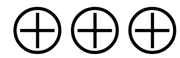
Fractions arise to express PART of a UNIT

1 What part of an HOUR is thirty minutes? Fifteen minutes? twelve minutes?
(The UNIT here is HOUR.)

2 What *fraction* of the children are happy? ☺ ☹ ☺ ☹ ☺ ☹ ☺ ☹
(The UNIT here is the entire GROUP or set of children.)
In this example, we may also perceive the fraction as a *ratio*.
One person might say *half* or *four-eighths* the children are happy.
Another might say *four* (are happy), of *eight altogether*, or *one of every two*.

3 It's 12 miles to Nana's house. We have driven 9 miles. What part of the drive have we covered?
(The UNIT here is the distance to Nana's house.)
We compare 9 to 12.

4 Four children must divide three pies among them. How much of a PIE does each one get?
(The UNIT here is a PIE.)



Most generally, a fraction is used to express part of a whole.
The denominator tells us into how many equal parts the whole was divided.
The numerator tells us how many of those parts we have.

$\frac{3}{4}$ 3 ← numerator, 4 ← denominator

♦**Definition:** The set of **rational numbers**, denoted ***Q***, is $\{{}^a/_b \mid a \in \mathbb{Z} \text{ and } b \in \mathbb{Z} \text{ and } b \neq 0\}$.
The **numerator** of ${}^a/_b$ is defined to be *a*, the **denominator** of ${}^a/_b$ is *b*.

The concept of equivalent fractions is encoded in the

♦**FUNDAMENTAL LAW OF FRACTIONS:** **if c ≠ 0,** $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

FRACTION STRIPS:

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

In grade school this is called RENAMING fractions.
For example, 1/2 = (1·4)/(2·4) = 4/8, illustrated at right:
1/2 is the *simplest* or *reduced* form of the fraction.
A rational fraction is said to be in **SIMPLEST** or **reduced form** when the *numerator and denominator are relatively prime.*
(Special case: if the denominator is 1, we usually omit writing the denominator.)

After we define multiplication !

♦**Reduce:** $\frac{72}{27} = \frac{8 \cdot 9}{3 \cdot 9} = \frac{8}{3} \cdot \frac{9}{9} = \frac{8}{3} \cdot 1 =$

$\frac{960}{420} =$

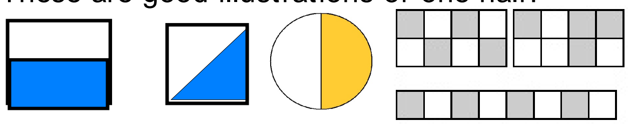
COMPARING FRACTIONS:

Are $\frac{3}{7}$ and $\frac{4}{7}$ equivalent? $\frac{3}{4}$ and $\frac{2}{3}$? $\frac{72}{27}$ and $\frac{480}{210}$? $\frac{130}{195}$ and $\frac{42}{63}$?

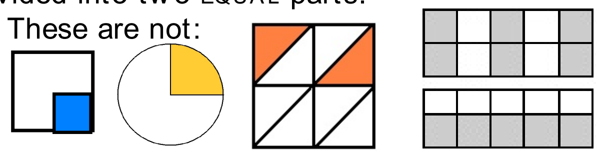
♦**How we know that a/b and c/d are equivalent: Reduce, express with common denominator.**

What about when they are NOT equivalent? How do we put them in order?
(1) use common denominator (then compare numerators)
(2) compare to easily recognized fractions
(3) if the numerators are the same, compare denominators. (Larger denominator⇒smaller fraction!)
(Would you rather have ½ or ⅛ of $1million?)

