3. All are algebraic expressions except for b, e and i.
Items a, d and h are numeric expressions, and therefore are algebraic expressions.
b. $3 + \div 7$ has no meaning, cannot be evaluated, does not represent a number.
e. $3x + 2 = 7$ is an equation. It does not represent a number.
i. $y \div 0$ for any choice of y, this does not represent a number (because $\div 0$ is undefined)

5. Fred is confused about the meaning of the equal sign. He wrote:
$3(x+2) = 3x + 6 - x = 2x + 6 + 8 = 2x + 14$ the first two "=" are incorrect...
...because $3(x+2) = 3x + 6$ and $3(x+2) \neq 3x + 6 - x$
...because $3x + 6 - x = 2x + 6$ and $3x + 6 - x \neq 2x + 6 + 8$.
Fred could have written: $3(x+2) - x + 8$ Or: $3(x+2) = 3x + 6$
$= 3x + 6 - x + 8$ $3x + 6 - x = 2x + 6$
$= 3x - x + 6 + 8$ $2x + 6 + 8 = 2x + 14$
$= 2x + 14$

6a. The number of inches in m feet:
There are 12m inches in m feet.

There are 12 inches in 1 foot,
24 inches in 2 feet,
____ inches in 10 feet....
____ Inches in m feet.

6b. The perimeter of a square with side s cm is $4s$ cm.

6c. The value in cents of x nickels and y dimes is $5x + 10y$.

6d. The number of pounds in 6z ounces....
is 6z/16

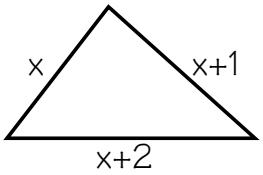
Start with what you know:
The number of pounds in 16 oz is 1
The number of pounds in 32 oz is 2
Work your way up, then ask yourself how you are getting the answer.
The number of pounds in 32 oz is 32/16
The number of pounds in w oz is w/16
(w stands for whatever)

6e. Three consecutive whole numbers,
the smallest of which is n... are n, n+1, n+2
(Note, these are THREE expressions, not one.)

6f. The average speed of a plane that travels 600 miles in 2 hours is 300 mph.
The average speed of a train that travels w miles in 5 hours is $w/5$ miles per hour.

6g. Ann is 18 years younger than Bill. $A = B - 18$ If Bill is B yr old.
Carmen is 1/5 as old as Ann. $C = A/5 = (B - 18)/5$
Dana is 4 yr older than Carmen. $D = C + 4 = (B - 18)/5 + 4$

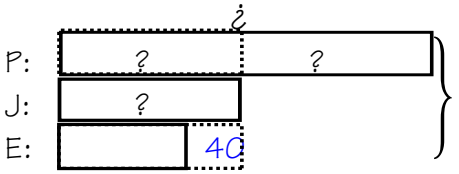
7. The lengths in inches of the sides of a triangle are consecutive integers, P is 27".



Let x be the number of inches length of the shortest side.
The next two are x+1 and x+2.
Perimeter = 27 (inches), and perimeter = x + (x+1) + (x+2) [inches], so...

$x + (x + 1) + (x + 2) = 27$
$3x + 3 = 27$
$3x = 24$
$x = 8$ The shortest side is 8 inches long.

8a. (Re PT5A p 25 #6): P stands for Peter's stickers, J for Jay's and E for Emily's.

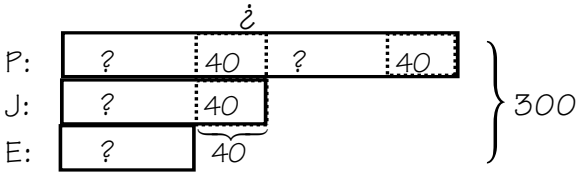


If we give E 40 more stickers, she'd have the same as J. And the total would be 40 more.
Then 4 units = 340,
So 1 unit = 340 ÷ 4 = 85
&... Peter has 2 × 85 = 170 stickers

USING ALGEBRA:
Let x = # stickers Jay has. Jay has x stickers,
so Peter has 2x stickers,
and Emily has x – 40.
Altogether they have 300 stickers, so:

$$x + 2x + x - 40 = 300$$
$$4x - 40 = 300$$
$$4x = 340$$
$$x = 85$$ Peter has 2·85 = 170

