Definition: DECIMAL NUMBERS

If x is any whole number, $x\,.\,d_1 d_2 d_3 d_4 \ldots$
represents the number: $x + d_1/10 + d_2/100 + d_3/1000 + d_4/10000 + d_5/100000 \ldots$
(which is also known as: $x + d_1{\times}10^{-1} + d_2{\times}10^{-2} + d_3{\times}10^{-3} + d_4{\times}10^{-4} + d_5{\times}10^{-5} \ldots$)
so, e.g.: 321.50678 = 321 + 5/10 + 0/100 + 6/1000 + 7/10000 + 8/100000
fully expanded: $3{\times}10^2 + 2{\times}10^1 + 1{\times}10^0 + 5{\times}10^{-1} + 0{\times}10^{-2} + 6{\times}10^{-3} + 7{\times}10^{-4} + 8{\times}10^{-5}$

The "**.**" (in 321.50678, e.g.) is called the **decimal point**.
The place values of the digits to the *right* of the decimal point are *tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, etc.* *(Not to be confused with tens, hundreds, thousands, etc.)*
In general: **the place value** of the digit in the **nth decimal place (*n* places after the decimal point) is 10^{-n}.**
Notice this is not symmetric... the first place to the left of the decimal point is $\mathbf{10^0}$, not $\mathbf{10^1}$ ...!

Read and expand into *long form* (words): 786.3007

OPERATIONS ON DECIMAL NUMBERS

We *know how* to add, subtract, multiply and divide decimals; we now look at *why*.

Add: 203.602 + 3.47

```
  203.602   =    200 + 3 + 6/10 + 0/100 + 2/1000
+   3.47    +          3 + 4/10 + 7/100
```

CHIP MODEL MAY HELP!

Subtract 23.5 – 3.47

```
  23.5          23.5         23 + 5/10 + 0/100
– 3.47      –    3.47     —(  3 + 4/10 + 7/100 )
```

...What's wrong with this picture?

Multiplication & Division by 10's:

2.17 x 10 = (2 + 1/10 + 7/100) x 10
= 2x10 + (1/10)x10 + (7/100)x10
= 2x10 + 1 + 7/10 = 21.7

Similarly, 2.17×10^2 = (2 + 1/10 + 7/100) x 100 = 2·100 + (1/10)·100 + (7/100)·100 = 200 + 10 + 7
= 217

21.7×10^{-1} = 21.7 x 1/10 = 21.7 ÷ 10
= (20 + 1 + 7/10) ÷ 10
= 20÷10 + 1÷10 + (7/10) ÷10
= 2 + .1 + .07 = 2.17

Similarly, 21.7×10^{-2} = 21.7 x 1/100 = 21.7 ÷ 100 = .217

MULTIPLICATION BY 10 INCREASES THE PLACE VALUE OF EACH DIGIT (BY ONE PLACE).

DIVISION BY 10 DOES THE OPPOSITE.

Making sense out of the multiplication and division algorithms:

```
   23.5      (235 x 1/10)
 x 2.17      x (217 x 1/100)
 1 6 4 5
 2 3 5
4 7 0
5 0 9 9 5    = (217 x 235 ) x (1/10) x (1/100)
             = (217 x 235 ) x (1/1000)
             = (217 x 235 ) / (10^3)
```

15.2)4680.08

$$\frac{4680.08}{15.2} \times \frac{10}{10} = \frac{46800.8}{152}$$

***Estimate** the total!*

Rounding decimals:
Round 4.367 to the nearest *hundredth*. The nearest two decimals in hundredths are 4.36 and 4.37.
The nearer is 4.37, since it is .003 greater than 4.367, as opposed to 4.36, which is .007 less than 4.367.
This is apparent from a number line view:

4.360	4.361	4.362	4.363	4.364	4.365	4.366	**4.367**	4.368	4.369	**4.370**
4.36										**4.37**

Now round 4.3645 to the nearest hundredth:
4.3645 either rounds down to 4.36, or up to 4.37: 4.3645

4.37
—————— 4.365 —
4.36

When a number is *rounded*, it has been replaced with a nearby, but less precise, value.
The *difference* between the value before rounding and the rounded-off value is called the *rounding error*.