Illustrating fractions with area models.
To illustrate one-half, it is important that the UNIT be divided into two EQUAL parts.
These are good illustrations of one-half:



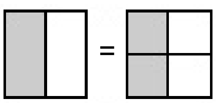
These are not:



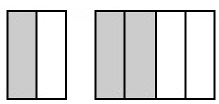
In different discussions of fractions the unit may vary, but one common unit should be used throughout a particular discussion. Diagrams used to illustrate addition, subtraction, multiplication and division, as well as fraction equivalence and comparison should follow suit. For instance, to show equivalence, use ONE unit and subdivide:

$^1/_2 = {}^2/_4$

use:

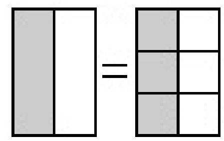


do NOT use:

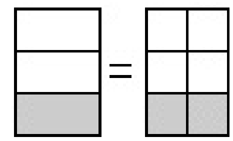


Rectangles aren't "glamorous", but they're easy to divide two ways!

$^1/_2 = {}^3/_6$

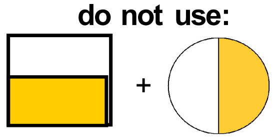


$^1/_3 = {}^2/_6$



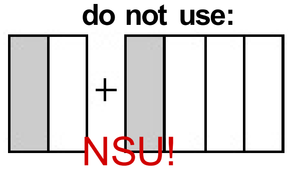
To model the addition of

½ + ½
do not use:

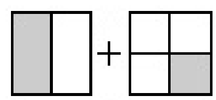


↖Not same unit!↗

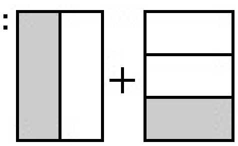
½ + ¼
do not use:



½ + ¼
use:



½ + ⅓
use:



↖Same unit!↗

Measurement models are very similar to area models, but are not necessarily 2-dimensional.

Set models are rarely used for comparisons or illustrating arithmetic operations. (Usually "find ⅔ of N")

☆☆☆☆ ☆☆☆☆ ☆☆☆☆
☆☆☆☆ ☆☆☆☆ ☆☆☆☆

Many fractions do not exist in W, the set of whole numbers.
They belong to the set of "rational numbers", which includes the whole numbers, and all these new ratios.
To save space, we will refer to the set of fractions as "Q".

Getting ready for addition & subtraction of fractions:

We start with common denominators:

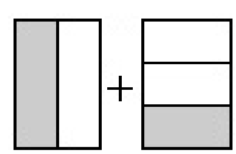
$\frac{1}{3} + \frac{1}{3}$ $\qquad$ $\frac{2}{5} - \frac{1}{5}$ $\qquad$ $\frac{1}{4} + \frac{2}{4}$ $\qquad$ $\frac{1}{2} + \frac{1}{2}$ $\qquad$ $\frac{1}{3} + \frac{1}{3}$

We step up to denominators that are "compatible", as one divides the other:

$\frac{1}{2} + \frac{1}{4}$ $\qquad$ $\frac{3}{4} - \frac{1}{2}$ $\qquad$ $\frac{1}{2} - \frac{1}{6}$

Until we are ready for the general case:

$\frac{1}{2} + \frac{1}{3}$ $\qquad$ $\frac{1}{2} - \frac{1}{3}$ $\qquad$ $\frac{1}{5} + \frac{2}{3}$



Then we will go beyond that, to:

$\frac{1}{6} + \frac{1}{9}$ $\qquad$ $\frac{1}{6} - \frac{1}{9}$ $\qquad$ $\frac{5}{6} - \frac{4}{9}$ $\qquad$ $\frac{1}{12} + \frac{1}{16}$

♦**Def'n ADDITION on Q**: if a,b,c,d,D∈W & $D \neq 0$ & $d \neq 0$

$$\frac{a}{D} + \frac{b}{D} = \frac{a+b}{D}$$ *in general, for rational numbers a/b & c/d* $$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

♦**Def'n SUBTRACTION on Q**: if a,b,c,d,D∈W & aD> bD or ad> bc & $D \neq 0$ & $d \neq 0$

$$\frac{a}{D} - \frac{b}{D} = \frac{a-b}{D}$$ *in general, for rational numbers a/b & c/d* $$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$$

