1a.
$$\begin{array}{r} 3\ 1\ 5 \\ +\ 6\ 7\ 2 \\ \hline 9\ 8\ 7 \end{array}$$

1b.
$$\begin{array}{r} 4\ 8\ 3 \\ +\ 8\ 3\ 2 \\ \hline 1\ 3\ 1\ 5 \end{array}$$

2. Easiest to hardest: $\begin{array}{r} 30 \\ +69 \end{array}$ $\begin{array}{r} 39 \\ +70 \end{array}$ $\begin{array}{r} 39 \\ +71 \end{array}$

4a. (Re 3A pp25-27 #2-9)... which place values are rebundled?

#2: 2048 + 2 requires rebundling ONES.

#3: 5840 + 60 requires rebundling TENS.

#4: 3700 + 300 requires rebundling HUNDREDS.

#5a: 1028+234 ONES

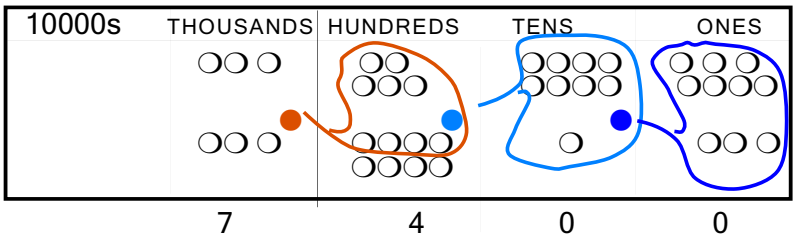
#5c. 4190+649 TENS

#5e. 6204+993 HUNDREDS

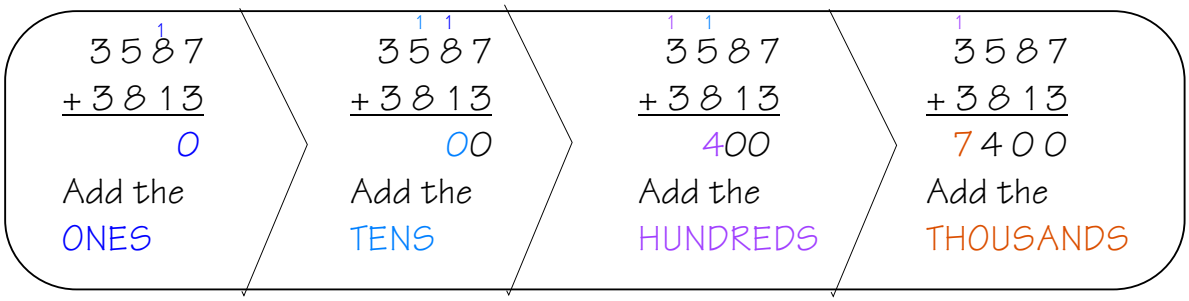
4d. (Re 3A p 27 #9b):

Chip Model:

3587+3813



Box with Arrows:



5. Sam: $\begin{array}{r} 25 \\ +89 \\ \hline 104 \end{array}$ Julie: $\begin{array}{r} 4 \\ 25 \\ +89 \\ \hline 141 \end{array}$ Frank: $\begin{array}{r} 25 \\ +89 \\ \hline 1014 \end{array}$

Sam lost the 1 ten he should have "carried" from the ones' column.
Julie "carried" a 4, and left 1 in the ones' column, when it should have been the reverse:
5+9= 14 , which is 10 + 4. So 4 (4 ones) should have been left, and 1 (1 ten) carried.
Frank ignores place value when writing his totals. The sum of 5 and 9 ones, which is 14, *should* have been bundled to make 1 ten, leaving 4 behind. He then compounds the error by adding the tens, then writing 10 in the hundreds place, thereby saying it is 10 hundreds, or a thousand.

1. The number 832 in expanded form is 800 & 30 & 2. To find 832 – 578, it is convenient to think of 832 as 7 hundreds, 12 tens and 12 ones.

