2. (Re PT4A pp42-49)
a. Re problem \#9, p45, parts a, b, c, f, l, j - ADD:
\#9a. $\frac{1}{2}+\frac{1}{8}=\frac{5}{8}$
\#9b. $\frac{1}{4}+\frac{2}{12}=\frac{5}{12}$
\#9c. $\frac{2}{3}+\frac{1}{9}=\frac{7}{9}$
\#9f. $\frac{2}{3}+\frac{1}{12}=\frac{9}{12}=\frac{3}{4}$ \#9i. $\frac{3}{4}+\frac{1}{12}=\frac{10}{12}=\frac{5}{6} \# 9 j \cdot \frac{1}{4}+\frac{1}{9}+\frac{3}{8}=\frac{6}{8}=\frac{3}{4}$
b. Re problem \#10, p49, parts a, b, c, f, l, j-SUBTRACT:
\#10a. $\frac{5}{9}-\frac{1}{3}=\frac{2}{9} \quad \# 10$ b. $\frac{3}{4}-\frac{3}{8}=\frac{3}{8} \quad \# 10$ c. $\frac{4}{5}-\frac{7}{10}=\frac{1}{10}$
\#10f. $\frac{7}{10}-\frac{1}{5}=\frac{5}{10}=\frac{1}{2} \# 10$ i. $\frac{5}{6}-\frac{7}{12}=\frac{3}{12}=\frac{1}{4} \# 10$ j. $1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$
c. Re problems \#4ab \& 5ab p50. Create bar diagrams similar to those on pp42-49.

\#4b. $\frac{5}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

\#5a. $\frac{3}{10}+\frac{3}{10}=\frac{6}{10}=\frac{3}{5}$
\#5b. $\frac{1}{12}+\frac{5}{12}=\frac{6}{12}=\frac{1}{2}$

3. (Re PT4A p50, problems \#6-9): Give "Teacher's Solutions" \& specify what type of subtraction. \#6. Sally ate one-eighth; sister ate three-eighths of the cake.
What fraction was eaten altogether?
\#7 Muthu spent four-ninths of his money, saved the rest. What fraction did he save? This can be viewed either as Part-Whole or as Take-away Subtraction.

$$
\frac{1}{8}+\frac{3}{8}=\frac{4}{8}=\frac{1}{2}
$$

Altogether they ate one half $\left(\frac{1}{2}\right)$ of the cake!


$$
1-\frac{4}{9}=\frac{5}{9}
$$

Muthu saved five ninths ( $\frac{5}{9}$ ) of his money.
3. Continued [\#8 is similar to another problem.]
\#9. Fatimah ate one-sixth of the pie, gave three-sixths away. What fraction was left?


This is Take-away Subtraction.

$$
\begin{aligned}
& 1-\left(\frac{1}{6}+\frac{3}{6}\right)= \\
& \frac{6}{6}-\frac{4}{6}=\frac{2}{6}=\frac{1}{3}
\end{aligned}
$$

One-third $\left(\frac{1}{3}\right)$ of the pie was left.
(Re PT4A p51, problems \#6-8):
\#6. Mary has $3 / 4$ liter of orange juice; she drinks $1 / 2$ liter of it. How much has she left?
This is Take-away subtraction.


$$
\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}
$$

Mary has one-fourth ( $1 / 4$ ) liter of orange juice left.
\#7. Mr J bought a can of paint. He used half of it to paint a table, one-eighth on a bookshelf. How much paint did he use altogether? [This problem is similar to some above problems.]


$$
\frac{1}{2}+\frac{1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}
$$

Mr. Johnson used five-eighths of a can of paint altogether.
\#8. Meredith bought two-fifths kg of shrimp. Courtney bought one-tenth kg less.
(a) Find the mass of shrimp bought by C. (b) Find total mass of shrimp they purchased.