E.g. $\frac{2}{3} + \frac{1}{2} = \frac{2\cdot 2 + 1\cdot 3}{3\cdot 2} =$ ___ $\quad 4\,^2/_3 = 4 + \frac{2}{3} = \frac{12+2}{3}$ ⇦*Dealing with "mixed numbers"*

Notice if the rational addends are integers, we obtain the accustomed results. $2+5 = \frac{2}{1} + \frac{5}{1} = \frac{2\cdot 1 + 5\cdot 1}{1\cdot 1}$

Also notice if b = d, the above general addition formula yields:

$\frac{a}{d} + \frac{c}{d} = \frac{ad+dc}{d\cdot d} = \frac{d(a+c)}{d\cdot d} = \frac{a+c}{d}$ *...as we expect.* E.g. $\frac{2}{7} + \frac{3}{7} =$ ________

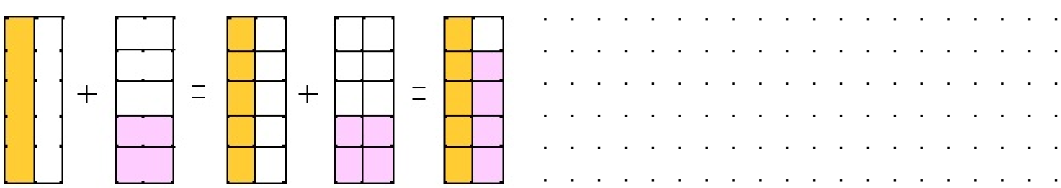
Although the given formula for + "works", it leads to unnecessarily large numbers plus extra work to reduce! The preferred method is to write equivalent fractions using the least common denominator, D— (LCD) ... and then appeal to the intuitively clear "a/D + b/D = (a + b)/D".

In short, rewrite the fractions with the LCD, then add numerators.

(Note the LCD is the actually the Least Common Multiple *of the denominators.)*

E.g. $\frac{5}{\underset{4\cdot 3\nearrow}{12}} + \frac{7}{\underset{4\cdot 4\nearrow}{16}} =$ __ + __ = __

Sketch a model to illustrate the *entire* process of adding 1/2 + 2/5:



$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

♦**Properties of + on Q**:

Closure: Given any rationals a/b and c/d, their sum, (ad + bc)/bd ∈ Q.
(Since: a,d,b,c ∈ Z guarantees ad+ bc and bd are both ∈ Z by closure of multiplication and addition on Z; and $b \neq 0$ & $d \neq 0$ guarantees $bd \neq 0$.

Commutativity: a/b + c/d = (**ad + bc**)/bd
c/d + a/b = (**cb + da**)/bd
The only difference between these is in the *numerators*. But *they* are equal, by commutativity of multiplication and addition on the set **W**[*or Z*] of **whole-numbers**[*or integers*]. Therefore, we might say commutativity of addition on Q is "inherited" from the commutativity of addition and multiplication on **W** [*or Z*].

Associativity:
Identity:
Inverses:
Are similarly "inherited" from the set of *W*, since, after rewriting as equivalent fractions with LCD's, we simply add the *whole numbers* in the numerators.

♦**Properties of – on Q**: *Properties are the same as for W.*

Mixed numbers. Common error. Counting up. Compensation.

Mixed numbers: $7 = 6 + 1 = 6 + {}^8/_8$

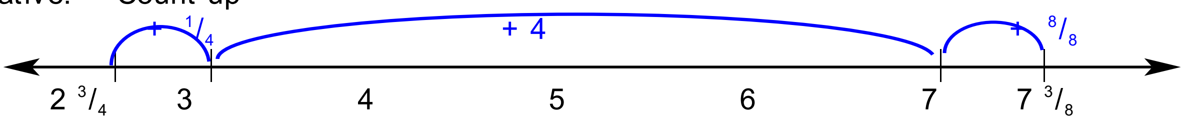
$$\begin{array}{r} 7\ {}^3/_8 \\ \underline{-\ 2\ {}^3/_4} \end{array} \qquad \begin{array}{r} 7\ {}^3/_8 \\ \underline{-\ 2\ {}^6/_8} \end{array} \qquad \begin{array}{r} 6\ {}^{11}/_8 \\ \underline{-\ 2\ {}^6/_8} \\ 4\ {}^5/_8 \end{array}$$

The traditional method:
Subtract the fraction from the fraction and
the whole number from the whole.
Exchange a unit for the appropriate fraction if needed.

