When comparing, note: Your problems may have different numbers, or be in different order.

/9 1. Write an algebraic expression for the following:

a. the number of inches in **m** feet

12m (inches)

1 ft = 12 in
2 ft = 24 in
10 ft = 129 in
m ft = ...

b. the cost in dollars of buying **x** CDs from a music club, if it costs \$10 to join the club, then \$15 for each CD purchased.

10 + 15x (dollars)

1 CD will cost \$10 + \$15
2 CDs cost \$10 + \$30
10 CDs cost \$10 + \$150....

c. the total value, in cents, of **d** dimes, **n** nickels and **p** "pennies".

10d + 5n + p (cents)

The value of
4 dimes, 3 nickels and 2 pennies
is $4\cdot10 + 3\cdot5 + 2$ cents

/8 2. Multiply each of the following, expressing your final result without parentheses.

a. $(a+b)^2 = (a+b)(a+b)$

$= (a+b)a + (a+b)b$

$= a\cdot a + b\cdot a + a\cdot b + b\cdot b$

$= \mathbf{a^2 + 2ab + b^2}$

b. $(a+b)(a-b) = (a+b)a - (a+b)b$

$= a^2 + ab - (ab + b^2) = \mathbf{a^2 - b^2}$

/8 3. Solve this problem algebraically.
Give a "teacher's solution", identifying the variables, showing the work, and stating the conclusion.
Problem: Sunny had three times as much money as Oscar. After Sunny spent \$350 and Oscar earned \$1000, Sunny had twice a much as Oscar. How much did they each have at the beginning?

In the beginning:	After:
Oscar had x	Oscar has x + \$1000
Sunny had 3x	Sunny has 3x – \$350

AFTER that (↑), Sunny had twice as much as Oscar....

3x – \$350 = 2 (x + \$1000)

We solve: $3x - 350 = 2x + 2000$

$x = 2350$

Oscar's original amount of money was \$2350.

Sunny had three times as much, so that was \$7050.

Check: 7050/3 = 2350. So Oscar had 1/3 as much as Sunny. (✔)

After Sunny spent \$350, she had 7050 – 350 = 6700 .

After Oscar earned \$1000, he had 2350 + 1000 = 3350. 2(3350) = 6700. (✔)

It all checks out to match the information given. ✔

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/6 4. Express $\frac{10^{15}\cdot 15^{20}}{50^{7}\cdot 9^{6}}$ as a product of prime numbers (each prime may appear only once).

$$\frac{10^{15}\cdot 15^{20}}{50^{7}\cdot 9^{6}} = \frac{(2\cdot5)^{15}\cdot(3\cdot5)^{20}}{(2\cdot5^2)^{7}\cdot(3^2)^{6}} = \frac{2^{15}\cdot5^{15}\cdot3^{20}\cdot5^{20}}{2^{7}\cdot5^{14}\cdot3^{12}} = \frac{2^{15}\cdot3^{20}\cdot5^{35}}{2^{7}\cdot3^{12}\cdot5^{14}} = \frac{2^{15}}{2^{7}}\,\frac{3^{20}}{3^{12}}\,\frac{5^{35}}{5^{14}} = \mathbf{2^{8}\cdot3^{8}\cdot5^{21}}$$

$$\frac{10^{13}\cdot 15^{20}}{50^{7}\cdot 9^{6}} = \frac{(2\cdot5)^{13}\cdot(3\cdot5)^{20}}{(2\cdot5^2)^{7}\cdot(3^2)^{6}} = \frac{2^{13}\cdot5^{13}\cdot3^{20}\cdot5^{20}}{2^{7}\cdot5^{14}\cdot3^{12}} = \frac{2^{13}\cdot3^{20}\cdot5^{33}}{2^{7}\cdot3^{12}\cdot5^{14}} = \frac{2^{13}}{2^{7}}\,\frac{3^{20}}{3^{12}}\,\frac{5^{33}}{5^{14}} = \mathbf{2^{6}\cdot3^{8}\cdot5^{19}}$$

/4 5. List all the digits that can replace "d" (that can be placed in the blank) so that:

a. 4 is a divisor of 123571d (123571_) — $4|12$ and $4|16$, so d may be **2 or 6**.

b. 11 is a factor of 8070d2 (8070_2) — Alternating sum is 8−0+7−0+d−2 = 13 + d, which is a multiple of 11 only if the digit d is **9**.