The *problems* with rounding: *How do you round 4.365 to the nearest hundredth?* The nearest numbers terminating in the hundredths' place are 4.36 and 4.37, both .005 from 4.365. There are three "rounding rules" that may be adopted to cover such instances:

1) **Round to nearest even:** The most widely adopted rule. Favored because about half the time you round up, half the time you round down; so a long series of rounding tends to average out the errors to zero.
2) **Round to nearest odd:** Same as above, but opposite, for contrary people.
3) **Round up:** This is the simplest because any decimal with "5" in the next place (after the one to which the number is being rounded) gets rounded up. The disadvantage to this rule is that by always rounding halfway-points up, totals are too high on the average. If you're the tax collector, you round up!

_Eg. Round 19539.98748 to the *nearest*: ______

thousandth	*tenth*	*unit*	*hundred*	*thousand*
19539.987	19540.0	19540.	19500	20000

Caution:
Don't round "piecemeal":

For example, Teddy rounded 4.97448 to 4.98 this way:
4.97448 ≈ 4.9745 ≈ 4.975 ≈ 4.98

Help Teddy: If 4.97448 is to be rounded to the *nearest* hundredth...
4.97448 will either truncate to 4.97 or round up to 4.98—which of these is closer to 4.97448?
What is the halfway point between 4.97 and 4.98? ...is 4.97448 *above* or *below* that "halfway point"?

Caution:
Don't round too soon: ...especially when using a calculator, don't round until the computation is complete! Calculators carry extra digits "for free", in most instances. This method minimizes error from rounding. ( )

For example, to compute 3.50 x $\sqrt{2}$, since 3.50 expresses 3 digits of precision, we wish to have three digits in the final result. If we round the square root of 2 too early, we risk an erroneous answer. $\sqrt{2} \approx$ 1.414213562. If we round to 1.41, then multiply by 3.50, we obtain 4.935, which we round to 4.94; however, if we carry the extra decimals and multiply by 3.50, we obtain 4.9497468, which rounds to 4.95 (even 1.414 x 3.50 will give this result). **Conclusion: *don't round too soon.***

Caution:
Express correctly! Round 8,498.98506 to the nearest *tenth*. 8,500.0 **Round 8,498.98506 to the nearest ten.** 8,500

A number which has been rounded to 3400 is between 3350 & 3450.
A number which has been rounded to 3400. or 3400 is between 3399.5 & 3400.5

this material was discussed weeks ago, but it makes sense to review it now

540000000000000 is difficult to *see*. We write, alternately, 540,000,000,000,000 to help count the zeroes and figure the place value. Even so, most do not fully comprehend the words "billion" and "trillion". Thus, to many of us, reading or hearing the number above is five hundred forty trillion is not illuminating. Very large numbers (as well as very small numbers) written in our traditional numeration system are cumbersome to write, read and use. For purposes of expressing, multiplying and dividing such numbers, alternate means are often employed, most commonly **scientific notation**—write as ***x.ddd... x*** $\boldsymbol{10^{p}}$.

(where x may not be 0.)

Use of scientific notation requires:

- recognition of place value expressed in exponential form,
- ability to round decimal numbers,
- full command of the arithmetic properties of multiplication and division,
- understanding of significant digits (this relates to rounding).

Write in scientific notation form. (Check by multiplying!)

Examples:

$7{,}000{,}000 = 7 \times 10^{6}$

7,650,000 = $\mathbf{7}.65 \mathbf{\times 10^{6}}$

$.000007 = 7 \times 10^{-6}$

.0**1**29 = $\mathbf{1}.29 \mathbf{\times 10^{-2}}$

Key idea: *Focus on the **leading digit**. If it is in the right place, the rest will follow!*

Try it: 540,000,000,000 = 5.4 x 10 ⇦ *Fill in the*

.00000713 = 7.13 x 10 ⇦ *exponents*

.0001 =

$10.02 \times 10^{4} =$

... And be able to convert from Scientific Notation to Standard Form:

$3.784 \times 10^{3} =$	$3.702 \times 10^{-3} =$	$4.50 \times 10^{-2} =$	$4.50 \times 10^{3} =$
~~3784~~	0.003702	0.0450	4500

Arithmetic in Scientific Notation:

$(3.12 \times 10^{7})\times(4.25 \times 10^{-4}) = (3.12 \times 4.25) \times (10^{7} \times 10^{-4}) \doteq$ ______ (1.326×10^{4})

* $(4.34 \times 10^{3})^{3} / (2.728 \times 10^{-2}) = (4.34^{3}/2.728) \times (10^{9}/10^{-2})$

$= 29.965727\cdots \times 10^{11} \doteq$ ______

30.0×10^{12}

⇨ ***No knees!***: * $5.67 \times 10^{14} + 3.33 \times 10^{15} \doteq$ ______

$3.897 \times 10^{15} \doteq 3.90 \times 10^{15}$

⇨ **Use your calculator** to assist in computing:

⇨ * $(5.407 \times 10^{7})^{2} \times (3.66 \times 10^{-4}) \div (1.3311 \times 10^{2})^{3} =$ ______

45.369268×10^{4}

⇨ Express the result with correct number of significant digits in scientific notation.

($45.369268 \times 10^{4} \doteq 4.54 \times 10^{5}$)

Note there are three forms of decimals: *terminating (.025)*
non-terminating repeating (.3333…) and
non-terminating non-repeating (.417411741117411117…)

★ **RATIONAL NUMBERS MAY BE EXPRESSED AS DECIMALS**

Converting rational fraction to decimal form: Use the *long division algorithm*. E.g.

$\frac{7}{10}$ = ______ $\frac{1}{2}$ = ______ $\frac{3}{4}$ = ______ $\frac{3}{25}$ = ______ $\frac{1}{125}$ = ______ $\frac{9}{16}$ = ______ $\frac{3}{64}$ = ______

$\frac{2}{3}$ = ______ $\frac{5}{9}$ = ______ $\frac{4}{7}$ = ______ $\frac{2}{11}$ = ______ $\frac{5}{13}$ = ______ $\frac{1}{75}$ = ______ $\frac{1}{19}$ = ______

A rational fraction in reduced form is equivalent to a terminating decimal if, and only if, the denominator (of the reduced fraction) has *only 2's and/or 5's as prime factors.* For example, 1/125 terminates in three places because

$\frac{1}{125} = \frac{1}{5\cdot5\cdot5} = \frac{1\cdot2\cdot2\cdot2}{5\cdot5\cdot5\cdot2\cdot2\cdot2} = \frac{8}{1000} = .008$ … which happens because 10 is a multiple of 5 (… and, thus, 1000 is a multiple of 5^3)

If a rational fraction in reduced form has denominator with prime factors other than 2 and 5, the decimal does not terminate. For example: Can $\frac{1}{3}$ be written as $\frac{x}{10}$? … as $\frac{x}{100}$? … $\frac{x}{1000}$?

Rationals written as decimals may be TERMINATING *(e.g. 3/5 = .6)* or NON-TERMINATING *(e.g. 3/11 = .27272⋯).*
Rationals which, reduced, have denominators with *prime factors 2 & 5 only* are **TERMINATING** decimals because:

Other rationals have NON-TERMINATING, **REPEATING** decimal expansions—because:

★ **A DECIMAL NUMERAL MAY BE EXPRESSED AS RATIO OF TWO INTEGERS, IF…**

★ **THE DECIMAL IS TERMINATING:**

A terminating decimal has a finite number of digits to the right of the decimal point.
Such decimals are easily expressed in rational form (ratio of two integers); just read them aloud.

E.g.: 3.47 = 3 + 4/10 + 7/100 = 300/100 + 40/100 + 7/100 = (300+40+7)/100 = 347/100

1.6 =

You can make even *shorter* work of such conversions: 52.0302 = 520302/______
If a decimal number terminates in the nth place after the decimal, it is equivalent to a fraction whose numerator is the number formed by the digits without the decimal point, and denominator 10^n (where n is number of digits after the decimal point) .