Alternative: Use ‘improper’ fractions.

$7\ {}^3/_8 - 2\ {}^3/_4 = (7 + {}^3/_8) - (2 + {}^3/_4) = {}^{59}/_8 - {}^{11}/_4 = {}^{59}/_8 - {}^{22}/_8 = {}^{37}/_8 = 4\ {}^5/_8$
Plus: avoids mixed numbers at the point of subtracting.
Minus: Look at those FUN big numbers, and the extra conversion steps at the beginning & end.

Alternative: “Count up”



So we see the difference is $4 + {}^1/_4 + {}^3/_8 = 4\ {}^5/_8$

Alternative: “compensation” (Based in part on some perceptions from the above picture):

$7\ {}^3/_8 \overset{+\ {}^2/_8}{} - 2\ {}^6/_8 \overset{+\ {}^2/_8}{} = 7{}^5/_8 - 3 = 4{}^5/_8$
shifts both #s up the
line the same amount.

↑Motivation: It’s so much easier to subtract 3!

Addition compensation:

$3{}^1/_3 + 2{}^5/_6 = 3{}^2/_6 \overset{-{}^1/_6}{} + 2{}^5/_6 \overset{+\ {}^1/_6}{} = 3{}^1/_6 + 3$

(Boost one, drop the other so the total is unchanged.)

Common error:

$\frac{2}{3} + \frac{3}{5} = \frac{5}{8}$

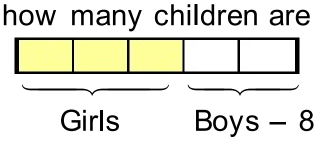
How silly is that? $> ½ + > ½ = < 1$????

$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$????

$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$

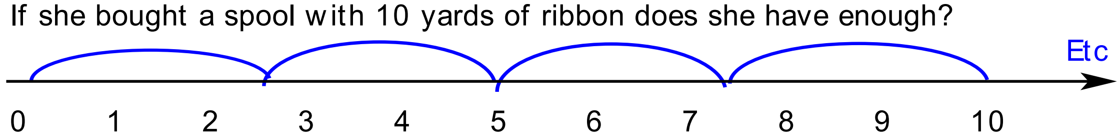
Word problems

Three-fifths of the children in Ms Jones’ class participate on school teams.
If 8 children in Ms Jones’ class do not participate on school teams,
how many children are in Ms. Jones’ class?



←Fifths partitions the class into 5 equal-size groups.
Since the 8 Boys make up 2 of those groups, we see that each group must contain 8/2 = 4 children.
So there are 5·4 = 20 children altogether in Ms. Jones’ class.

Mabel is wrapping gifts for the teachers. Each gift needs 2½ feet of ribbon.
If there are 20 teachers getting gifts, how much ribbon does Mabel need?
If she bought a spool with 10 yards of ribbon does she have enough?



Introduction to multiplication:

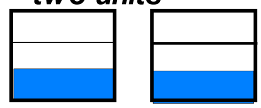
Whole number × Fraction via REPEATED ADDITION:

EG $2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ $\quad 3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

We see the whole # multiplies the numerator.