2. To find 1221 – 888, regroup 1221 as 11 hundreds, 11 tens, 11 ones. 1221 = 1100 + 110 + 11

3. 1000 is "9 hundred ninety ten". So 1000 – 888 is $\underset{100}{900-800} + \underset{10}{90-80} + \underset{2}{10-8}$

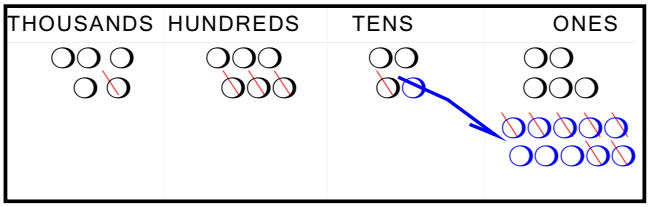
4. Order these easiest to hardest:

8256	8256	8003	(No unbundling; Unbundling; Unbundling across 0)
–7 145	–6589	–6007	

5a. (Re 3A p 30 #5b):
Chip Model for 5645 – 1317:

$$\begin{array}{r} {\scriptstyle 3\,15} \\ 5\,6\,\not{4}\,\not{5} \\ -\,1\,3\,1\,7 \\ \hline 4\,3\,2\,8 \end{array}$$

5645→
– 1317

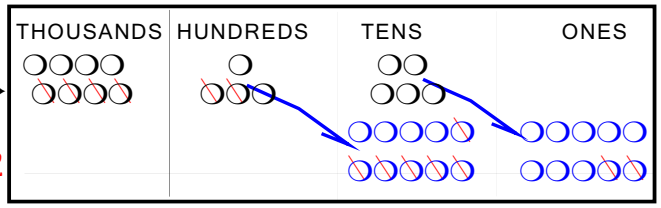


4 3 2 8 ← the DIFFERENCE

5b. (Re 3A p 30 #7b)
Chip Model for 8450–4262:

$$\begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 4\,10} \\ 8\,\not{4}\,\not{5}\,0 \\ -\,4\,2\,6\,2 \\ \hline 4\,1\,8\,8 \end{array}$$

8450→
– 4262

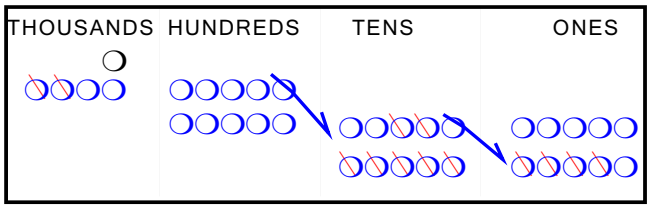


4 1 8 8

5c. (Re 3A p 32 #12d)
Chip Model for 5000 – 2074:

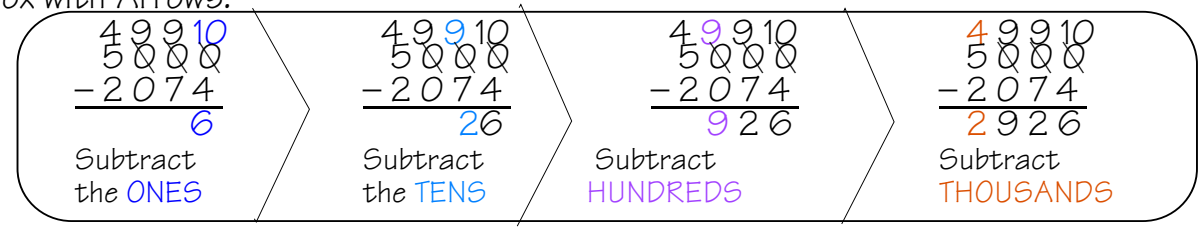
$$\begin{array}{r} 4\ 9\ 9\ {\scriptstyle 10} \\ 5\ \not{0}\ \not{0}\ \not{0} \\ -\,2\,0\,7\,4 \\ \hline 2\,9\,2\,6 \end{array}$$

5000→
– 2074



2 9 2 6

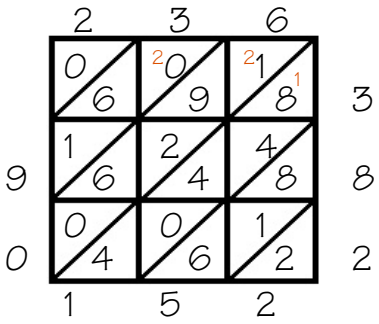
Box with Arrows:



6. (Re 3A p29 #1-14) which place values are rebundled? For the illustrated problems only:
#2 shows a ten unbundled. #3 a hundred is unbundled #4 thousand unbundled
#6 a ten, then a hundred #8 ten, hundred, thousand
#10 regroups across zeroes: 1000 becomes 9 hundreds, 9 tens, and 10 ones.
#13 regroups across zero: 100 is exchanged for 9 tens & 10 ones. A thousand is unbundled.