★ **THE DECIMAL IS NON-TERMINATING REPEATING:**

Call the repeating ("unknown") number *x*.
Multiply *x* by 10^n where *n* is the length of the repetend (the number of digits that repeat).
Subtract *x* from $10^n x$ to obtain $(10^n-1)x$.
Solve for *x*; multiply resulting fraction by $10^p/10^p$ if necessary to clear a decimal.

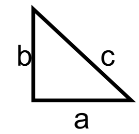
For example: *x* = 52.1909090909090…

$100x$ = 5219.090909090…
$-\ x$ = 52.190909090…
$99x$ =

so $x = \frac{5166.9}{99}$ = ______ = ______

By the way, notice the expression of a rational number as a decimal is not unique. $.\overline{9}$ = ______
Every terminating decimal has an alternate form. EG 5.687 = 5.686999999999⋯

THE SET OF TERMINATING & REPEATING DECIMALS IS EQUIVALENT TO THE SET OF RATIONAL NUMBERS



The Pythagoreans[1] believed that "Number rules the universe"; *everything* can be described in terms of numbers, and, further, that all numbers are rational– integers, or ratios of integers.
Pythagoras proved[2] what is called the **Pythagorean Theorem**[3]:
$a^2 = b^2 + c^2$ **the sum of the squares of the sides of a right triangle is the square of the hypotenuse.** (and this happens only in a right triangle.)

Pythagoras' view of the describability of the universe in terms of rational numbers was contradicted–*destroyed*–by the very theorem which now bears his name. For if we construct a right triangle with sides of equal length 1, then we can demonstrate that the hypotenuse (*whose length ought to correspond to* ***some*** *number*), is $\sqrt{2}$, and $\sqrt{2}$ **cannot be expressed as a rational number**—cannot be expressed as the ratio of two integers. As we argue below:

First we note that every whole number has a unique prime factorization. $w = p_1^{r_1} p_2^{r_2} p_3^{r_3} p_4^{r_4} \cdots p_k^{r_k}$

Then w^2 must have a prime factorization with the same primes to even powers: $w^2 = p_1^{2r_1} p_2^{2r_2} p_3^{2r_3} p_4^{2r_4} \cdots p_k^{2r_k}$

Suppose $\sqrt{2} = p/q$

where $p,q \in Z$, $q \neq 0$ [after all, that's what it would mean for $\sqrt{2}$ to be rational].

... and we may assume p/q *has been reduced to lowest terms* (so p and q have NO common factors).

Multiplying both sides by q and squaring gives us $2q^2 = p^2$.

Which means 2 must be a factor of p^2, which in turn implies 2 must be a factor of p.

So p^2 has an odd power of 2 in its prime factorization... This is not possible.

Since every part of our argument is true, the only flaw must be in the assumption we made in the first place, that $\sqrt{2}$ can be written in rational form.]

Pythagoras and his followers were devastated. Even today, we are troubled to discover there are numbers
which cannot be written in the friendly form of a ratio of integers—but the irritation is diminished by the comforting thought that although such numbers exist, they are relatively few, and may be mostly ignored. NOT!

Let's make a small digression. We now know these irrationals exist, and $\sqrt{2}$ is one of them, and can *not* be written as a ratio of integers; but what *is* the nature of an irrational? What is the decimal representation of an irrational number? We have previously seen that a *rational* number can be expressed in decimal form using the division algorithm, and that the decimal form either terminates or repeats.
Furthermore, any terminating or repeating decimal can be expressed as a ratio of two integers.

Therefore: **The set of rational numbers is exactly the set of terminating and repeating decimals**.

The Irrationals (*reals not expressible as ratio of two integers*) **are non-terminating, non-repeating decimals**.

Consider, e.g. 1.10200300040000500000600000070000000800000000900000000010000000000011...
... or 1.12123123412345123456123456712345678123456789123456789101234567891011...

