(18) 1a. Write the number 1426 in Egyptian. (Hint: | ∩ ϛ 𝄔)

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1b. Write 1 9 8 4 as a Roman numeral.

M CM LXXX IX

1c. Write the Hindu-Arabic numeral that is equivalent to the Roman numeral M DCC XL VII.

1 7 4 7

1d. Write the Hindu-Arabic numeral for the number we state as:
"ten trillion, six hundred four million, seventy-three thousand, eight hundred ninety-two"

10,000,000,000,000 + 604,000,000 + etc = **10,000,604,073,892**

1e. What is the place value of the 7 in 1,270,968 ?

The 7 represents 70,000. It is in the **ten-thousands** place.

1f. Using each of the digits in 12,407 only once, write the largest number :

The largest number using the digits 1&2&4&0&7 each once is **74210**

1g. Write the number $6\times10^6 + 3\times10^4 + 5\times10^3 + 7\times10^0$ in standard Hindu-Arabic form.

That's 6×1,000,000 + 3×10,000 + 5×1,000 + 7 = **6,035,007**

1h. Write the number 4,307 in expanded exponential form.

4000 + 300 + 7 = 4×1000 + 3×100 + 7×1 = **$4\times10^3 + 3\times10^2 + 7\times10^0$**

1i. Write the number 73,406 in ℓong form (in words, as you would read it).

Seventy-three thousand, four hundred six . **Notice there is no mention of "and".**

(15) 2. **State the property** illustrated by each statement. Do not use the term "any-order property".
State the full description of the property, including the operation: e.g. "commutative property of addition".

a. x·5 = 5x

a. **COMMUTATIVE property of MULTIPLICATION**

b. 78·99 = 78(100 − 1) = 78·100 − 78·1

b. **DISTRIBUTIVE property of MULTIPLICATION over SUBTRACTION**

c. 72 + (26 + 8) = 72 + (8 + 26)

c. **COMMUTATIVE property of ADDITION**

d. 1· 47,327,676 = 47,327,676

d. **IDENTITY property of MULTIPLICATION**

e. 999·1 = 999

e. **IDENTITY property of MULTIPLICATION**

(6) 3. Name the property that is being illustrated in each example:

a. The **COMMUTATIVE** property of **MULTIPLICATION**   b. The **COMMUTATIVE** property of **ADDITION**

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(4) 4. Give a specific counterexample, using only whole numbers, **15 – (8 – 7)   ≠   (15 – 8) – 7**
         to show that subtraction is not associative.
         Your example should fully demonstrate the different results. **15 – (1)   ≠   (7) – 7**

                                                                      14   ≠   0

(3) 5. Draw rectangular arrays illustrating the distributive property for 2×(3+4) = 2×3 + 2×4

2 × (3 + 4)   **2 × 3**   **2 × 4**
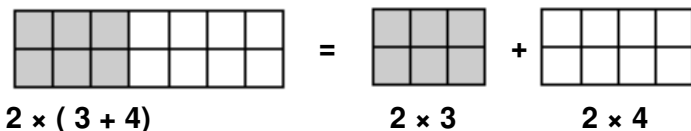
(3) 6. For each word problem given, state the interpretation of subtraction illustrated.
         Possible answers are: Part-Whole (missing addend)   Take-away   Comparison

     a. Amazingly, 15 of Ms. Smith's third-grade class of 27 were on the honor roll last month.
         How many were not on the honor roll last month?   **27**
         Type of problem:
                               Part-Whole (missing addend)   **15**   **?**
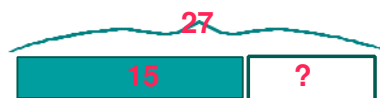
     b. Joni put a dozen apples on the back porch. Squirrels came in and stole five of the apples.
         How many are left?
         Type of problem: **Take-away**

     c. Frank earned 85 on the spelling test, and Ernest earned 60. How much higher is Frank's score than Ernest's?
         Type of problem:
                               Comparison
                                                                               ?
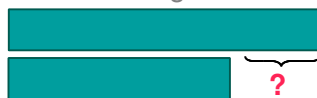
(2) 7. For each word problem given, state the interpretation of division illustrated.
         Possible answers are: Partitive Division (PD)   Measurement Division (MD)

     a. 72 eggs is how many dozen?
         Type of problem: **Measurement Division (MD)**   *"How many 12s in 72?"*

   b. We drove 3200 miles across the US in 5 days. What was our average distance per day?