8. Sam subtracts the smaller digit from the larger digit in each column.
Julie exchanged a ten for ten ones, but left "10" in the tens column, instead of reducing to 9.
Frank: a hundred is ten TENS, not ten ONES

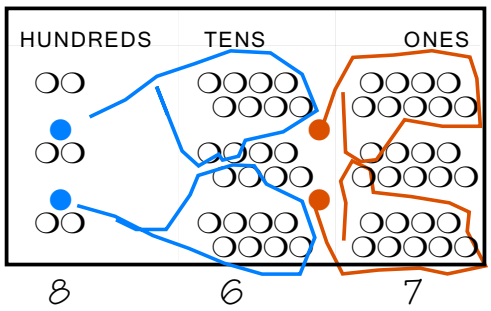
1c. Compute 236 x 382 via lattice:



The product is 90152

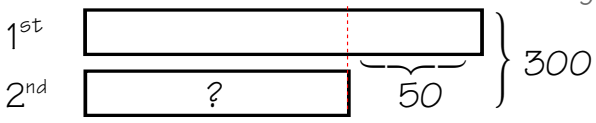
3. (Re 3A p53 #9g)

$$\begin{array}{r} {\scriptstyle 2\ \ 2\ \ } \\ 289 \\ \times\ \ 3 \\ \hline 867 \end{array}$$



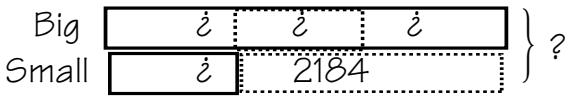
6. (Re 4A Practice 2B 8 only)

#8: 300 children are divided into two groups. 50 more in the 1st than 2nd. How many in 2nd?



The numbers to the left of the red line sum to 300– 50, which is 250. 250 ÷2 = 125
The second group contains 125 children.
Check: 1st group contains 125+50 = 175; 125+175 = 300 ✔

#9 Difference between two numbers is 2184. Bigger number is 3 times the smaller. Find sum.



Two units (¿) = 2184
Four units = 2x2184 = 4368
The sum of the two numbers is 4368.

8. Sam:

$$\begin{array}{r} 32 \\ \times\ 7 \\ \hline 2114 \end{array}$$

Julie:

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ 27 \\ \times\ 4 \\ \hline 88 \end{array}$$

Frank:

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ 37 \\ \times\ 4 \\ \hline 118 \end{array}$$

Sam is ignoring place value. He is multiplying 7x2 and 7x3 and writing those products without regard to place value. When he multiplied 7x2, and got 14, he should have "carried" the 1 (1 ten) to the tens column to add onto the product of 7x3. That 7x3 product is really 7x30 which is 210, not 2100 ... 7 x 3 tens = 21 tens, not 21 hundreds!

Julie apparently forgot to add the "carried" 2 to the product of 4x2.

Frank may have multiplied 4x3 incorrectly, then added two, or not. It is hard to tell just what he did.

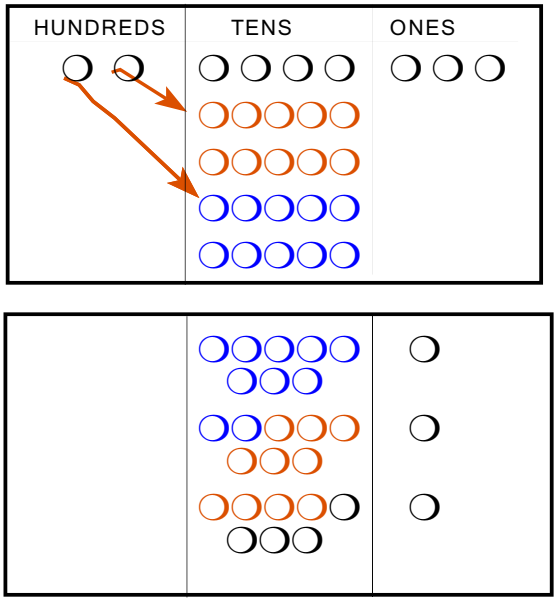
1. (Re 3A p 57 examples) make a word problem using quotient and remainder.
Gery has 67 hard candies, which he is putting into party favor bags, 6 in each bag. How many party favor bags can be filled, and how many candies will be left over? 67 ÷ 6 = 11
67 = 6 x 11 + 1 Gery can fill 11 party favor bags with 6 candies each, and 1 candy will be left.