The sum of two rationals is rational. What about the sum of two irrationals?
This can be a little difficult to explore, since we have no general arithmetic algorithms for irrationals.
What is $\sqrt{2} + \sqrt{3}$? (Hint: square that!)
Is the sum of two irrationals always irrational? ...How about $\sqrt{2} + \sqrt{8}$? $-3\sqrt{2} + \sqrt{18}$?
What is the sum of a rational number and an irrational? Consider, for example, $\sqrt{2} + ½$.
Intuitively, we have a non-terminating non-repeating decimal plus .5, the total of which should be non-terminating non-repeating. However, we look for proof....

[1] students/disciples of the school of Pythagoras at Crotona in Southern Italy

[2] ... or so he is credited. Pythagoras may have proved it, perhaps one or some of his student followers.

[3] though it was known to the Babylonians & Egyptians at least centuries earlier

We can easily *prove* $\sqrt{2}$ + ½ is irrational:

Suppose $\sqrt{2}$ + ½ = a/b … where a/b is rational.
Then $\sqrt{2}$ = a/b – ½ which, since it is the difference of two rational numbers, is a rational number.
This contradicts the known status of $\sqrt{2}$ – it's irrational.

The argument above showing $\sqrt{2}$ is irrational may be used to show that the square root of any prime (or of any composite number whose prime factors are not all to an even power) is not rational. Furthermore, there are other irrational numbers (*pi* and *e* for instance) that "occur" in numerous circumstances. In addition, for every irrational, we can construct a whole family of irrationals by adding any rational. (Irrational + rational is ...) Thus we can easily see there are at least as many irrational numbers as rationals. It was not until the end of the nineteenth century that a means of tackling the cardinality of infinite sets was devised to settle this question: **How does the size of the set of irrationals compare with the size of the set of rationals?**

Georg Cantor[4] was primarily responsible for the development of theory of sets & classes, with particular emphasis on the infinite. He devised the means of comparing cardinality of sets by mapping, or establishing a 1-1 correspondence between two sets to show they are of the same cardinality. (Recall our demonstrations that the cardinality of N is equivalent to that of Z.) Although the rationals are *dense* in the number line, the cardinality of Q is the same as that of N! But the cardinality of the irrationals is greater! Here is Cantor's ingeniously simple proof:

Any set whose cardinality is that of N can be *listed*. (The act of *listing* establishes a 1-1 correspondence with N.) Suppose a "list" of all irrationals is presented. We will show that the list does not—*CANNOT*—contain all the irrationals, by constructing an irrational number that is not in the list.

Suppose the first few numbers in the alleged list are:	$m.d_1d_2d_3d_4d_5\ldots$	*e.g.*	*10.**1**450368…*	**.3**
	$n.e_1e_2e_3e_4e_5\ldots$		*4.2**9**07863…*	**7**
	$o.f_1f_2f_3f_4f_5\ldots$		*.00**0**6721…*	**2**
	$p.g_1g_2g_3g_4g_5\ldots$		*.033**2**737…*	**4**
				⋱

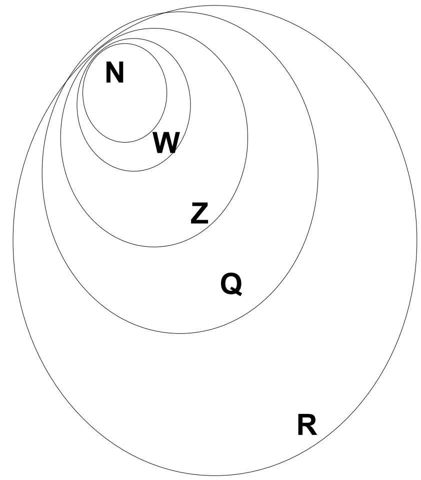
We select a digit different from d_1 as the first decimal digit of our number; select a digit different from e_2 as the second digit of our number; select a digit different from f_3 as the third digit, and so on. The number so constructed cannot be the first number in the list because it differs in the first decimal place; cannot be the second number in the list because it differs in the second decimal place from that number; and so on. So it is *not in the list at all!* Thus the list is not complete.