         Type of problem: **Partitive Division (PD)**   *"3200 divided into 5 parts."*

(3) 8. This illustration shows measurement division for the problem 23 ÷ 5.
         Fill in the blanks with appropriate numbers to illustrate the Quotient-Remainder Theorem,
         as it applies to this division example.
         25 = 4 ·5 + 3   0   5   10   15   20 **23** 25
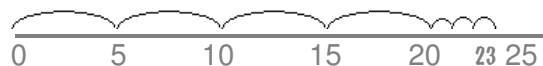
(4) 9. Calculate each of the following divisions. Use the definition of division ("missing factor") to show why that
         answer is correct. If the division is undefined, say so, and use the definition of division to explain why.

     a. 6 ÷ 2 = **3**   because: **2 · 3 = 6**

     b. 0 ÷ 2 = **0**   because: **2 · 0 = 0**

     c. 2 ÷ 0 **is undefined** because: **0 · ? = 2 has no solution, no answer**

     d. 0 ÷ 0 **is undefined** because: **0 · ? = 0 has many solutions, no unique answer; so it is meaningless**

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(12) 10. Illustrate Mental Math techniques to compute the following. Write your answers in a way that shows clearly the manner in which you computed the answer mentally.

a. $47 + 78 + 201 + 22 + 53$

$= 47 + 53 + 78 + 22 + 201$ **rearranging, using commutative & associative properties**
$= \quad 100 \quad + \quad 100 \quad + 201$ **to get these nice compatible combinations**
$= \qquad\qquad 301$

b. $847 - 298$

$= \quad 847^{+2} - \; 298^{+2}$ **compensation for subtraction**
$= \quad 849 \; - \; 300$
$= \quad 549$

c. $345 + 597$

$= \quad 345 \overset{\curvearrowright 3}{} + \; 597$ **compensation for addition**
$= \quad 342 \; + \; 600$
$= \quad 942$

d. 802×998

$= \quad 802 \times (1000 - 2)$
$= \quad 802000 - 1604$ **Distributive property (of × over −)**
$= \quad 800396$

e. 242×5 Alternately:

$= \quad 242 \times 10 \div 2$ **Special relationship between 5, 2 and 10** $= 242 \div 2 \times 10$
$= \quad 2420 \div 2$ **$5 = 10 \div 2$**
$= \quad 1210$ **So × 5 = × 10 ÷ 2**

f. $225 \div 5$

$= \quad 225 \; \times 2 \; \div \; 10$ **Special relationship between 5, 2 and 10** $= 242 \div 10 \times 2$
$= \quad 450 \; \div 10$ Notice the action just reverses the above
$= \quad 45$ **i.e. ÷ 5 = × 2 ÷ 10**

g. $4 \times 632 \times 25$

$= \quad (4 \times 25) \times \; 632$ **rearranging, using commutative & associative properties**
$= \quad 100 \times 632$
$= \quad\; 63200$

(7) 11. Use a Chip Model to illustrate fully the subtraction: $304 - 125$. Show the standard arithmetic alongside.

HUNDREDS	TENS	ONES
− 1	2	5
1	**7**	**9**

$$\begin{array}{r} \overset{9}{\cancel{2}\,\overset{10}{\cancel{\;0\;}}\,\overset{14}{\cancel{4}}} \\ - \; 1 \;\; 2 \;\; 5 \\ \hline 1 \;\; 7 \;\; 9 \end{array}$$

