Our text tells us doing "mental math" is valuable, lists techniques, and tells us we should practice each technique. Below is a list of techniques and problems. To understand why these techniques work, we need to know Place Value, the commutative and associative properties of addition and multiplication, and the distributive properties.

Rearranging:

$8 + 9 + 11 + 3 + 2 =$ $8+3 + 9+2 + 11 = 11 + 11 + 11 = 33$

$164 + 72 + 36 + 5 =$ $164 + 36 + 72+5 = 200 + 77 = 277$

$8 + 5 + 15 + 32 + 12 =$

$4 \times 3 \times 11 \times 5 =$

Splitting numbers:

$9 \times 70 =$ $9 \times 7 \times 10 =$ (You probably do this all the time, automatically.)

$5 \times 311 = 5 \times (300 + 11) =$

$5 \times 220 = 5 \times 2 \times 110 =$

Compatible numbers:

...a bit redundant, since in many of the examples above we combine certain numbers to get "nice" results.

$6 \times 55 =$ $2 \times 3 \times 5 \times 11 = 2 \times 5 \times 3 \times 11$

$5 + 37 + 35 + 18 =$ $5 + 35 + 37 + 18$

$14 \times 16 + 7 \times 8 =$ $7 \times 32 + 7 \times 8$

$8 \times 35 \times 55 =$

$4^3 + 6 \cdot 4^2 =$

$1 + 2 + 3 + 4 + \cdots + 96 + 97 + 98 + 99 + 100 =$

Compensation:

Addition & Multiplication – one number gives to the other.
Subtraction & Division– do the same thing to each number (so that the second one is nice).

ADDITION: $96 + 46 =$ $96 + 4 + 42 =$ (Steal part of 46, to boost 96 up to 100)

SUBTRACTION: $96 - 37 =$ $99 - 40 =$ (Shift both numbers up or down, to make the subtrahend "nice".)

MULTIPLICATION: $55 \times 72 =$ $55 \times 2 \times 36 = 110 \times 36 =$ (Steal a factor 2 from 72 to boost 55 .)

DIVISION: $840 \div 14 =$ $420 \div 7 =$ (Reduce both dividend & divisor by common factor, or

$730 \div 50 =$ boost both dividend and divisor by common factor.)

$140 \div 5 =$

Left-to-right calculation:

$239 + 647 =$ $800 + 70 + 16 =$

$508 + 407 =$

(with anticipation):

$$\begin{array}{r} 239 \\ +\ 647 \end{array}$$

$$\begin{array}{r} 653 \\ +\ 364 \end{array}$$

The fact that our numeration system is place value base TEN means that we can use certain "tricks" for numbers that are close to powers of ten. For example:

9 x 34 = (10 - 1) x 34 = 340 - 34 =

11 x 34 = (10 + 1) x 34 = 340 + 34 =

99 x 12 =

Notice we are relying heavily on the distributive property in the above examples. More:

165 ÷ 15 =

286 ÷ 13 =

135 ÷ 15 =

121 ÷ 11 =

More tricks of the trade:

2 x 5 = 10 so 5 = 10 ÷ 2. To multiply by 5, multiply by 10, then halve that result.
To divide by 5, double the number, then divide by 10.

Doubling and halving is relatively easy. Since 4 = 2 x2 , to multiply by 4, double the number twice.
To divide by 4, halve the number twice.

More examples, mixed:

120 ÷ 5 =

4 x 13 =

80 ÷ 5 =

184 ÷ 8 =

900 + 120 =

90 x 400 =

37 x 101 =

48 x 1002 =

34 x 98 =

108 ÷ 9 =

47 + 25 + 87 + 53 + 75 =

2 x 3 x 25 x 66 =