Therefore, it is **not possible to list** all the irrationals, and thus they cannot be put into 1-1 correspondence with the natural numbers. **The cardinality of the set of irrationals is greater than the cardinality of N.** *Thus although the rationals are dense in the number line (no interval gaps), there are other numbers (irrationals) in the number line, and they far outnumber the rationals!*

Properties of Arithmetic Operations on REAL numbers:
Addition and multiplication on the set of all real numbers have the following properties:
CLOSURE; COMMUTATIVITY; ASSOCIATIVITY; IDENTITY (0 for + ; 1 for x);
INVERSES for + : for any decimal a, -a is the additive inverse.
INVERSES for x:
for each decimal b other than 0, there is another decimal number, 1/b such that bx(1/b) = 1
Subtraction and Division have the closure property except for division by 0.

[4] *1845-1918* Cantor's theory of the infinite so rocked the scientific and, particularly, mathematical world of his time that some antagonists were able to block his advancement; he was viewed as subversive by some, unbalanced by others. The bitterness of his life, coupled with insecurity bred in Cantor, and perhaps some genetic predisposition, led to a number of breakdowns for which Cantor was hospitalized. In the early twentieth century his work began to be recognized as the profoundly real work of a genius; this recognition was too little and too late.

‼ Our new Universe: $N \subsetneq W \subsetneq Z \subsetneq Q \subsetneq R$



‼ N = $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \cdots \}$

‼ W = $\{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \cdots \}$

‼ Z = $\{ \cdots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \cdots \}$

‼ Q = $\{ {}^{p}/_{q} \mid p \in Z, q \in Z, \text{ and } q \neq 0 \}$
= $\{ 0, {}^{1}/_{1}, {}^{-1}/_{1}, {}^{-1}/_{2}, {}^{1}/_{2}, {}^{1}/_{3}, {}^{-1}/_{3}, {}^{2}/_{3}, {}^{2}/_{1}, {}^{-2}/_{1}, {}^{-2}/_{3}, \cdots \}$
= The set of all terminating and repeating decimals

‼ R = The set of all decimals
= The set containing all rationals and all irrationals
= $Q \cup \mathcal{I}$
= Set of all decimals that terminate or repeat $\cup$ Set of all nonterminating nonrepeating decimals
= { Set of all decimals that *can* be written as a ratio of integers } $\cup$ { Set of all decimals that *cannot* be written as a ratio of integers }

Note:
If we think of R as our new Universe, $\mathcal{I}$, the set of irrationals, is the complement of Q, the set of rationals.
Every real number is rational or irrational– one or the other/

Trichotomy:
Given any two real numbers *a* and *b*, exactly one of the following holds: $a < b$ or $a > b$ or $a = b$.
In particular, given any real number *r*, then either *r* is positive ($r > 0$), or *r* is negative ($r < 0$), or *r* is 0.

Some important details:

- The irrationals form a set disjoint (separate) from the rationals.
- Together the rationals and irrationals make up R, the set of real numbers.
- $N \subseteq W \subseteq Z \subseteq Q \subseteq R$. (Each of these is contained in —is part of —the next.)
- The set of real numbers is **larger** than the set of rationals, and is dense in the number line!
- The set of real numbers can be thought of as corresponding to the points (all points) on a line.

(The set of real numbers may also be thought of as the set of all decimal numbers, including those which terminate or repeat (rationals) and those which neither terminate nor repeat (irrationals).)

Radicals & Fractional exponents:
√, called **radical**, stands for the *principle square root*—the *positive root* of a positive number.
To indicate the negative of the square root, we write $-\sqrt{\ }$. For instance, $\sqrt{16} = 4$. $-\sqrt{16} = -4$.
$\sqrt{x}$ may be written in the form $x^{1/2}$.