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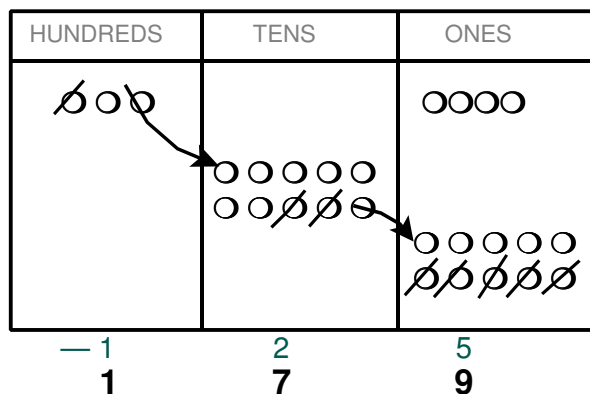
(6) 12. Determine whether each of the following is true (T) or false (F). $(24 \div 4) \div 2 \neq 24 \div (4 \div 2)$
If false, use a counterexample to demonstrate that it is not true.
a. Division is associative. T (F) $(6) \div 2 \neq 24 \div (2)$

FALSE, as demonstrated at right $3 \neq 12$

b. For all numbers, $(a+b)^2 = a^2 + b^2$ T (F) $(1+2)^2 \neq 1^2 + 2^2$

    $(\quad 3\quad)^2 \neq 1 + 4$

    $9 \quad \neq \quad 5$

c. Addition is closed on the set of odd numbers.   T (F)

1 is an odd number, and 3 is an odd number, but 1+3 is 4, not an odd number.
So to get the sum of these two odd numbers, we have to go outside the set of odd numbers.

(6) 13. When computing 37 × 62 using the standard multiplication algorithm, show what the two lines indicated represent (what are they)?   Show the same calculation using the lattice method. Show ALL steps.

```
  6 2
x 3 7
4 3 4  ←  7 × 62
1 8 6 0  ←  30 × 62
2 2 9 4
```

Lattice (6 2 across top; 2, 2 down right side):
```
      6     2
   1 / | 6 /
2 /  8 | /  6   3
   4 / | 1 /
2 /  2 | /  4   7
      9     4
```
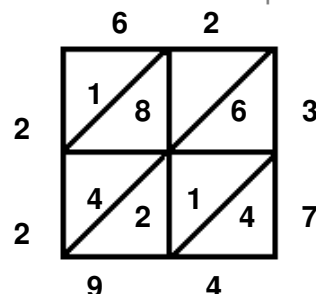
(5) 14. Compute 4362 ÷ 47 using long division. Show an estimation for the first digit of the quotient. (Even if the firs estimate is not correct, one estimate is all you need to show.) Show the correct Quotient and Remainder.

```
        9 2   ← Quotient
   47 ) 4 3 6 2
        4 2 3
        1 3 2
          9 4
          3 8   ← Remainder
```
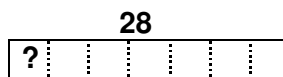
You can write: 92 R 38

Estimate:
```
        9
   50 ) 450
```

(5) 15. Make a word problem using partitive division for 28 ÷ 7. Illustrate the complete solution with a bar diagram.

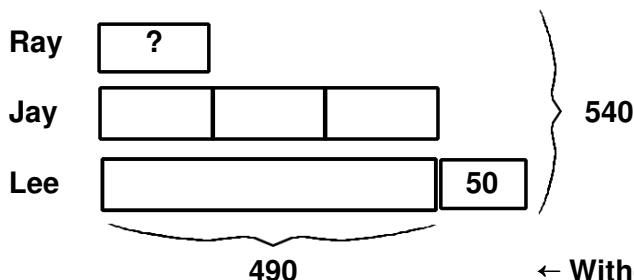**Word Problem:**
28 toys were divided equally among 7 tots.
How many toys did each tot receive?

28

? [bar diagram]

$28 \div 7 = 4$

Each tot received 4 toys.

(5) 16. Give a full "teacher's solution":   Jay, Ray and Lee collect stickers. Jay has 3 times as many stickers as Ray. Lee has 50 more than Jay.   They have 540 stickers altogether. How many stickers does Ray have?

Ray [?]

Jay [   ]   } 540

Lee [   ] 50

490 ← Without Lee's extra 50,
they have 540 – 50 stickers

```
  540
–  50
  490
```

$490 \div 7 = 70$

Ray has 70 stickers.

Check: **Ray has 70**
        Jay has 210
        Lee has 260

        Total: 540 ✔