4. (Re 3A p 62 example) draw chip model for 243 ÷3

$$\begin{array}{r} 081 \\ 3\overline{)243} \\ \underline{24} \\ 3 \end{array}$$

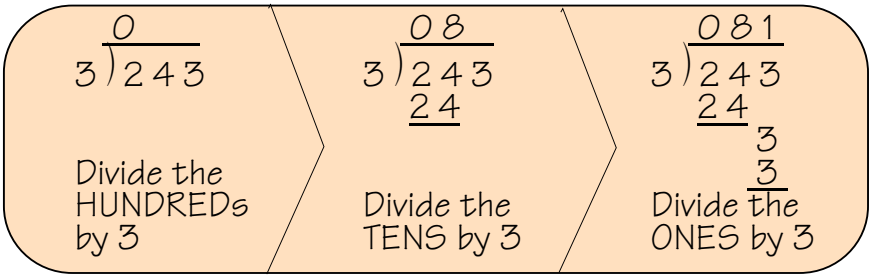
2 cannot be split into 3 equal parts, so we exchange or unbundle 2 100s for/to 20 tens.

Then split the resulting 24 tens into 3 equal parts...



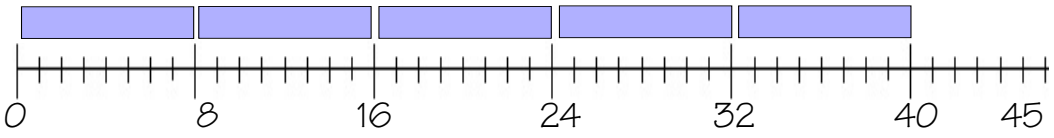
Showing 243 = 3 x 0 8 1

Box with Arrows:



5. Cupcakes are put into boxes of 8 each for the bake sale. How many boxes can be filled if there are 45 cupcakes? How many cupcakes will be left over?

How many 8s are in 45?



$$\begin{array}{r} 5 \\ 8\overline{)45} \\ \underline{40} \\ 5 \end{array}$$

← number of segments
← 5 segments of length 8
← units remaining to complete 45.

6. Show 17456 ÷ 8 in the manner of example 4.7
This is shown at right .

$$\begin{array}{r} 2187 \\ 8\overline{)17456} \\ \underline{16000} \\ 1456 \\ \underline{800} \\ 656 \\ \underline{640} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

← 2 segments of 8000
← left over, still to go
← 1 segment of 800
← units still to go
← 8 segments of 80
← remaining
← 7 segments of 8
← happiness is 0 remainder.

Estimation, like mental math, is an art. Techniques vary, and what occurs to one person at one point in time might well differ from the approach seen by someone else, or by the same person at another time.

3. (Re WB 5A [p14-17] Ex 5 & Ex 6):

Exercise 5 (pp14-15)

#1*** These are all essentially multiplication by powers of 10.
Do them mentally & use a calculator to check. [That's appropriate use of calculator.]
EG 392 x 800 = (392x8) x 100 = (784x4) x 100 = (1568x2) x 100 = 313,600

#2a. 326 x 47 ≈ 300 x 50 = 3 x 5 x 100 x 10 = 150,000

#2b. 78 x 586 ≈ 80 x 600 = 48,000
[Since both factors were rounded up, this is high. Calc. says 45,708.]

#2d. 4165 x 53 ≈ 4200 x 50 = 210,000

#3. 28 x 229 ≈ 30 x 230 = 6900 ...both were rounded up, so this is definitely high.
28 x 229 ≈ 30 x 200 = 6000 ...rounded 229 down substantially, so this is low.
Cost of radio sets is between $6000 and $6900.... Say $6450.

#4. 114 x 92 ≈ 110 x 90 = 9900 low. Area is close to, and above, 9900 square inches.

Exercise 6 (pp16-17)

#1 Mental math – see comment*** at #1 above.