Notice that

$\sqrt{x}\,\sqrt{x} = x$ (provided $\sqrt{x}$ exists)... And this can also be written
$x^{1/2}\ x^{1/2} = x$

(STRICTLY Optional !!) More Radicals & Fractional exponents:

The √, or radical, symbol is used to denote other roots of numbers, such as cube root: $\sqrt[3]{27} = 3$

We define: $\sqrt[n]{\ }$ denotes the **nth root** of a number. For instance, $\sqrt[6]{64} = 2$ because $2^6 = 64$.
Just as $\sqrt{x}$ may be written in the form $x^{1/2}$, $\sqrt[3]{x}$ is $x^{1/3}$ and in general: $x^{1/q} = \sqrt[q]{x}$.

...and *more* generally: $x^{p/q} = (x^{1/q})^p = (\sqrt[q]{x})^p$.

E.g. $8^{1/3} = \sqrt[3]{8} = 2$ $\qquad 32^{2/5} = (32^{1/5})^2 = (2)^2 = 4$

$8^{-1/3} = 1/\sqrt[3]{8} = 1/2$ $\qquad 32^{-2/5} = 1/(32^{1/5})^2 = 1/(2)^2 = 1/4$

QUIZ YOURSELF! (1-15 simplify; 16-20 answer rational, irrational, unpredictable)(* = *optional !)*

1*. $16^{1/4} =$ ____ **2*.** $32^{1/5} =$ ____ **3*.** $27^{1/3} =$ ____ **4*.** $64^{1/3} =$ ____

5. $4^{-1/2} =$ ____ **6.** $36 \cdot 36^{-1/2} =$ ____ **7.** $7^{1/2} \cdot 7^{1/2} =$ ____ **8*.** $4^{3/2} =$ ____

9*. $({}^a/_b)^{-1/2} =$ ____ **10*.** $(16/9)^{-1/2} =$ ____ **11.** $\sqrt{x^6} =$ ____ **12.** $\sqrt{72a^6b^7} =$ __

13. $\sqrt{72} + \sqrt{18} =$ ____ **14.** $\sqrt{36 \cdot 2^3 \cdot x^{12} y^3} =$ ____ **15.** $\{ 64x^5y^6 \cdot (x+y)^8 \}^{1/2} =$ _______

16. The sum of a rational and an irrational is:

17. The product of a rational and irrational is:

18. The sum (difference) of two rationals is:

19. The sum (difference) of two irrationals is:

20. The product (quotient) of two rationals is:

21. The product (quotient) of two irrationals is:

¿Answers?:

1. 2 **2**. 2 **3**. 3 **4**. 4 **5**. 1/2 **6**. 6 **7**. 7 **8**. 8 **9**. $({}^b/_a)^{1/2}$ or ${}^{\sqrt{b}}/_{\sqrt{a}}$ **10**. 3/4 **11**. $\sqrt{(x^3)^2} = x^3$

12. $\sqrt{8 \cdot 9a^6b^7} = \sqrt{4 \cdot 9a^6b^6 2b} = 2 \cdot 3a^3b^3\sqrt{2b}$ **13**. $\sqrt{9 \cdot 4 \cdot 2} + \sqrt{9 \cdot 2} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

14. $\sqrt{2^2 \cdot 3^2 \cdot 2^3 \cdot x^{12} y^3} = \sqrt{2^5 \cdot 3^2 \cdot (x^6)^2 \cdot y^3} = 4 \cdot 3x^6y\sqrt{2y} = 12x^6y\sqrt{2y}$

15. $\{ 64x^5y^6 \cdot (x+y)^6 \}^{1/2} = \{ 2^6x^5y^6 \cdot (x+y)^6 \}^{1/2} = 8x^2y^3(x+y)^4\{x\}^{1/2}$ **16**. irrational (see prior discussion)

17. if the rational is zero, the product is rational because it's 0; otherwise, irrational.
(*Consider*, *eg*, $2 \cdot \sqrt{2}$, and.... If $q \in Q$ and $q \cdot b = p \in Q$ then $b = p/q$ must be rational unless q is zero.)

18. rational (+ or −) **19**. unpredictable (+ or −) (*Consider* $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$; $(9 - \sqrt{2}) + (\sqrt{2} - 5) = 4$)

20. rational (· or ÷) **21**. unpredictable (· or ÷) (*Consider* $\sqrt{2} \cdot \sqrt{2} = 2$; $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$)