Alternate solution:



Here we think of the basic unit as E's amount,
see that J has 40 more, etc.
3× 40 = 120
300 – 120 = 180
4 units = 180
1 unit = 180 ÷ 4 = 45
So Peter has 45+40+45+40 = 170 stickers

USING ALGEBRA:
Let x = # stickers Emily has. Then...
Jay has x + 40
Peter has 2(Jay's #) = 2(x+40)
They have 300 altogether, so:

$$2(x+40) + x + 40 + x = 300$$
$$2x + 80 + x + 40 + x = 300$$
$$4x + 120 = 300$$
$$4x = 180$$
$$x = 45$$

So Peter has 2(45+40) = 170 stickers.

9. (Re PT 6A pp 7-11)
#1. Alan is 8 yr old. a. How old will he be in 5 yr? b. How old will he be in x years?
13, x+8 (yr); B [Although it may seem in part a that you are evaluating, that is more of a "warm-up" for building the expression requested in part b.]

#2 Jim has $2 more than Travis. a. If Jim has $10, how much does Travis have? b. If Jim has $m....
$8, m – 8 ($); B [Very similar to #1]

#3 Tracy bought w kg of flour, used 5 kg. a. Express amount left. b. Tracy bought 8 kg. What's left?
w – 5 ←B, 8– 5 = 3 ←E; B,E [part a is Building, part b is Evaluating.]

#4 There are 4 apples in each packet. a. How many in n packets? b. n = 8, # apples =? c. c. n= 11....
4n (apples) ←B, 32 (apples)←E, 44 (apples)←E; B, E, E

#5. 3 boxes, each containing p wings. a. Express total of wings. B. If each box contains 7 wings....
3p, 7p (wings) ; B, B

#6. A rectangular tile is k cm by 8 cm. Express area in terms of k.
8k (sq cm) ; B

#7. Ali has 8 boxes, each to have equal # of marbles. a. # = 96 How many in each box? b. # = x....
96/8 = 12, x/8 ; B [This is another where it may seem we start off E, but the purpose is B.]

9. Continued

#8. Meihua bought 3 books. a. Total cost \$12, find average cost. b. If total cost is \$m....
\$12/3 = \$4, m/3 (\$); B [see comment above.]

#9 Find the value of each of the following when n = 6.
6+4 = 10, 10+6 = 16, 9, 0, 24, 60, 3, 1, ½ ; all parts are E

#10. Tyrone puts x marbles in each bag, 5 bags plus 3 marbles. a. total marbles? b. If x = 10....
5x+3←B 50+3 = 53←E

#11. Find the value of 2x – 3 when x = 5.
7; E

#12. Jeff had \$50, gave \$y to his son. 2 daughters split rest equally. a. Girls get ? b. If y=12....
(50 - y)/2 ←B, 19 ←E; B,E [part a is Building, part b is Evaluating.]

#13. Find the value of (x–4)/2 when x = 12.
4; E

#14. a. Find the value of (4n+3)/5 when n=8 b. ...of (45 – 3r)/3 when r = 5.
7, 10; E,E

10c. (Re PT 6A p14 #6–9)

#6. The admission fee to a bird park is \$y, to an amusement park is \$1 more.
a. Express the admission fee to the amusement park in terms of y. \$(y + 1)
b. If admission fee to bird park is \$8, find admission fee to amusement park. \$9

#7. Rope is x m long. Iron rod is 3 times as long.
a. Express length of iron rod in terms of x. 3x
b. If the rope is 9 m long, how long is the iron rod? 27m

#8. Henry is x yr old. Betty is 3 times as old as Henry. Peter is 4 yr older than Betty.
a. Express Peter's age in terms of x. 4 + 3x yr
b. If Henry is 4 yr old, how old is Peter? 16 yr

#9 Huili bought cartons of milk at \$2 each. She gave the cashier \$50 and received \$y change.
a. Express the number of cartons of milk Huili bought in terms of y. (50 - y)/2
b. If y = 38, how many cartons of milk did Huili buy? 6

11. Write a short word problem which builds the given expression in the given context.
Make clear what each letter represents.