#2a. 282 ÷ 52 ≈ 300 ÷ 50 = 30 ÷ 5 = 6

#2b. 324 ÷ 42 ≈ 320 ÷ 40 = 32 ÷ 4 = 8

#2c. 4406 ÷ 49 ≈ 4400 ÷ 50 = 440 ÷ 5 = 88

#2d. 1705 ÷ 31 ≈ 1800 ÷ 30 = 60
1705 ÷ 31 ≈ 1710 ÷ 30 = 171 ÷ 3 = 57 (and this is still high)

#3. 805 ÷ 28 ≈ 800 ÷ 28 = 200 ÷ 7 ≈ 30 Each CD cost him approximately $30.

#4. 1044 ÷ 36 ≈ 261 ÷ 9 ≈ 270 ÷ 9 = 30 The width of the hall is about 30 m.

5. 351 + 456 if rounded to the nearest hundred, before addition, we get 400 + 500 = 900.
But in fact 351 + 456 is very close to 350 + 450 = 800. The true total is 807.

6a. To get a high estimate for 1556 – 371,
round 1556 up to 1600, and 371 down to, say, 350, get 1250.

6b. To underestimate 3462 ÷ 28, round 28 up to 30.
3462 ÷ 28 ≈ 3462 ÷ 30 ≈ 346 ÷ 3 ≈ 115

6c. 5750 ÷ 800 ≈ 5600 ÷ 800 = 7 and that's an UNDERestimate, so we can SAFELY say the elevator should lift 7 of those 800-pound gorillas.

2a. (Re PT5A p 29 #5adgj):

#5a. $17\overline{)63}$ = 3 R 12 (51 / 12) Estimation: $63 \div 17 \approx 60 \div 20 = 3$

#5d. $34\overline{)76}$ = 2 R 8 (68 / 8) Estimation: $\approx 70 \div 35 = 2$ (Or just think 2x34 = 68)

#5g. $67\overline{)149}$ = 2 R 15 (134 / 15) Estimation: $\approx 140 \div 70 = 2$ (Or just think 3x67 > 180)

#5j. $72\overline{)668}$ = 9 R 20 (648 / 20) Estimation: $\approx 630 \div 70 = 9$ (Or just think 10x72 = 720 > 668)

2b. On pages 29-30, what component skill is emphasized?
Answer: a lot of estimation is being flaunted, such as in problem 1: 14Ø ÷2Ø

2c. Why no chip models here?
The purpose of the chip model is to make sense of the algorithm, and is feasible with very small divisors. Further, at this point, the students have had man y opportunities to understand the division algorithm. The division process should be nearly automatic. Finally, the divisors are growing quite large, too unwieldy for the chip model.

2d. (Re 5A p 31 #16abd)

#16a. $28\overline{)6692}$ = 239 R 0 (56 / 109 / 84 / 252 / 252)
➪ Estimation: $66 \div 28 \approx 70 \div 30 = 2^{+}$ (Or 3x28 > 3x25 = 75>66)
➪ Est: $109 \div 28 \approx 100 \div 25$ but 4 x 28 = 4x(25 +3) = 100 +12 >109 So reduce quotient to 3
➪ Est: 280 ÷ 28 = 10, so 252 ÷ 28 might be close to 9...

#16b. $18\overline{)2409}$ = 133 R 15 (18 / 60 / 54 / 69 / 54 / 15)
➪ Estimation: $24 \div 18 \approx 1$
Est: $60 \div 20 = 3$
Est: Since 3x18 = 54, and 54+18 > 69, this quotient is also 3.
or $70 \div 20 = 3+$, not 4.

#16d.
```
     107   R 15
56 ) 6008
     56
     40
      0
     408
     392
      18
```
➪ Estimation: $60 \div 54 \approx 1$

Est: $40 \div 54$ is less than 1, so must be 0

Est: $400 \div 60 \approx 7$ (6 or 7)

4. What do you tell Tracy when she writes the following?

```
      7  5   R5
6 ) 4 2 3 5
    4 2
      0 3 5
        3 0
          5
```

```
      7  0  5   R5
6 ) 4 2 3 5
    4 2
      0 3 5
        3 0
          5
```

I would tell Tracy that the "7" must be in the hundreds place, because she was dividing 6 into the 42 hundreds; and an easy way to make sure it is right is to always place the quotient digit in the ending column of the division taking place.

I would then point out that when she "brought the 3 down", she should have completed the step, that is to divide 6 into "03". The result is 0 and that goes above the 3.

Every time we bring a digit down, we should complete the process, which is to find the correct digit in the quotient. This makes sure that all the quotient digits keep their correct place value.