Note- This is not a "teacher's solution".
Call it "Backwards comparison subtraction" because they give the difference between two quantities... then ask for one of them.
Regardless of the type of subtraction problem, we end up with the subtraction:

$$
\frac{2}{5}-\frac{1}{10}=\frac{4}{10}-\frac{1}{10}=\frac{3}{10}
$$

The total mass of shrimp purchased is two-fifths plus three-tenths $\mathrm{kg}=7 / 10 \mathrm{~kg}$ total.
4. (Re PT4A. Review B pp 66-69 \#12-16)

4a. (Answer Problems \#12-16):
\#12: fraction shaded is... $\quad \frac{5}{8}, \quad \frac{1}{3}, \quad \frac{1}{6}, \quad \frac{1}{2}, \quad \frac{3}{4}, \quad \frac{1}{2}$
\#13: fraction shaded is... $\frac{4}{9}, \frac{2}{3}$
\#14: in increasing order... $\frac{1}{12}<\frac{1}{3}<\frac{3}{5}<\frac{4}{4}<\frac{3}{2}$
\#15: missing part... 6, 18, 2, 3
\#16 in simplest form...
$\frac{8}{10}=\frac{4}{5}, \quad \frac{2}{12}=\frac{1}{6}, \quad 1 \frac{2}{6}=\frac{4}{3}, \quad 2 \frac{3}{12}=\frac{9}{4}$
$4 b$. The above problems are intended to evaluate and consolidate knowledge of EQUIVALENT FRACTIONS.
5. A "teacher's explanation" for: $\frac{4}{5}=\frac{8}{10}$

These two fractions represent the same position

on the number line. When the interval between 0 and 1 is divided into 5 equal parts, taking 4 of those is the same as... taking 8 parts, after the interval has been divided into 10 equal parts. $\frac{4}{5}$ of the way from $O$ to 1 is the same as $\frac{8}{10}$ of the way from $O$ to 1 .
6. Give Teacher's Solutions to the following problems using a diagram based on the model:

6a. MS used 3/10 of a bottle of cooking oil; that measured 150 mL . How much did the whole bottle hold?


Area Model
$\frac{3}{10}=150 \mathrm{~mL}$ so $\frac{1}{10}=50 \mathrm{~mL}$
Then $\frac{10}{10}=10 \times 50 \mathrm{~mL}=500 \mathrm{~mL}$
The whole bottle held 500 mL .

6b. Four-fifths of the choir are girls. If 8 boys in choir, how many altogether? Use measurement model.


The choir is made up of 40 members.

Since Girls make up 4 fifths of the total, the Boys comprise $(1-4 / 5)$ or $1 / 5$ of the total. Think of $1 / 5$ of the total as a "unit".
The Boys form one unit. So one unit $=8 \mathrm{members}$.
The choir is made up of five units. $5 \cdot 8=40$.

6c. Jim had 15 stamps. He gave two-fifths of them to Jill. How many stamps did he give to Jill?


Two parts of five parts went to Jill. One part = 3 stamps.
Two parts $=2.3$ stamps $=6$ stamps.
Jim gave Jill 6 stamps.
6d. Beth made 12 bows. She used $1 / 5$ meter of ribbon for each bow. How much ribbon did she use?


One unit = one fifth meter.
12 units $=12$ fifths meter $=22 / 5 \mathrm{~m}$.
Beth used $2 / 5 \mathrm{~m}$ of ribbon.
6e. A shopkeeper had 150 kg of rice. He sold $2 / 5$ of it, and packed the remainder equally into 5 bags. Find the weight of rice in each bag.
Steo 1:


$$
1-\frac{2}{5}=\frac{3}{5} \quad \frac{3}{5} \cdot 150 \mathrm{~kg}=90 \mathrm{~kg}
$$

OR: 5 parts $=150 \mathrm{~kg}$
2 part $=30 \mathrm{~kg}$
3 parts $=90 \mathrm{~kg}$
Step 2: $\quad 90 \mathrm{~kg}$

$90 \mathrm{~kg} \div 5=18 \mathrm{~kg}$.

Each bag should contain 18 kg .

1. ( $\operatorname{Re}$ PT 4A, pp 52-57)

1a. Illustrate Problems \#8a and \#8b (p 57) using a number line.

$$
\frac{1}{5} \frac{2}{5} \frac{3}{5} \frac{4}{5} \frac{5}{5} \frac{6}{5} \frac{7}{5} \frac{8}{5} \frac{9}{5} \frac{10}{5}
$$

\#8a.