/16 6. In the statements below ***a***, ***b*** and ***c*** represent whole numbers.
Circle T if the statement is true, **F** if it is false.

a. T (F) If $4|a$ and $6|a$ then $24|a$. — Suppose a is 6 or 12 or 36 ! $4|6$ & $6|6$ but $24\nmid 6$!

b. (T) F If ***a*** is divisible by 2 & by 3, then ***a*** is divisible by 6. — OK because 2 & 3 are relatively prime.

c. (T) F If ***a*** is any integer, then 0 is a multiple of ***a***. — 0 is on every list of multiples. ···, −2, 0, 2,···

d. (T) F The numbers 32 and 15 are relatively prime. — GCF(32, 15) = 1

e. (T) F 13 is a divisor of $2^4\cdot5^4\cdot7\cdot13^2\cdot17^5\cdot3 + 26$. — 13 clearly divides both terms.

f. T (F) $2\cdot3^2\cdot5^4\cdot11\cdot17^5\cdot43 + 24$ is a multiple of 11. — 11 divides the first term, but not 24.

g. (T) F If 5 | a and 5 | b then 5 | a+b. — If a & b are both multiples of 5, then a+b is too.

h. T (F) If $5\nmid a$ and $5\nmid b$ then $5\nmid a+b$. — See problem #9a.

/10 7. To test whether the numbers 211 & 221 are prime...

a. we need to test for divisibility by certain primes. List all the primes we need to test.

2 3 5 7 11 13 — There's no need to test further, because $17^2 > N$.
If N is composite, at least one factor must be $< \sqrt{N}$.

b. Is 221 prime or composite? Explain or show how you know.

$2\nmid 221$ ($2\nmid 1$) — $7\nmid 221$ (210 + 11)
$3\nmid 221$ (2+2+1=5) — $11\nmid 221$ (220 + 1)
$5\nmid 221$ ($5\nmid 1$) — $13|221$!

221 = 13·17 ; therefore 221 is composite.

c. Is 211 prime or composite? Explain or show how you know.

$2\nmid 211$ ($2\nmid 1$, the last digit, so $2\nmid 210 + 1$)
$3\nmid 211$ (sum of digits = 2+1+1 = 4 and $3\nmid 4$)
$5\nmid 211$ ($5\nmid 1$, the last digit, so $5\nmid 210+1$)
$7\nmid 211$ ($7\nmid 210 + 1$)
$11\nmid 211$ ($11\nmid 220 - 9$)
$13\nmid 211$ ($13\nmid 260 - 49$)

None of the smaller prime factors divides 211; therefore 211 is prime.

As stated in part a, if N is composite, it must have a prime factor less than $\sqrt{N}$.
So if 211 were composite, 211 would have a prime factor less than 14.something.
But none of the prime numbers less than 17 divide 211,
so 211 cannot be composite, therefore must be prime.

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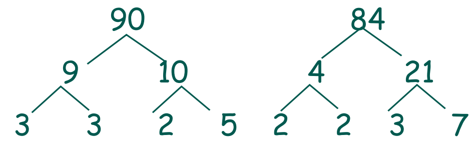
ALL ANSWERS ON THIS PAGE MAY BE LEFT IN FACTORED FORM.

/12 8. a. Find the Greatest Common Factor (GCF) of 90 and 84.



$90 = 2 \cdot 3^2 \cdot 5$
$84 = 2^2 \cdot 3 \cdot 7$ } So **GCF(90, 84) = 2 · 3**

b. Find GCF ($2^3 \cdot 3^2 \cdot 7 \cdot 13^2$, $2^2 \cdot 5^3 \cdot 7^2 \cdot 13$, $2^2 \cdot 3^2 \cdot 5 \cdot 13^2$)

Picking up the minimum power of each prime listed: $2^2 \cdot 3^0 \cdot 5^0 \cdot 7^0 \cdot 13^1 = \mathbf{2^2 \cdot 13}$ or 52

c. Find LCM ($2^3 \cdot 3^2 \cdot 7 \cdot 13^2$, $2^2 \cdot 5^3 \cdot 7^2 \cdot 13$, $2^2 \cdot 3^2 \cdot 5 \cdot 13^2$)

Picking up the maximum power of each prime listed: $\mathbf{2^3 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 13^2}$ or 74,529,000

/8 9. In each of the following, ***show*** the statement is false with a *fully demonstrated* **counterexample**.

a. If $3 \nmid a$ & $3 \nmid b$ then $3 \nmid (a + b)$. Let a = **4** b = **5**

The above claims: **3** $\nmid$ 4 **& 3** $\nmid$ 5 **so 3** $\nmid$ **(**4 **+** 5**)**

But 4+5 = 9 and 3|9. So the conclusion of this statement is false; thus the statement is false.

b. If the sum of the digits of a number is divisible by 2, then the number is divisible by 2.