Fraction × Whole number via THE WORD "OF": ***two units***

EG $\frac{1}{3} \times 2 = \frac{1}{3}$ of $2 = \;\;?$

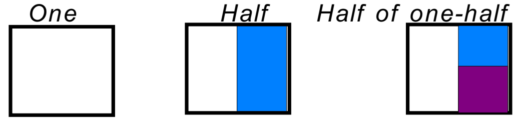
$\frac{1}{3}$ of  is $\frac{2}{3}$

With this & a few more examples we can see that this multiplication is commutative, just like in W

Fraction × Fraction

E.g. $\frac{1}{2} \times \frac{1}{2} =$

One *Half* *Half of one-half is:*



← Half of one-half

Consider:

$\frac{1}{3} \times \frac{1}{2} \qquad \frac{1}{3} \times \frac{3}{5} \qquad \frac{2}{3} \times \frac{3}{5}$

Which lead to our...

♦Def'n. MULTIPLICATION on Q: $\frac{a}{b} \times \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{a\,c}{b\,d}$

Make your own Models of multiplication for: $\frac{1}{4}$ of $\frac{1}{2}$ $\quad \frac{3}{4}$ of $\frac{1}{2}$ $\quad \frac{1}{4}$ of $\frac{1}{3}$ $\quad \frac{3}{4}$ of $\frac{1}{3}$

Multiply: $\frac{15}{8}\,\frac{24}{25} = \frac{3{\cdot}5 \cdot 3{\cdot}8}{8 \cdot 5{\cdot}5} = \frac{3{\cdot}3}{5}$

Since we know all the factors in the num & den before we multiply, we know we can "cancel" BEFORE we multiply.... I prefer what's shown.

What about "Mixed Numbers" (they should call these "Mixed Up")

$10\frac{2}{3} \times 4\frac{1}{2} = (10 + \frac{2}{3}) \times (4 + \frac{1}{2}) = \;\cdots$ *(FOIL away)*

BAD– Common Error: multiply wholes, multiply fractions, add.

"Improper" fractions are NOT bad:

$10\frac{2}{3} \times 4\frac{1}{2} = (10 + \frac{2}{3}) \times (4 + \frac{1}{2}) = \frac{32}{3} \cdot \frac{9}{2} = \frac{32{\cdot}9}{3{\cdot}2} = 48$

APP: One fourth of the students at WHS are freshmen. Of these, 3 fourths are in a geometry class. What part of the WHS students are freshmen taking geometry?

♦Properties of Multiplication on Q: (assume a/b. c/d, e/f are rationals)

Closure: ac/bd $\in$ Q since ac and bd are in W[Z], and bd $\neq$ 0 because b $\neq$ 0 and d $\neq$ 0.

Commutativity: semi-inherited from W[Z]: a/b · c/d = ac/bd = ca/db = c/d · a/b

Associativity: ditto: (a/b · c/d) · e/f = ac/bd · e/f = (ac)e/(bd)f = a(ce)/b(df) = *etc!*

Identity: is still 1 (*aka* 1/1): a/b · 1 = a/b · 1/1 = a·1/b·1 = a/b.

Inverses *for all non-zero rationals a/b!*: If a $\neq$ 0 and b $\neq$ 0, then a/b x b/a = 1.

(The multiplicative inverse of a/b, provided a $\neq$ 0, is b/a .)

Annihilator: 0 still takes all! 0· whatever = 0

Distributive property of multiplication over addition: still holds.

Notice that:

½ of 8 is the same as 8 ÷ 2

¼ of 12 is 12÷ 4 and so on.

Further, consider that:

1 ÷ ½ = 2, 1÷ ⅓ = 3, 1÷ ¼ = 4

2 ÷ ½ = 4, 3 ÷ ½ = 6, and so on.

Division by a whole number seems to be multiplication by the reciprocal.

Division by 1/n seems to be multiplication by n, the reciprocal.

