Section 1.1 some answers

1. a. 38, 3213, 2012, 2030

   b. |||| ||||, ∩∩∩|||| |||, 𓍢𓍢𓍢𓍢𓍢𓍢∩∩∩∩|||| ||||, 𓆼𓍢𓍢𓍢 ∩∩∩∩||||||||

   (Egyptian characters may be stacked two high, as shown below. See #3b.)

   c. 3733, 1970, 2059, 444

   d. LXXXVI, CXLIX, CCLXXXIV, MMMCMXLII

3. a. ,

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   𓍢  ∩∩∩  || |||             135
+ 𓍢𓍢𓍢 ∩∩∩ ∩∩∩ |            + 361
  𓍢𓍢𓍢𓍢∩∩∩∩∩∩∩∩∩ ||| |||       496
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   b.

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  𓍢𓍢∩∩∩∩∩∩∩ | | |            273
+ 𓍢  ∩∩  | | | | |          + 125
     ∩∩∩∩   | | | |           398
𓍢𓍢𓍢 ∩∩∩∩∩   | | | |
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5. a. 6,000,003,408 (Note: "four hundred and 8" is incorrect form.)

   b. 2,000,003,000,009,506

   c. 23,326

   d. 54,026

   e. 13,205

6. a. One thousand, three hundred forty-seven

   b. Five thousand, nine hundred

   c. Seven thousand, fifty-eight

   d. Seven billion

   e. Sixty-seven trillion, three hundred forty-five billion, eight hundred ninety-two million, eight hundred sixty-eight thousand, seven hundred thirty-six

7. a. | ·TEN = ∩

   b. ∩·TEN = 𓍢

   c. 𓍢 ·TEN = 𓆼

   d. 𓍢 ∩|·TEN = 𓆼𓍢∩

   e. ∩∩|||·TEN = 𓍢𓍢∩∩∩

   f. 𓍢𓍢∩|||·TEN = 𓆼𓆼𓍢∩∩∩

Section 1.2 some answers

1. a. Expanded
   b. Hundreds
   c. Look at the second (next) digit; in general, progress until they don't match. Using the same digits directs the student to see the importance of place value.
   f. (9) 5073, 4973
      (10) 1000, 9999
      (11) 4123, 3412, 3142, 2431
      (12) 913, 1703, 1892, 9003
   g. (10) 1000, 9999
      (13) 540, 405

2. a. (Refers to 3A p 11 problem 14a) Put the largest digit in the thousands place; the next largest in the hundreds place and the third largest in the tens place.
   b. 2078
   c. (refers to 3A p 12 problem 5) 1736, 7504, 90, 800, 3, 900
   e. subtract 1000, thousands

3. a. (refers to 4A p 9 problem 3) the teacher
   b. 8000, 60,000
   c. Textbook 4A, p. 10, problem 7a:

Ten Thousands	Thousands	Hundreds	Tens	Ones
10000 10000	1000 1000 1000 1000 1000 1000	100 100 100	10 10 10 10	1 1 1 1 1

   d. (8a) A 5100, B 5300, C 5700, D 5900, E 6400
   e. (refers to of 4A, p 11, problem 10):
      (c) 15, 000 – 6, 000 = (15–6) thousands = 9 thousands = 9, 000
      (e) (7 x 4) thousands = 28, 000
      (g) (12 ÷ 3) thousands = 4, 000

4. (refers to 5A page 10)
   (1a) 11,012
   (1c) 700,013
   (2e) Six million, twenty thousand
   (3e) 8,000
   (4c) 184,900
   (5c) 103,002, 113,002
   (5e) 7,742,000, 5,742,000
   (6c) 425,700, 2,357,000, 2,537,000, 3,257,000

6. a. $324_{FIVE} = 89_{TEN}$, $1440_{FIVE} = 245_{TEN}$
   b. 321_{FIVE}, 2133_{FIVE}
   c. 1101_{FIVE}

Section 1.3 some answers

1. a. ♣♣♣♣♣♣♣♦♦♦ = ♦♦♦♣♣♣♣♣♣♣ (Total amounts equal)
   b.  (Total lengths equal)

2. a. Compensation: $86 \overset{14}{\frown} + 34 = 100 + 20$. Takes 14 from 34, adds it to 86.
   b. Commutative property of addition (says a+b = b+a)
   c. Additive Identity property
   d. Both associative & commutative properties of + are needed to get (34+66) +82
   e. Commutative property of addition