1b. Illustrate problems \#10a and \#10b (p 57) using an area model. Start with one unit. [\#10b not shown]

3. (Re PT 5A pp 37-43)

3a. Illustrate problems 1c, [2c, and 3c] of Practice 3B using pictures similar to example 2.4

Solutions for $2 c \& 3 c$ are NOT shown


Yes, the least common denominator of 6 and 8 is 24 , not 48 . However, the point of these illustrations is that we cannot intuitively add most fractions without a common denominator, and we can add any two fractions by restating them as equivalent fractions with a common denominator, and that is always possible... leading to the definition of addition of fractions:

$$
\frac{a}{b}+\frac{c}{d}=\frac{a d+b c}{b d}
$$

This "formula" tells us how to add any two fractions, regardless of the particular numbers involved.
3b. Give a Teacher's Solution to Problems 7 and 8 of Practice 3B (p 40).
\#7 Mary ate $1 / 8$ of a cake. Peter ate another $1 / 4$ of it.
\#7a What fraction of the cake did they eat altogether?


Notice both of the above diagrams assume a little more sophisticated view of addition, that is, they both use the least common denominator and assume the student will see the compatibility of the denominators 8 and 4 .
\#7b What part of the cake did Peter eat more than Mary?

$$
\frac{1}{4}-\frac{1}{8}=\frac{2}{8}-\frac{1}{8}=\frac{1}{8}
$$


$3 b$ continued
\#8 Ali spent $3 / 5$ of his money on books and $1 / 4$ of it on a pen.
\#8a What fraction of his money did he spend?


He spent 17 twentieths of his money.
\#8b What fraction of his money had he left? The unspent part of the picture: $\frac{3}{20}$
3c. Illustrate problems1c, 2c, \& 3c of Practice 3C (p43) in the manner of Ex. 2.7.


Unsolicited comment: This is not nearly as satisfying as "counting up" in subtraction problems.
\#2c $4 \frac{5}{6}-1 \frac{1}{4}=\frac{3}{4}+2+\frac{5}{6}=2+\frac{3}{4}+\frac{5}{6}=2+\frac{9}{12}+\frac{10}{12}=2 \frac{19}{12}=3 \frac{7}{12}$

4. Mental Math (Show your intermediate steps):
[4b, 4d, 4e not shown]
4a. $28 \frac{2}{7}-3 \frac{6^{+1 / 7}}{7}=28 \frac{3}{7}-4=24 \frac{3}{7}$
4c. $\left(1 \frac{3}{4}+4 \frac{5}{11}\right)+\left(2 \frac{8}{11}+5 \frac{1}{4}\right)=(1+4+2+5)+\frac{3}{4}+\frac{1}{4}+\frac{5}{11}+\frac{8}{11}=12+1+\frac{13}{11}=13+1+\frac{2}{11}=14 \frac{2}{11}$
7. Use long division to convert to mixed numbers:
[7b not shown]
$7 a$.

$$
\begin{aligned}
& 3 7 \longdiv { \frac { 1 9 } { 7 3 5 } } \quad \frac { 7 3 5 } { 3 7 } = 1 9 \frac { 3 2 } { 3 7 } \\
& \frac{37}{365} \\
& \frac{333}{32}
\end{aligned}
$$

8. A student claims $\frac{46}{6}$ can't be the same as $\frac{23}{3}$ because $46 \div 6$ is 7 R 3 , while $23 \div 3$ is 7 R 2 .

Answer:
The remainders must be compared to the divisors.
$46 \div 6=\frac{46}{6}=7 \frac{4}{6} \quad 23 \div 3=\frac{23}{3}=7 \frac{2}{3}$
Just as you cannot judge the size of a fraction by its numerator or denominator alone, you cannot judge a fraction by the remainder that results when the division is performed. You can also illustrate this or use an example with simpler numbers. EG $4 / 3$ versus $8 / 6$ (remainders are 1 and 2 ).