11a. The expression 12c in the context of baking cookies:
Margo has 12 cookie sheets. She can put c cookies on each baking sheet.
How many cookies will her cookie sheets hold?

11b. The expression 13r+3s in the context of shopping:
Incredible Bargains has ruanas on sale for \$13 each, and scarves for \$3 each.
At these prices, what will be the total cost for r ruanas and s scarves?

11c. 2w + 13, allowance: John had \$13 when he began saving two dollars each week from allowance. How much will he have after saving \$2 per week for w weeks?

11d. (240–x)/50 time to complete trip: It is 240 miles to San Francisco. We have driven x mi, at a steady 50 mph. If we continue at this rate, how long will it take to get to San Francisco?

3. Illustrate the addition of x^2+3x+4 and $2x^2+x+7$...

3a. Using a picture involving rectangular arrays of areas x^2, x and 1:

x^2+3x+4 : x^2 + ▬ + ▪▪▪▪

$2x^2+x+7$: x^2 x^2 + ▬ + ▪▪▪▪▪▪▪

Sum is $3x^2+4x+11$

3b. $x^2+3x+4+2x^2+x+7$
$= x^2+2x^2+3x+x+4+7$
$= (1+2)x^2+(3+1)x+(4+7)$
$= 3x^2+4x+11$

By repeated use of commutative and associative properties,
⇦ we know these terms can be rearranged in any order.
...by the distributive property applied to x^2+2x^2 and to $3x+x$

4. $(a+b)^2 = a^2+2ab+b^2$ specifically where $b=1$ says: $(a+1)^2 = a^2+2a+1$
so $21^2 = 20^2+2\cdot20+1 = 400+40+1 = 441$
similarly $31^2 = 30^2+2\cdot30+1 = 900+60+1 = 961$
and 41^2 = (do it mentally) $= 1600+80+1 =$
and 51^2 = (do it mentally) = __00 + __0 + 1 = 2601

$(a-b)^2 = a^2-2ab+b^2$ specifically where $b=1$ says: $(a-1)^2 = a^2-2a+1$
so $19^2 = 20^2+2\cdot20+1 = 400-40+1 = 361$
similarly $29^2 = 30^2+2\cdot30+1 = 900-60+1 = 841$
and 39^2 = (do it mentally) $= 1600-80+1 =$
and 49^2 = (do it mentally) = __00 − __0 + 1 = 2401

5. $15^2 = (10+5)^2 = 100+2\cdot50+25 = 225$
$150^2 = (15\cdot10)^2 = 15^2\cdot10^2 = 225\cdot100 = 22500$
$151^2 = (150+1)^2 = 150^2+2\cdot150+1 = 22500+300+1 = 22801$

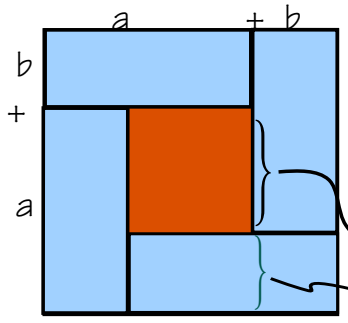
6a. $14\cdot16 = (15-1)(15+1) = 15^2-1 = 225-1 = 224$

6b. $13\cdot17 = (15-2)(15+2) = 15^2-4 = 225-4 = 221$

The product of any two numbers that differ by 2 is $(n-1)(n+1) = n^2-1$
(and n is the average of the two)

8. $(a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2+2ab+b^2)$
$= (a+b)a^2+(a+b)2ab+(a+b)b^2$
$= a^3+a^2b+2a^2b+2ab^2+ab^2+b^3$
$= a^3+3a^2b+3ab^2+b^3$

Viewing $(a+b)$ as a number, get by the distributive property.
also by dist. prop.
combining like terms,
...which also uses the D.P.

9. 

The area of the entire large square at left is $(a+b)^2$.
The area of the smaller square in the middle is $(a-b)^2$.
The DIFFERENCE between these areas is made up of the four a-by-b rectangles. So: $(a+b)^2-(a-b)^2 = 4ab$
This can be verified using the distributive property of × over + on the integers.

Decreasing amounts of work details are shown on higher-numbered problems.
If unclear, review lower-numbered problems.