3. a. $91 + 15 + 9 = (91 + 9) + 15 = 100 + 15 = 15$
   b. $4+17+32+23+36+20 = (36+4)+(17+23)+20+32 = 40+40+20+32 = 132$
   c. $75 + 13 + 4 + 25 = (75 + 25) + (13 + 4) = 100 + 17 = 117$
   d. $28 + 32 + 35 + 7 = (28 + 32) + (35 + 7) = 60 + 42 = 102$
   e. $34 + 17 + 6 + 23 = (34 + 6) + (23 + 17) = 40 + 40 = 80$

4. a. $7 + 9 =$ twice $8 = 16$
   b. $19 + 21 =$ twice $20 = 40$
   c. $24 + 26 =$ twice $25 = 50$
   d. $6 + 4 =$ twice $5 = 10$

5. (refers to PM3A p 22 #1,2):

   (1a) $509 \overset{9}{\frown} + 365 = 500 + 374 = 874$

   (1b) $128 \overset{20}{\frown} + 280 = 108 + 300 = 408$

   (1c) $384 \overset{16}{\frown} + 418 = 400 + 402 = 802$

   (2a) $746 \overset{46}{\frown} + 254 = 700 + 300 = 1000$

   (2b) $262 \overset{38}{\frown} + 138 = 300 + 100 = 400$

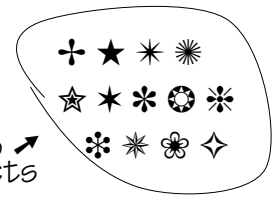
   (2c) $432 \overset{32}{\frown} + 368 = 400 + 400 = 800.$

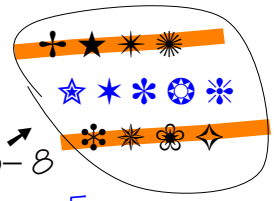
7. a. $\leq$
   b. $=$
   c. $\geq$
   d. $\approx$
   e. $\neq$

8. a. "Ryan = $2" means that Ryan IS $2. The student meant "Ryan's money is $2"
   b. $4.8203 \approx 4.8$ or $4.8203 \doteq 4.8$ (the latter is actually more correct).
   c. $(3 + 15) \div 2 + 6 = 18 \div 2 + 6 = 9 + 6 = 15.$

Section 1.4 some answers

1. a. Illustrate 13 – 8 using take-away set model:





b. Illustrate 16 – 7 on the number line

16 – 7 = ? Where 7 + ? = 16

(This is Part-whole or "missing addend")

2. a. Helper defines difference of two numbers as the LARGER – the smaller.
The difference between 9 & 3 = the difference between 3 & 9 = 9 – 3 = 6.

b-i. (Refers to 3A p20 # 4,6,7):
(4) Part-whole
(6) Comparison
(7) Part-whole

b-ii (refers to 3A p 23 #6, 8b, 9a):
(6) Part-whole
(8b) Take away ("sold"= "taken away")
Or Part-whole (sold + left = total)
(9a) Part-whole

3. a. 34 – 10 + 17 – 27 = 10 – 10 = 0 *(This problem is NOT in the spirit of this chapter;*
b. 36 – 16 + 28 – 4 = 20 + 24 = 44 *it requires rearranging with "–" and negative #s.)*

4. Counting up:
a. 14 – 8:
8 →10→ 14
2 + 4 = 6

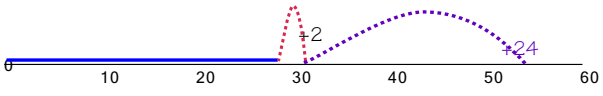
b. 178 – 96:
96 → 100 → 178
4 + 78 = 82

c. 425 – 292:
292→300→400→425
8 + 100 + 25 =132

5. Compensation *(change both the same way, difference won't change)*:
a. 57 – 19 = 58 – 20 = 38
b. 86 – 18 = 88 – 20 = 68
c. 95 – 47 = 98 – 50 = 48
d. 173 – 129 = 174 – 130 = 44

6. a. Illustrate 54 – 28 using a take-away model:
54 can be represented by ❿❿❿❿❿ ①①①①
But one ❿ = ten ①,
so 54 can also be: ①①①①① ❿❿❿❿ ①①①①①
Now we can take away 28: ①①①①① ❿❿❿❿ ①①①①①
... and we see that what's left is 26

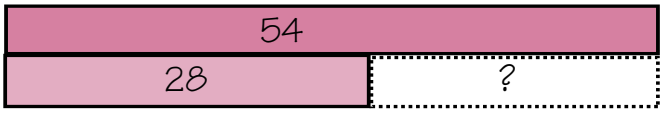
b. Counting-up:



$28 \underbrace{+2+24}_{26} = 54$

c. Comparison:
54 can be represented as ❿❿❿❿❿ ①①①①
Which is the same value as ❿❿❿❿ ①①①①①①①① ①① ①①①①
While 28 can be... ❿❿ ①①①①①①①①
So the difference is ❿❿ and ①① ①①①① = 26

Section 1.4 a few more answers

6. d. 