1. Mental Math. Since $25=\frac{100}{4}$, we can easily multiply:

1a. $25 \times 64=\frac{100}{4} \times 64=100 \times \frac{64}{4}=100 \times 16=1600$
1d.
$3212 \times 25=3212 \times \frac{100}{4}=80300$
2. Mental Math. Show your thinking; use arithmetic properties...
[\#2d not shown]
2a. $44 \cdot \frac{3}{8}+44 \cdot \frac{7}{8}=44 \cdot\left(\frac{3}{8}+\frac{7}{8}\right)=44 \cdot \frac{10}{8}=44\left(1+\frac{1}{4}\right)=44+11 \cdots 55$
2b. $\left(\frac{4}{7}+\frac{7}{9}\right)-\frac{3}{7}=\frac{4}{7}-\frac{3}{7}+\frac{7}{9}=\frac{1}{7}+\frac{7}{9}=\frac{9+49}{63}=\frac{58}{63} \quad$ Note: almost certainly the
2c. $48 \times 99 \frac{5}{12}=48\left(100-\frac{7}{12}\right)=4800-\frac{487}{12}=4800-28=4772$
3. Estimate to the nearest whole number:
a. $59 \times 1 / 3 \approx 60 \times 1 / 3=20$...this answer is off by only a third of a unit.
b. $24 \frac{1 / 4}{} \times 12 / 3 \approx 24 \times 12 / 3=24 \cdot 1+24 \cdot 2 / 3=24+16=40 \quad \ldots$ off by $1 / 4 \times 12 / 3$,
4. (RePT5A pp 44-45, 48)
which is less than $1 / 4 \times 2$
4a. $(p 48, \# 2) \quad 24 \quad \frac{80}{3} \quad \frac{25}{3} \quad(p 48, \# 3) \quad 49 \quad 52 \quad 45$
4b. Use Method 3 to find: $48 \times \frac{23}{12}=4^{48} \times \frac{23}{12}=4 \cdot 23=92$

$$
320 \times \frac{13}{80}=4320 \times \frac{13}{80}=4 \cdot 13=52
$$

5. (Re PT 5A, pp49-52 Practice 3E \#5-10 no diagrams necessary)
[\#7 \& \#8 not shown] \#5a. $\frac{20}{7} \cdot \frac{7}{4}=5$ \#5c. 5
\#6. $\frac{1}{3}$ of $\frac{1}{2} m=\frac{1}{6} m$
\#9. She gave away $\frac{1}{5}$ of $\left(1-\frac{1}{6}\right)=\frac{1}{5} \cdot \frac{5}{6}=\frac{1}{6}$ of the cake.
\#10. The area of a rectangle which measures $\frac{5}{8} m$ by $\frac{3}{5} m=\frac{5}{8} \cdot \frac{3}{5} m^{2}=\frac{3}{8} m^{2}$
6. (Re PW 5A, p 57-59, Exercise 22 \& 23)

6a. Illustrate the solution to Exercise 22, \#2a, 2b)
\#2a. $\frac{4}{9} \times \frac{1}{2}=\frac{4 \cdot 1}{9 \cdot 2}=\frac{4}{18}=\frac{2}{9}$
$\frac{1}{2}$
[ \#2b of Exercise 22 is not shown.]

The middle illustration is the critical one.




6b. Give Teacher's Solutions to Problems 2, 3 and 4 of Exercise 23 (PW 5A p 59).
\#2 Mrs. Smith bought $\frac{5}{6}$ lb of meat. She cooked $\frac{2}{3}$ of it. How much meat did she cook?
First show:


Mrs. Smith cooked $\frac{5}{9}$ lb of meat.

$$
\frac{2}{3} \times \frac{5}{6}=\frac{10}{18}=\frac{5}{9}
$$

\#3 A rectangle measures $\frac{3}{4}$ yd by $\frac{2}{5}$ yd. Find its area.