1. a. $32 \cdot 32 = 2^5 \cdot 2^5 = 2^{5+5} = 2^{10} = 1024$

   b. $1024 \div 256 = 2^{10} \div 2^8 = 2^{10-8} = 2^2 = 4$

   c. $4096 \div 32 = 4 \cdot 1024 \div 32 = 2^2\, 2^{10} \div 2^5 = 2^{12} \div 2^5 = 2^{12-5} = 2^7 = 2^{8-1} = 2^8 \div 2 = 256 \div 2 = 128$

2. a. $2^8 \cdot 2^7 \div 2^{11} = 2^{15} \div 2^{11} = 2^4 = 16$

   b. $(2^3)^5 \div 2^9 = 2^{15} \div 2^9 = 2_6 = 64$

   c. $256 \cdot 128 \div 2048 = 2^8 \cdot 2^7 \div 2^{11}$

3. (Re PT 6A p 11 15, 16, 17ghi):

   #15. 9, 18, 9/5

   #16 8, 9, 12

   #17 47, 130, 120

4. a. $(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2 = (5 \cdot 5)(5 \cdot 5)(5 \cdot 5) = 5^{2 \cdot 3} = 5^6$

   b. $(5^2)^m = 5^2 \cdot 5^2 \cdot 5^2 \cdot \ldots \cdot 5^2 = (5 \cdot 5)(5 \cdot 5)(5 \cdot 5) \cdot \ldots \cdot (5 \cdot 5) = 5^{2m}$

   c. $(a^2)^3 = a^2 \cdot a^2 \cdot a^2 = (a \cdot a)(a \cdot a)(a \cdot a) = a^{2 \cdot 3} = a^6$

5. a. $3^4\, 5^4 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (5 \cdot 5 \cdot 5 \cdot 5) = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)(3 \cdot 5) = (3 \cdot 5)^4 = 15^4$

   b. $3^4\, b^4 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (b \cdot b \cdot b \cdot b) = (3 \cdot b)(3 \cdot b)(3 \cdot b)(3 \cdot b) = (3b)^4$

6. a. $\dfrac{2^5\, 6^2\, (18)^2}{3^4 \cdot 4^2} = \dfrac{2^5\, (2 \cdot 3)^2\, (2 \cdot 3^2)^2}{3^4\, (2^2)^2} = \dfrac{2^5\, 2^2\, 3^2\, 2^2\, 3^4}{3^4\, 2^4} = \dfrac{2^9}{2^4}\,\dfrac{3^6}{3^4} = 2^5\, 3^2$

   b. $\dfrac{2^5\, (2b)^2\, (2b^2)^2}{b^4 \cdot (4)^2} = \dfrac{2^5\, 2^2\, b^2\, 2^2\, b^4}{b^4\, (2^2)^2} = \dfrac{2^5\, 2^2\, b^2\, 2^2\, b^4}{b^4\, 2^4} = \dfrac{2^9}{2^4}\,\dfrac{b^6}{b^4} = 2^5\, b^2$

   c. $\dfrac{a^5\, (ab)^2\, (ab^2)^2}{b^4\, (a^2)^2} = \dfrac{a^5\; a^2\, b^2\; a^2\, b^4}{b^4\, a^4} = \dfrac{a^9}{a^4}\,\dfrac{b^6}{b^4} = a^5\; b^2$

   d. Letting 3=b and 2=a makes all the above identical. Interesting to note part c is shortest.

7. a. $\dfrac{5^3 \cdot 24^2 \cdot 10^0}{8 \cdot 15^2 \cdot 3} = \dfrac{5^3\, (2^3 \cdot 3)^2 \cdot 1}{2^3\, (3 \cdot 5)^2 \cdot 3} = \dfrac{5^3\, 2^6\, 3^2}{2^3\, 3^3\, 5^2} = \dfrac{2^6}{2^3}\,\dfrac{3^2}{3^3}\,\dfrac{5^3}{5^2} = \dfrac{2^3\, 5}{3}$

   b. $\dfrac{a^3 \cdot (bc^3)^2 \cdot (ac)^0}{c^3 \cdot (ab)^2 \cdot b} = \dfrac{a^3\, (c^3 \cdot b)^2 \cdot 1}{c^3\, (b \cdot a)^2 \cdot b} = \dfrac{a^3\, c^6\, b^2}{c^3\, b^3\, a^2} = \dfrac{c^6}{c^3}\,\dfrac{b^2}{b^3}\,\dfrac{a^3}{a^2} = \dfrac{c^3\, a}{b}$

   c. Replace a with 5, b with 3, and c with 2.