Measurement model for 54 – 28
Comparison interpretation

7. Make up first grade word problems for...
   a. Take-away interpretation for 15 – 7:
      Jill has 15 stickers. If she gives away 7 of them, how many will she have left?

   b. Part-whole interpretation for 26 – 4.
      Principal Orasco has 26 pencils. Four of them are used. How many are new?

   c. Comparison interpretation for 17 – 5.
      Juana's older sister is 17. Juana is five. How much older is her older sister?
      Well this is not such a great FIRST-grade problem— not quite tangible enough.

8. a. In which grade should teaching of subtraction facts begin?
      Subtraction should be learned along with addition.
   b. What is subtraction within 20?
      Subtraction problems from 1 – 1 (or possibly 0 – 0), up to 19 – 9.

Section 1.5 some answers

#1 & #2 all refer to parts of Primary Text 3A.

1a. (Re p40) They are illustrating the commutative property of multiplication.

1b. i. (Re pp41-2 #3-5) What model is being used in each?

(3) Mostly Set model, but parts b and c could be interpreted as arrays.

(4) Measurement model, since each ring is "worth" 2 points, forcing a "measure".

(5) Set model. But one can make an argument for rectangular array:

The illustration seems to be inviting us to view the question as shown at left!

ii. (Re p 46 #3,4) The word problems describe set models, but illustrate them using bar diagrams– a type of measurement model.

iii . (Re p47 # 6 & 9)

(6) "...6 rows of chairs, 30 chairs altogether; how many chairs in each row?"

Looks like a rectangular array problem to me.

(9) "Harry weighs 38 kg... four times as heavy as his brother. "

Measurement model. This looks more like division than multiplication!

iv. (Re p49)

Here they are clearly showing rectangular arrays.

1c. *What is the purpose of four-fact families such as those in #2 on page 40?*

The purpose is to reinforce the relationship between multiplication and division, and make students realize that division facts are related to multiplication facts.

2a. *What are the students asked to make on pp 68-69? What ... for?*

Multiplication "flash cards", for multiplication facts drill.

2b. *What model in #1 p 71? What property is illustrated in #1b?*

Repeated addition, though one can also argue for the rectangular array.

#1b illustrates the commutative property of multiplication with an array.

2c. *(Re pp71-72 #2) uses what property?*

Illustrates use of distributive property to obtain 6x6 and 6x7 from 6x5.

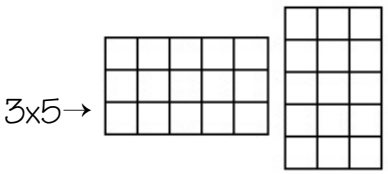
2d. *Draw a rectangular array illustrating how 6x6=36 can be used to find 6x12.*

2e. *(Re p73 #3)*

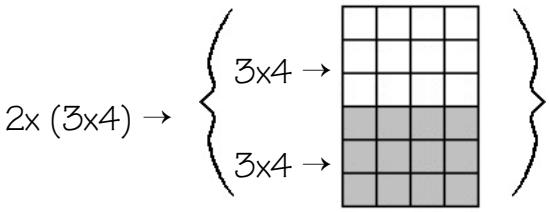
commutative property of multiplication

Section 1.5 more answers

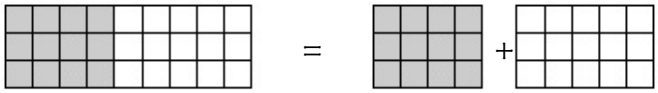
3a. Rectangular array showing 3x5 = 5x3 :



3b. R. Array showing 2x(3x4) = (2x3)x4 :



3c. R. Array showing 3x(4+5) = 3x4 + 3x5:



3 x (4+5) = 3x4 + 3x5

4. *Identify the properties:*

4a. 7 x 5 = 5 x 7 uses the Commutative Property of Multiplication

4b. 6 + 0 = 6 uses the Identity Property of Addition

4c. 3+(5+2) = (3+5)+2 uses the Associative Property of Addition

4d. I x ∩∩II = ∩∩II uses the Identity Property of Multiplication (1x22 = 22)

4e. 3+4 = 1(3+4) uses the Identity Property of Multiplication (1x7 = 7)

The point of #4d & #4e is that the property is inherent to multiplication; the property does not depend on how the numbers are written as numerals, or even as more complex expressions.