A $\frac{3}{4}$ yd by $\frac{2}{5}$ yd rectangle occupies
$\frac{3}{4}$ of $\frac{2}{5}$ of a square yard
$\frac{3}{4} \cdot \frac{2}{5}=\frac{6}{20}=\frac{3}{10}$


The area is $6 / 20$, or $3 / 10$, of a square yard.
\#4 Susan spent $\frac{3}{5}$ of her money on a calculator, and $\frac{2}{3}$ of the remainder on a pen. What fraction


She spent $\frac{2}{3}$ of $\left.\begin{array}{|l|l|l|l|l|}\hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline\end{array}\right\} \begin{aligned} & \text { the } 2 / 5 \text { remainder } \\ & \frac{2}{3} \text { of } \frac{2}{5}=\frac{4}{15}\end{aligned}$

So altogether she spent:


Susan had $1-\frac{13}{15}=\frac{2}{15}$ of her money left.
This can be done with one diagram, but the progression is shown here to make the presentation clear:

$\rightarrow$


Another style of presentation:


In each case, the presentation must be completed with the statement:
Susan had $1-\frac{13}{15}=\frac{2}{15}$ of her money left.
10. (Re PT 5A p56-59 \& PW 5A pp68-71)

Give Teacher's Solutions for Exercises 28 \& 29 (in PW 5A)
Exercise 28:
\#1. Tracy bought 120 eggs. She used $\frac{2}{3}$ of them for cakes, $\frac{1}{4}$ of the remainder for cookies. How many had she left? 120


Tracy had 30 eggs left.
$2 / 3$ used $\Rightarrow 1 / 3$ left. $1 / 3$ of $120=40$
She used $1 / 4$ of the remainder (40) for cookies. $1 / 4$ of $40=10.3 / 4$ of $40=30$ eggs are left.

Comment: She used $2 / 3$ and $1 / 4$ of $1 / 3$, $\square$ so $3 / 4$ of $1 / 3$ of 120 eggs are left.
\#4 Sam packed 42 kg of rice into one Big and 6 Small, equal bags. Big bag got 3/7 of the rice.
How many kg of rice in each Small bag?


Exercise 29:
\#1 Lindsey sold $\frac{3}{5}$ of her tarts in the AM, $\frac{1}{4}$ of the rest in the PM. She had 300 left. How many?


If 2 parts $=400$
Then 1 part = 200 \& 5 parts must be 1000 .

Lindsey had made 1000 tarts.
\#2 MrsK sold $\frac{3}{5}$ of her tarts in the AM, $\frac{1}{4}$ of the rest in the PM.
She sold 200 more in the AM than in the PM. How many had she made?

\#3 and \#4 are not shown.
11. Keep in mind we have TWO tests. Try them both on each problem!
(1) Do we know how many parts we want? (Partitive) or do we know the size of the parts? (Measure)
(2) When we divide, do the units match? No $\rightarrow$ Partitive Yes $\rightarrow$ Measure.

11a. MD 11b. MD 11c. PD 11d. PD 11e. MD 11f. PD

1. (Re PT 5A pp 53-55) Give Teacher's Solutions to Practice 3F, Problems 4-9

Teacher's Solutions not shown for \#4, 6, 7,8,9
\#4 $2 / 5 \mathrm{~m}$
\#5 (Rephrased) $\frac{4}{5}$ of the money collected at a garage sale was divided equally among 4 clubs. What fraction of the [garage sale proceeds] did each club receive?

Proceeds

\#6 $\quad 1 / 20 \mathrm{~kg}$
\#7 1/10 pt
\#8 $3 / 16 \mathrm{~m}$
\#9 $1 / 8 \mathrm{~kg}$
3. Give a Teacher's Solution using bar diagram:

3a. After spending $\frac{4}{7}$ of her money on a jacket, Rita had $\$ 36$ left. How much money did she have at first?