The above statement says that since the sum of the digits in the number 121 is 4, which is divisible by 2, therefore the number 121 must be divisible by 2.
This is nonsense, since 121 = 11^2 (prime factorization), and is clearly not a multiple of 2.

/10 10. a. *Use the Euclidean algorithm* to **find** the **GCF** of 3150 and 306;
b. then use the result from part a to find the **LCM** of 3150 and 306.
Show the work (of course) !

3150 = 10 · 306 + 90 (3150 – 3060)

306 = 3 · 90 + 36 (306 – 270)

90 = 2 · 36 + 18 (90 – 72)

36 = 2 · 18 + 0 !!!! **So GCF (3150, 306) = 18**

Since GCF (a,b) · LCM (a, b) = a · b for all whole numbers a and b, we know:

$$LCM(a, b) = \frac{a \cdot b}{GCF(a, b)}$$

$$LCM(3150, 306) = \frac{3150 \cdot 306}{GCF(3150, 306)}$$

$$\mathbf{LCM(3150, 306) = \frac{3150 \cdot 306}{18}}$$

$$= \frac{3150 \cdot \cancel{18} \cdot 17}{\cancel{18}}$$

$$= 53550$$

/6 11. Marie checks in at the office at 9:00 AM and every 72 minutes thereafter. Jean, from Qwik Copy Service, makes his first pickup at 9:00 AM and returns every 45 minutes. Both businesses close at 5 PM. Is there any other time during the day (after 9:00 AM) when Marie and Jean will be at the office simultaneously? (If so, name the time.)

Marie will be in the office at 9:00 10:12 11:24 12:36 13:48 etc.
Jean will be in the office at 9:00 9:45 10:30 11:15 12:30 etc.

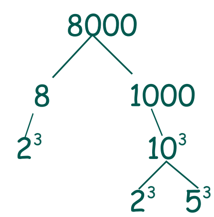
To find when they will be in the office again simultaneously, we need the LCM(72, 45)

72 = $2^3 \cdot 3^2$
45 = $3^2 \cdot 5$
LCM = $2^3 \cdot 3^2 \cdot 5 = 4 \cdot 9 \cdot 10 = 360$ Well that's nice: 360 minutes = 6 hours.

Marie & Jean will be in the office simultaneously again at 9:00 + 6 hours = 15:00 or 3 PM

/3 12. How many factors has 8000? Make a table listing them all.

$8000 = 2^6 \cdot 5^3$ So 8000 has (6+1) (3+1) = 7·4 = **28 factors.**



They are:

	2^0	2^1	2^2	2^3	2^4	2^5	2^6
5^0	**1**	**2**	**4**	**8**	**16**	**32**	**64**
5^1	**5**	**10**	**20**	**40**	**80**	**160**	**320**
5^2	**25**	**50**	**100**	**200**	**400**	**800**	**1600**
5^3	**125**	**250**	**500**	**1000**	**2000**	**4000**	**8000**

← **Factors of 8000**

/3 13. Write two composite numbers that are relatively prime.

Answers will vary. Here are some possibilities:

4 and 9
12 and 25
12 and 35
24 and 25

/3 14. If x and y and p and z are all whole numbers, and $2^x 3^3 5^y 7^8 = 6 \cdot 15^2 7^z$,
Find x and y and z.

By the Fundamental Theorem of Arithmetic, the whole numbers on each side, being equal, must have the same prime factorization. So we look at this through prime factors.

$2^X 3^3 5^y 7^8 = 6 \cdot 15^2 7^Z$ break the 6 and 15 into prime factors...

$2^X 3^3 5^y 7^8 = 2 \cdot 3 \cdot 3^2 5^2 7^Z$ then equate the powers on each side

$2^X = 2$ requires that **x = 1**

$5^y = 5^2$ says **y = 2**

$7^8 = 7^Z$ says **z = 8**

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