Just as 6 ÷ 2 = 3 ...since 2 x 3 = 6

♦Def'n. Division on Q: IF $c \neq 0$ (& $b,d \neq 0$) $\quad \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ad}{bc} = \dfrac{a}{b} \times \dfrac{d}{c}$

...since $\dfrac{c}{d} \times \dfrac{ad}{bc} = \dfrac{acd}{bcd} = \dfrac{a}{b}$

i.e. $\dfrac{ad}{bc}$ *is the "magic number" that solves* $\dfrac{c}{d} \bullet ? = \dfrac{a}{b}$

E.g. $\dfrac{2}{3} \div \dfrac{c}{d} = \dfrac{2}{3} \cdot \dfrac{d}{c} = \dfrac{2d}{3c}$ since $\dfrac{c}{d} \cdot \dfrac{2d}{3c} = \dfrac{2}{3}$

Other views: $\dfrac{3}{4} \div \dfrac{1}{5} = \dfrac{15}{20} \div \dfrac{4}{20} = 15 \div 4 = \dfrac{15}{4} = \dfrac{3 \cdot 5}{4 \cdot 1}$

$\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a\mathbf{d}}{b\mathbf{d}} \div \dfrac{\mathbf{b}c}{\mathbf{b}d} = \dfrac{ad}{bc} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

$\dfrac{a}{b} \div \dfrac{c}{d} = x$ ...x must satisfy: $\dfrac{c}{d} \cdot x = \dfrac{a}{b}$ (Then solve for x.)

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot bd}{\frac{c}{d} \cdot bd} = \frac{a \cdot d}{c \cdot b}$$ Provided $c,b,d \neq 0$

A mixed number example:

$$10\frac{2}{3} \div 1\frac{3}{5} = \frac{32}{3} \div \frac{8}{5} = \frac{\frac{32}{3}}{\frac{8}{5}} = \frac{\frac{32}{3} \times 15}{\frac{8}{5} \times 15} = \frac{32 \cdot 5}{8 \cdot 3} =$$

"Cancellation" (?)

$$1\frac{7}{8} \div 1\frac{1}{24} = \frac{15}{8} \cdot \frac{24}{25} = \frac{15 \cdot 24}{8 \cdot 25} = \frac{5 \cdot 3 \cdot 8 \cdot 3}{8 \cdot 5 \cdot 5} =$$

~~Properties~~ of division on Q:

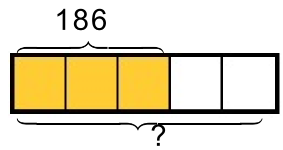
Almost closure: Closure, except for division by 0. $a/b \div c/d = ad/bc$, a rational, provided $c \neq 0$.

The ability to divide any number by almost any number expands the problems for which we can compute answers. This property goes hand in hand with the property of INVERSES for multiplication. 1 is the multiplicative identity. If we ask, in multiplication, is there a way to "get back to the identity, 1" using multiplication, the answer is now yes, just multiply by the reciprocal.

$\dfrac{a}{b} \cdot ? = 1$ has an answer for every fraction except 0.

But that, in turn means that $1 \div \dfrac{a}{b}$ has an answer except when a = 0.

Word problems put the arithmetic to use.
However, you may also observe the truth of the "rules" via the setting of a word problem.
Take, for instance, this problem from the homework:
After reading 186 pages, Jennifer had read 3/5 of her book. How many pages has the book?



Solution by scaling: 186 is 3 parts (of 5 parts).
Each part is $186 \div 3 = 62$.
The whole is 5 parts, or $5 \cdot 62 = 310$ pages.

Look at the algebra: Let X = total pages $\frac{3}{5}$ of $X = 186$

So $X = 186 \div {}^{3}/_{5}$ See process above!!

Also important: Notice that the problem $\frac{a}{b}$ of $X = \#$ results in the division $X = \# \div {}^{a}/_{b}$

You will be asked to make up a word problem yielding (for example) $22 \div {}^{3}/_{5}$. The above observation indicates that a problem of the form: "John used 3/5 of his fuel, or 22 gallons. How much fuel did he have to start?" will have that result.

(4a) Make a word problem for $135 \div 5$ using measurement division.

135 athletes were assigned to practice in teams of 5. How many teams ?

(4b) ...Ditto, but this time partitive division.

135 athletes were assigned equally to 5 service committees. How many on each committee?