4f. 3(8x6) = (3x8)6 uses the Associative Property of Multiplication.

The omission of the "x" symbol does not change the fact that the operation is multiplication!

4g. (7x5) + (2x5) = (7+2)x5 uses the Distributive Property of Multiplication over Addition.

Note that the parentheses on the left side are not required. ie 7x5 + 2x5 means (7x5) + (2x5)

5. Says the text: " To multiply a number by 5, take half the number, then multiply by 10. "
P.S. From me to you: You can also multiply by 10, then take half !

Steps must be shown

6 x 5 = (6 ÷ 2) x 10 = 3 x 10 = 30 or 6 x 5 = 6 x 10 ÷ 2 = 60 ÷ 2 = 30

[*Problem says do the same for 8, 7, 12, 23, 84 (not shown) and 321....*]

321 x 5 = 320 x 5 + 5 = (320 ÷ 2) x 10 + 5 = 160 x 10 + 5 = 1600 + 5

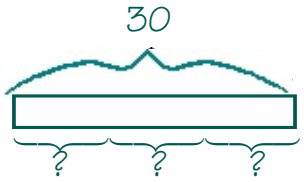
or 321 x 5 = 321 x 10 ÷ 2 = 3210 ÷ 2 = 1605

6. *Compute 25x16 in your head by thinking of 15 as 10+5.*

24 x 15 = 24 x (10 + 5)
= 240 x 10 + 24 x 10 ÷ 2
= 2400 + 2400 ÷ 2

Section 1.6 some answers

1. *Identify as measurement (MD) or partitive ({D) division.*



1a. PD "Partioning 30 into 3 bundles results in what size bundles?"

1b. MD "24 balls packed into boxes [bundles] of 6 [each]. How many [bundles]?"
"How many 6s are in 24?"

1c. MD

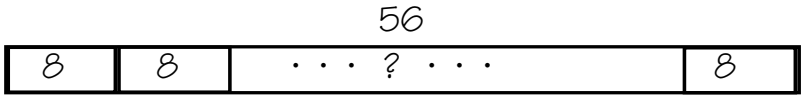
1d. PD

1e. PD

1f. PD

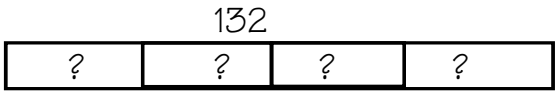
2a.	(Re 3A p43)	PD, PD	#10 is PD #11 is PD also.
2b.	(Re 3A p65)	MD, PD, MD, MD, MD	#6 MD #7 PD #8 MD #9 MD #10 MD
2c.	(Re 4A p35)	PD, PD, MD	#7 PD #8 PD #9 MD

3a. *Illustrate with bar diagrams: MD for 56 ÷ 8 :*



The 8s can be shown inside or outside. The question, which should be made clear, is: How many 8s, or groups of 8, is 56?

3b. *Illustrate Partitive division for 132 ÷ 4 :*

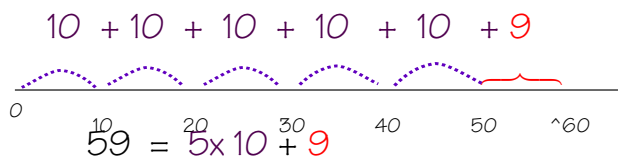


Does this picture say "If 132 is partitioned into 4 equal parts, How big is each part?

4a. *Make up a word problem for MD for 84 ÷ 21.*
Using MD, the question is: 84 is how many groups of 21 ?
Every classroom has 21 students. The 3rd and 4th grade classes had a meeting l the auditorium; there were 84 students. How many classes participated in that meeting?

4b. *Make up a word problem for PD for 91÷ 5.*
Using PD, the question is: 91 is how many groups of 5? (And how many are left over?)
The Principal had to put 91 children into 5 classrooms. To make the classes as equal in size as possible, how many children are in each classroom? How many children are left over, and still need a classroom?

5a. *Illustrate the Quotient-Remainder Theorem: for 59 ÷ 10 on a number line:*



5b. *...using a set model for 14÷ 4 :*