3b. While filling her backyard pool, Anita watched level rise from $\frac{1}{9}$ full to $\frac{1}{3}$ full in $\frac{2}{3} \mathrm{hr}$. Find time to fill it.


$$
\begin{aligned}
& 2 \text { parts }=\frac{2}{3} h r \\
& 1 \text { part }=\frac{1}{3} \mathrm{hr} \\
& \text { The whole }=9 \text { parts }=9 \cdot \frac{1}{3} \mathrm{hr}=3 \mathrm{hr}
\end{aligned}
$$

The pool will fill in 3 hours.
4. Give a Teacher's Solution using scaling:

4a. After reading 186 pages, Jennifer had read $\frac{3}{5}$ of her book. How many pages has the book?

$$
\left.\begin{array}{l}
186 \text { pages }=\frac{3}{5} \text { of the book } \\
62 \text { pages }=\frac{1}{5} \text { of the book } \\
310 \text { pages }=\frac{5}{5} \text { of the book }
\end{array}\right\} \text { Multiply by } 5
$$

4b. A dump truck contains $\frac{2}{3}$ of a ton of dirt, but is only $\frac{3}{10}$ - full. How many tons of dirt can the truck hold?
$\left.\begin{array}{l}\frac{2}{3} \text { of a ton }=\frac{3}{10} \text { of a truck-full } \\ \frac{2}{9} \text { of a ton }=\frac{1}{10} \text { of a truck-full } \\ \frac{20}{9} \text { of a ton }=\frac{10}{10} \text { of a truck-full }\end{array}\right\}$ Multiply by 10
The truck can hold $\frac{20}{9}$, or $2 \frac{2}{9}$, tons of dirt.
5. Give a Teacher's Solution using algebra:
$5 a$. $\frac{3}{7}$ of the coins in a box are nickels; the rest, pennies. If there are 36 pennies, how many coins total?

$$
\begin{array}{rlrl}
\left(1-\frac{3}{7}\right) \times=36 \text { or } \quad \frac{4}{7} x & =36 \\
4 x & =7 \cdot 36 \\
x & =7 \cdot 9 \quad & & \text { Multiply both sides by } 7 \\
& \text { Divide both sides by } 4
\end{array}
$$

The box contains 63 coins [ 36 pennies, and 27 nickels].
5b. A farmer took $\frac{3}{4}$ hr to plow $\frac{2}{5}$ of his corn field. How many hours needed to plow the whole field? We know doing $\frac{2}{5}$ of the whole field took $\frac{3}{4}$ hours. So $\frac{3}{4}$ hrs $=\frac{2}{5}$ of (time for the whole job)

$$
\begin{aligned}
\frac{2}{5} T & =\frac{3}{4}[h r] \\
T & =\frac{5}{2} \cdot \frac{3}{4}[h r]=\frac{15}{8}[h r] \text { The whole field will take } \frac{15}{8} \text {, or } 1 \frac{7}{8} \text { hours }
\end{aligned}
$$

6. Give a Teacher's Solution to each of the following problems.

Ga. Michelle spent $\frac{3}{5}$ of her $\$$ on a backpack. With the rest she got 3 CDs at $\$ 12$ ea. Backpack cost $\qquad$ $?$


2 parts $=\$ 36$
1 part $=\$ 18$
3 parts $=\$ 54 \quad$ The backpack cost $\$ 54$.
bb. Whitney sold $\frac{2}{3}$ of her cookies \& gave away $\frac{1}{5}$ of the rest. 60 cookies were left. How many to start?


4 little parts $=60$ cookies
1 little part = 15 cookies
5 little parts $=75$ cookies
1 third part $=75$ cookies
3 thirds $=3.75$ cookies $=225$ cookies
Whitney had 225 cookies to start.
bc. Tony spent $\frac{2}{5}$ of his money on running shoes. He also bought a coat, which cost $\$ 6$ less than
 the shoes. He then had $\$ 37$ left. How much had he at first?

1 part $=\$ 37-\$ 6$ ( $\$ 6$ of that $\$ 37$ belongs to the $4^{\text {th }}$ fifth.) 5 parts $=5 \cdot \$ 31=\$ 155$

Tony had $\$ 155$ to start
Gd. Fish tank weighs 11.5 lbs when $1 / 8$ full, 34 lbs when $3 / 4$ full. What is weight of empty tank?
Total $=$ tank + water $\quad 11.5 \mathrm{lb} \quad 34-11.5 \quad 5$ parts water $=22.5 \mathrm{lb}$


The empty tank weighs 7 lb .