(4c) ...Measurement division $32 \div 3\frac{3}{4}$

It takes $3\frac{3}{4}$ minutes to grade 1 page of a test, how many pages can be graded in 32 minutes?

(4d) ...Partitive division for $72 \div {}^{3}/_{5}$

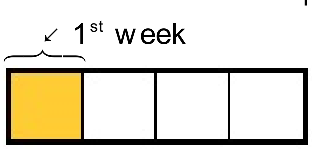
After 72 hours grading, Mary found she was ${}^{3}/_{5}$ done. How long to grade all the papers?

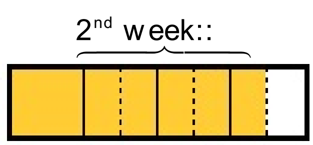
Bar diagrams can be especially helpful when Fractions are involved.

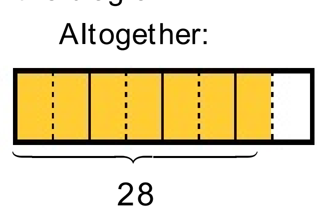
Consider solving this problem without bar diagrams, without algebra:

(5e) A pet shop sold ${}^{1}/_{4}$ of its puppies in the 1st week, ${}^{5}/_{6}$ of the rest in the 2nd week. The pet shop sold 28 puppies altogether during those two weeks. How many puppies had they to start?

Our text emphasizes using ONE bar diagram.
That's fine for this problem, but we show here the development of the diagram.







$28 \div 7 = 4$ (each part)
$8 \cdot 4 = 32$ puppies to start.

Using algebra:

Let X = number of puppies to start

$$({}^{1}/_{4})X + ({}^{5}/_{6})({}^{3}/_{4})X = 28$$

$$({}^{7}/_{8})X = 28$$

$$X = 28({}^{8}/_{7}) = 32$$

There were 32 puppies to start.

Fraction "Rules" Summary

(Fundamental Law): $\frac{a}{b} = \frac{a\,c}{b\,c}$

(+ – Definition): $\frac{a}{b} \pm \frac{c}{d} = \frac{a\,d}{b\,d} \pm \frac{b\,c}{b\,d} = \frac{a\,d \pm b\,c}{b\,d}$

Fraction as ÷ : $a \div b = \frac{a}{b}$ $(\, b \neq 0 \,)$

Multiplication : $\frac{a}{b} \cdot \frac{c}{d} = \frac{a\,c}{b\,d}$

Division : $\frac{a}{b} \div \frac{c}{d} = \frac{a\,d}{b\,c}$ $(\, c \neq 0 \,)$

The set of fractions (all numbers that can be written as ${}^{a}/_{b}$) has the following properties:
All the arithmetic rules for the Whole numbers apply.
In addition:

For every member x of Q, except if x = 0, there is another member of Q, called ${}^{1}/_{x}$, such that

$$x \cdot \frac{1}{X} = 1$$

$\frac{1}{X}$ is called the **reciprocal** of x, and is also called the **multiplicative inverse** of x.

For instance, if x is a whole number, 1, 2, 3, ··· then $\frac{1}{X}$ is $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots$ (respectively).

Other fractions such as $\frac{a}{b}$ can be thought of as $a \cdot (\frac{1}{b})$ when it suits.

What is the multiplicative inverse of $\frac{2}{3}$? $\boxed{\frac{2}{3} \cdot \qquad = 1}$

See text for proofs of that these numbers satisfy the "rules" stated in the summary above.

