears to be 5½ inches?

(5) 10. John spent 2/5 of his money on dinner, then 1/3 of the remainder on dessert.
   a.   What fraction of his money did John spend altogether?

   b.   What fraction of his money does he have left over?

(5) 11. Dean spent 3/8 of his money on a gift; after that he had only $45 left.
   How much did Dean have when he went shopping? Give a "teacher's solution" on this one.

(5) 12. The ratio of girls to boys in the 5th grade is 5:6. If there are 132 children in the 5th grade, how many are girls?

(5) 13. Three sevenths of the children at a party are under 5 yrs old. There are 10 more children over 5 than under 5. How many children are at this party?

(5) 14. Jane bought a house in 1996 for $240,000. The value of the house increased 150% before she sold it in 1999. How much did she get for her house when she sold it?

(5) 15. Cliff bought his house for $225,000, then sold it for $300,000 six months later. By what percent did the house value appreciate?

(5) 16. A computer is on sale at 40% off! If the price now is $900, what was the original price?

(5) 17. Glen took 1 and a half hours to grade 2 fifths of the quizzes. At that rate, what is the total time Glen will need to grade all the quizzes?

(5) 1. *Order* the following rational numbers (least to greatest) : $\dfrac{2}{3}$ $\dfrac{14}{25}$ $0.\overline{67}$ $0.6\overline{7}$

$$\dfrac{14}{25} \ < \ \dfrac{2}{3} \ < \ .\overline{67} \ < \ .6\overline{7}$$

$\dfrac{50}{75}$ $\dfrac{42}{75}$ $.6767\cdots < .67777\cdots$

and $\tfrac{2}{3} = .6666\cdots < .6767\cdots$

(5) 2. Calculate these:

a. $\dfrac{3}{8}\left(\dfrac{2}{3} \div \dfrac{1}{6} \right) - \dfrac{1}{3} \ =$

$\dfrac{3}{8}\left(\dfrac{2}{3} \cdot \dfrac{6}{1} \right) - \dfrac{1}{3} \ =$

$\dfrac{3}{2\cdot 2\cdot 2}\cdot \dfrac{2}{3} \cdot \dfrac{6}{1} - \dfrac{1}{3} \ =$

$\dfrac{3}{2} - \dfrac{1}{3} \ = \ \dfrac{9-2}{6}$

b. $3\dfrac{3}{4} \times 1\dfrac{1}{5} \ =$

$\dfrac{15}{4} \times \dfrac{6}{5} \ =$

$\dfrac{3\cdot 5}{2\cdot 2} \times \dfrac{2\cdot 3}{5} \ =$

$\dfrac{3}{2} \times \dfrac{3}{1} \ = \ \dfrac{9}{2} \ = \ 4\dfrac{1}{2}$

(5) 3. Express the repeating decimal $0.\overline{67}$ as a fraction in lowest terms.

$$\begin{array}{rl} 100x \ = & 67.67676767\cdots \\ x \ = & .67676767\cdots \\ \hline 99x \ = & 67 \end{array}$$

We multiply by 100 to "move the decimal point" 2 places, so the repetends will align exactly.

so $x \ = \ \dfrac{67}{99}$ (This is reduced form already.)

(5) 4. True or false:

a. " **If** $\dfrac{a}{b}$ and $\dfrac{c}{d}$ **are both rational** then $\dfrac{ac}{bd}$ **must be rational also.** "   True, this is closure of multiplication on Q.

b. $3 + \sqrt{5}$ is irrational.   True.   The sum of an irrational number and rational number must be irrational. If $3 + \sqrt{5} =$ (a rational number), then $\sqrt{5} =$ (a rational number) $- 3$, which is rational. But $\sqrt{5}$ is irrational. So $3 + \sqrt{5}$ cannot be a rational number.

c. $\sqrt{5} \times \sqrt{20}$ is irrational.   False. $\sqrt{5} \times \sqrt{20} = \sqrt{100} = 10$, which is rational.

d. $1.02002000200002\cdots$ is irrational. True, assuming that what is meant at left is a decimal number in which the sequences of 0s keep increasing in length.