Steps are combined for brevity. If unclear, review lower-numbered problems.

8. a. $\dfrac{6^{21}\cdot 10^{18}\cdot 15^{22}}{30^{11}\cdot 16^{7}} = \dfrac{(2\cdot 3)^{21}\cdot(2\cdot 5)^{18}\cdot(3\cdot 5)^{22}}{(2\cdot 3\cdot 5)^{11}\cdot(2^4)^{7}} = \dfrac{2^{21+18}}{2^{11+28}}\dfrac{3^{21+22}}{3^{11}}\dfrac{5^{18+22}}{5^{11}} = \dfrac{2^{39}}{2^{39}}\dfrac{3^{43}}{3^{11}}\dfrac{5^{40}}{5^{11}} = 3^{32}\,5^{29}$

7 is replaced by n:

b. $\dfrac{6^{3n}\cdot 10^{n+11}\cdot 15^{22}}{30^{11}\cdot 16^{n}} = \dfrac{(2\cdot 3)^{3n}\cdot(2\cdot 5)^{n+11}\cdot(3\cdot 5)^{22}}{(2\cdot 3\cdot 5)^{11}\cdot(2^4)^{n}} = \dfrac{2^{4n+11}}{2^{11+4n}}\dfrac{3^{3n+22}}{3^{11}}\dfrac{5^{n+33}}{5^{11}} = 2^{0}\,3^{3n+11}\,5^{n+22} = 3^{3n+11}\,5^{n+22}$

11 is replaced by m:

c. $\dfrac{6^{3n}\cdot 10^{n+m}\cdot 15^{2m}}{30^{m}\cdot 16^{n}} = \dfrac{(2\cdot 3)^{3n}\cdot(2\cdot 5)^{n+m}\cdot(3\cdot 5)^{2m}}{(2\cdot 3\cdot 5)^{m}\cdot(2^4)^{n}} = \dfrac{2^{4n+m}}{2^{m+4n}}\dfrac{3^{3n+2m}}{3^{m}}\dfrac{5^{n+3m}}{5^{m}} = 2^{0}\,3^{3n+m}\,5^{n+2m} = 3^{3n+m}\,5^{n+2m}$

10. Write each number in scientific notation:

a. $1030 = 1.03 \times 10^3$

$15600 = 1.56 \times 10^4$

$345{,}000{,}000 = 3.45 \times 10^8$

Distributive property!

b. $3.4 \times 10^7 + 5.2 \times 10^7 = (3.4 + 5.2) \times 10^7 = 8.6 \times 10^7$

$6 \times 10^8 + 9.3 \times 10^8 = (6 + 9.3) \times 10^8 = 15.3 \times 10^8 = 1.53 \times 10^9$

c. $(2 \times 10^4) \times (3.2 \times 10^5) = 2 \times 3.2 \times 10^4 \times 10^5 = 6.4 \times 10^9$

$(8 \times 10^4) \times (96 \times 10^{23}) = 8 \times 96 \times 10^4 \times 10^{23} = 768 \times 10^{27} = 7.68 \times 10^{29}$

d. $\dfrac{6 \times 10^9}{3 \times 10^4} = \dfrac{6}{3} \times \dfrac{10^9}{10^4} = 2 \times 10^5$

$\dfrac{5.4 \times 10^8}{9 \times 10^5} = \dfrac{5.4}{9} \times \dfrac{10^8}{10^5} = .6 \times 10^3 = 6 \times 10^2$

e. $(2 \times 10^7)^3 = 2^3 \cdot (10^7)^3 = 8 \times 10^{21}$

$(5 \times 10^4)^3 = 5^3 \cdot (10^4)^3 = 125 \times 10^{12} = 1.25 \times 10^{14}$

Then there's the one they didn't warn you about: $5.67 \times 10^{14} + 3.33 \times 10^{15}$

How do you add these?

Try it and see if you get the answer below.

$3.897 \times 10^{15} \doteq 3.90 \times 10^{15}$