Complex fractions:

$$\left[\left(\frac{2}{3} \cdot \frac{1}{3}\right) + \left(\frac{3}{2} \div \frac{9}{2}\right)\right] \div \frac{10}{27}$$

$$a\,(\,b^2 \div c) + c \div \frac{a}{2}$$

$$\frac{\frac{2}{9} + \frac{2}{27}}{\frac{2}{9} - \frac{2}{27}}$$

$$\frac{3\frac{1}{4} - 2}{1\frac{2}{3} + 1}$$

$$\frac{20}{21} \times \frac{27}{5} \div \frac{18}{35} = \frac{\overset{\overset{2}{\cancel{4}}\times 5}{\cancel{20}} \times \overset{3\times\cancel{9}}{\cancel{27}} \times \overset{5\times\cancel{7}}{\cancel{35}}}{\underset{3\times\cancel{7}}{\cancel{21}} \times \cancel{5} \times \underset{\cancel{2}\times\cancel{9}}{\cancel{18}}}$$

$$\frac{20}{21} \times \frac{27}{5} \div \frac{18}{35} = \frac{2\cdot 10\cdot 3\cdot 9\cdot 5\cdot 7}{3\cdot 7\cdot 5\cdot 2\cdot 9} =$$

6-6A Complex fractions work:

$$\left[\left(\frac{2}{3}\cdot\frac{1}{3}\right)+\left(\frac{3}{2}\div\frac{9}{2}\right)\right]\div\frac{10}{27}$$

Order of operations: parentheses first!

$$\left[\frac{2}{9}+\frac{3}{9}\right]\cdot\frac{27}{10}$$

Parentheses still first, but we can get ready to do the division, by inverting....

$$\frac{5}{9}\cdot\frac{27}{10}=\frac{3}{2}$$

Then multiplying

$$a\left(b^2\div c\right)+c\div\frac{a}{2}$$

Write the divisions in fraction form.

$$a\left(\frac{b^2}{c}\right)+c\cdot\frac{2}{a}$$

Ready to multiply and add...

$$\frac{a}{a}\,\frac{a\,b^2}{c}+\frac{c}{c}\,\frac{2\,c}{a}$$

Get a common denominator — ac.

$$\frac{a^2b^2+2c^2}{a\,c}$$

& Add

$$\frac{\frac{2}{9}+\frac{2}{27}}{\frac{2}{9}-\frac{2}{27}}=\frac{\left(\frac{2}{9}+\frac{2}{27}\right)27}{\left(\frac{2}{9}-\frac{2}{27}\right)27}$$

The easy way to clean this up—multiply numerator & denominator by LCM(9,27), which is 27.

$$=\frac{6+2}{6-2}$$

Get: no more fractions within fraction !

$$=2$$

Do the same thing here—multiply numerator & denominator by 12.

$$\frac{3\frac{1}{4}-2}{1\frac{2}{3}+1}\quad\text{Get}\quad\frac{36+3-24}{12+8+12}=\frac{15}{32}$$

Alternately, combine the whole numbers first, then multiply by 12:

$$\frac{3\frac{1}{4}-2}{1\frac{2}{3}+1}=\frac{1+\frac{1}{4}}{2+\frac{2}{3}}=\frac{12+3}{24+8}=\frac{15}{32}$$

This is problem #5 in St 29 (§6.6 p 165).
Although not assigned, it illustrates the problem created by disorganized "cancellation".

Here is Kyle's work, if deciphered correctly: It's hard to see what's been "cancelled" and what's left.

$$\frac{20}{21}\times\frac{27}{5}\div\frac{18}{35}=\frac{\overset{\overset{2}{\cancel{4\times5}}}{\cancel{20}}\times\overset{\cancel{3\times9}}{\cancel{27}}\times\overset{5\times\cancel{7}}{\cancel{35}}}{\underset{3\times\cancel{7}}{\cancel{21}}\times\cancel{5}\times\underset{\cancel{2}\times\cancel{9}}{\cancel{18}}}$$

Here is the same work, organized. Anticipating reducing the fraction, 20 was factored into 2·10, & 18 into 2·9, so the 2s can be reduced. Etc. Try the "cancellation" yourself here:

$$\frac{20}{21}\times\frac{27}{5}\div\frac{18}{35}=\frac{2\cdot10\cdot3\cdot9\cdot5\cdot7}{3\cdot7\cdot5\cdot2\cdot9}$$

$$=10$$