(5) 5. Circle those fractions which have a terminating decimal form:

$\dfrac{7}{3^2}$ $\boxed{\dfrac{2^{19}}{5^9}}$ $\boxed{\dfrac{3}{8^{11}}}$ $\boxed{\dfrac{60}{150}}$ $\dfrac{2}{5}$

(5) 6. a.   Round 34.9782 to the nearest tenth.

35.0

34.9782 is above 34.95
35.0
34.9

b. Round 1952.788 to the nearest hundred.

2000

The underline on the hundreds digit signals this was rounded to the nearest hundred.

c. Write $7 \cdot 10^3 + 3 \cdot 10^0 + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}$ in standard decimal form.

$$7000 \quad + 3 \quad + .06 \quad + .008$$
$$7003.068$$

(5) 7. When you multiply 52.03 by 3.12, you multiply the whole numbers 5203 ·312 (& get 1623336). Then you place a decimal point 4 spaces from the right side of 1623336. Write the fraction product which justifies that part of the process. That is, use fractions to show WHY we need to put that decimal point in that place.
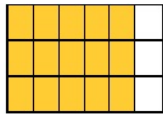
$$3.12 \times 523.03 \quad = \quad \frac{312}{100} \times \frac{52303}{100} \quad = \quad \frac{1624446}{10000}$$

Moving the decimal point four places to the left is dividing by ← 10000.

(5) 8. Write a statement of the multiplication fact illustrated here.
(The first two diagrams just give background.)
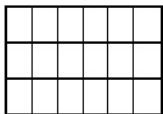
If this is a unit (1), then this is $\frac{5}{6}$

OR

$$\frac{2}{3} \cdot \frac{5}{6} = \frac{10}{18}$$

& this represents ... $\frac{2}{3}$ of $\frac{5}{6}$

(5) 9. The legend on a map shows that $^3/_4$ of an inch on the map corresponds to 12 miles. *How far* do we need to drive if the distance on the map appears to be 5½ inches?
Scaling method:

$\frac{3}{4}$ inch ⇨ 12 miles ⎞ ÷ 3
$\frac{1}{4}$ inch ⇨ 4 miles ⎠
                        ⎞ × 4
1 inch ⇨ 16 miles ⎠
                        × 5 ½
5½ in ⇨ 88 miles ⎠

$$\frac{X \text{ real mi}}{5\frac{1}{2} \text{ map in}} = \frac{12 \text{ real mi}}{\frac{3}{4} \text{ map in}}$$

$$X = \frac{12 \cdot 5\frac{1}{2}}{\frac{3}{4}} = \frac{66}{\frac{3}{4}} = 88$$

We need to drive 88 miles.

(5) 10. John spent $\frac{2}{5}$ of his money on dinner, then $\frac{1}{3}$ of the remainder on dessert.
   a. What fraction of his money did John spend altogether?

$$\frac{2}{5} + \frac{1}{3} \cdot \frac{3}{5} = \frac{3}{5}$$

John spent $\frac{3}{5}$ of his money altogether.

   b. What fraction of his money does he have left over?

$$1 - \frac{3}{5} = \frac{2}{5}$$

John has $\frac{2}{5}$ of his money left.

(5) 11. Dean spent $\frac{3}{8}$ of his money on a gift; after that he had only $45 left.
   How much did Dean have when he went shopping? Give a "teacher's solution" on this one.

Teacher solution with bar diagram:

= ???

$45
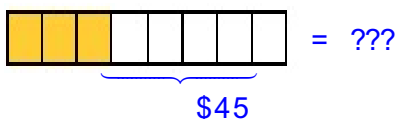
5 parts = $45
1 part = $9

8 parts = $72

Teacher solution using algebra:

Let X represent how much Dean had. After shopping, he had x − $\frac{3}{8}$ x , and we're told that is $45.
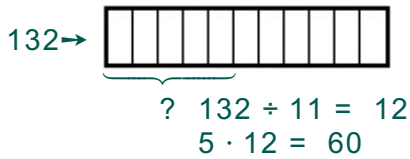
$$x - \frac{3}{8} x = \$45$$

$$\frac{5}{8} x = \$45$$

Dean had $72 when he went shopping.
(Check: then he spent $\frac{3}{8}$ of that money: $\frac{3}{8}$ of $72 = $27. That left $72 − $27 = $45. ✔)

(5) 12. The ratio of girls to boys in the 5th grade is 5:6. If 132 children in the 5th grade, how many are girls?

Without diagram:

132 → [diagram of 11 boxes]

Ratio 5:6 tells us to think of 5 parts & 6 parts, 11 parts.
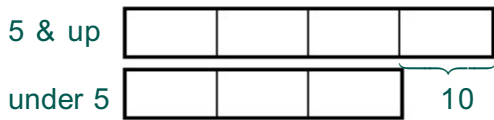
? 132 ÷ 11 = 12
5 · 12 = 60

11 parts → 132
1 part → 12
5 parts → 60

60 of the 5th graders are girls.

(5) 13. Three sevenths of the children at a party are under 5 yrs old.
There are 10 more children over 5 than under 5. ← This tells me to make a comparative diagram.
How many children are at this party?

5 & up [diagram of 4 boxes]

under 5 [diagram of 3 boxes]   10

This diagram exposes the truth. ↘

Without diagram:

Fraction $\frac{3}{7}$ tells us the whole is divided into 7 parts—
3 parts and 4 parts, the difference being 1 part.
1 part → 10 children
7 parts → 70 children.
↓

There are 70 children at the party.

(5) 14. Jane bought a house in 1996 for $240,000. The value of the house increased 150% before she sold it in 1999. How much did she get for her house when she sold it?

House was worth: $240,000
Increased 150%: + 360,000   ←150% of 240,000 = 1.50 · 240,000
1999 worth: $600,000 ↘
She got $600,000 for her house.

(5) 15. Cliff bought his house for $225,000, then sold it for $300,000 six months later.
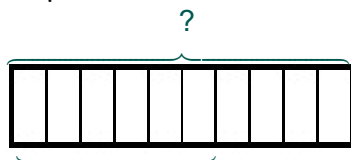By what percent did the house value appreciate?   "Appreciate" here means "increase".

Increase: 300,000 − 225,000 = 75,000

% increase = $\frac{\text{Increase}}{\text{Original}}$ = $\frac{75000}{225000}$ = $\frac{1}{3}$ = $.33\overline{3}$ = $33.\overline{3}$ %

The house appreciated in value one-third, or $33.\overline{3}$ %.

(5) 16. A computer is on sale at 40% off! If the price now is $900, what was the original price?

?
[diagram of 10 boxes]

$900
In this diagram, 40% is
represented as 4 tenths.
6 parts → $900
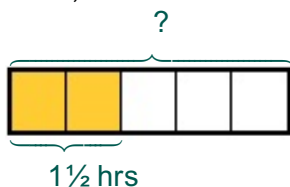1 part → $150
10 parts → $1500

Let x be the original price.

x − 40% of x = $900

60% of x   = $900

x = $900 ÷ .6 = $1500.

(5) 17. Glen took 1 and a half hours to grade 2 fifths of the quizzes.
At that rate, what is the total time Glen will need to grade all the quizzes?

?
[diagram of 5 boxes, 2 shaded]

1½ hrs

2 parts → 1.5 hrs
1 part → .75 or $\frac{3}{4}$ hr
5 parts → 5 · $\frac{3}{4}$ hr = $3\frac{3}{4}$ hr or 3.75 hr.

Glen will need 3 hours and 45 minutes to grade all.