7.1 Text HW Ratio & Proportion   Math 210 pbf8

1. (Re PT5A pp71-79 Practice 5A)

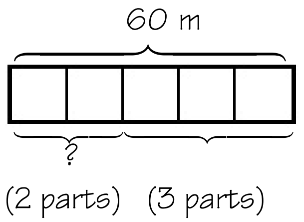
#1a,b. 3 : 6 = 1 : 2   6 : 4 = 3 : 2

#2. L:W is 16 : 12 = 4 : 3

#3. Saved: Spent = 35 : 15

#4. 2 : 7 = 4 : ? ? = 2·7 = 14 She used 14 liters of water.

#5 60 m is divided into two parts, ratio 2: 3. Hmm... 5 parts altogether ... each part is 12 m....



5 parts = 60 m
1 part = 12 m
3 parts = 36 m
2 parts = 24 m
} Checks out, adds up to 60 m

The shorter piece is 24 m long.

#6. Aw : Jw is 6 : 5. If Aw is 48 kg, find Jw.
48 kg is 6 parts, so 1 part is 8 kg, and 5 parts is 40 kg. John's 'weight' is 40 kg.

#7. The ratio of B : G is 2 : 5 . If there are 100 B, how many B&G altogether?

Algebraic-type solution:

2 : 5 = 100 : ?
Solve for ?, get 250.
So there are 100+250 = 350 altogether.

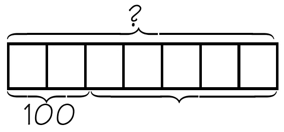
By the way, to solve for ?, I'd write $\frac{x}{100} = \frac{5}{2}$

OR: 2 : 7 = 100 : ?
Solve for ? Get 350, getting the total number of children directly.

"Scaling" solution:

OR 2 parts = 100
1 part = 50 ⇒ So there are 7 parts = 350 altogether.
OR 5 parts = 250 ⇒ There are 100 + 250 = 350 altogether.

"Bar-diagram" solution:



Once the bar diagram is drawn, the problem proceeds pretty much as shown via the "scaling method" above.

2. (Re PT 5A, pp 80-82, Practice 5B #4-9)

#4. B : G is 7 : 4 If 121 total, the number of boy is 7·11 = 77
(11 parts total = 121, 1 part is 11, 7 parts is 7·11)

#5. Wm has $120. Steve has $20 less. What is ratio of Steve's $ to Wm's. $? 100 : 120 = 5 : 6

#6. 90 cm pole is painted G, W & B, in ratio 3 : 4 : 2 . a. What length is G? b. What length is B?

3 : 4 : 2 ⇒ 9 parts total.

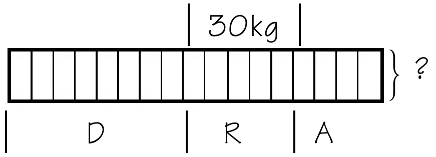
9 parts = 90 cm
1 part = 10 cm
3 parts = 30 cm

a. 30 cm length painted Green.
b. 20 cm length painted Black.

#7. Cement, Sand & Stone are mixed in the ratio 1 : 2 : 4.
If Sand & Stone total is 24 m^3.... C = ?   [In class we misread the question!!!]
Sand & Stone together make 6 parts. So 6 parts = 24 m^3
and 1 part = 4 m^3
Cement is "1 part", so we need 4 m^3 of Cement.

2 cont'd. Teacher's Solution required for #8 & 9.

#8. "The ratio of David's weight to Ryan's weight to Ali's weight is 8 : 5 : 4.
If Ryan weighs 30 kg, find the total weight of the 3 boys."



÷5: 5 parts = 30 kg
1 part = 6 kg
· 17: 17 parts = 6·17 = 102 kg

The total weight of the three boys is 102 kg.

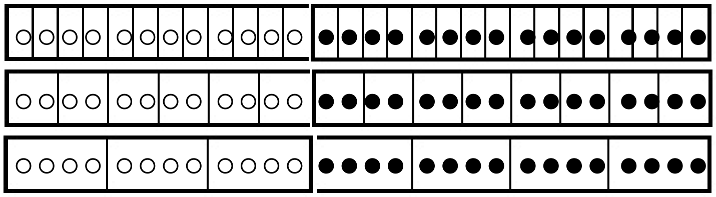
#9. "3 boys share a sum of money in the ratio 5 : 3 : 2.
If the smallest share is $30, find the biggest share."

The ratio of the smallest share to the biggest share is 2 : 5.

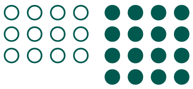
÷2: 2 parts = $30
1 part = $15
· 5: 5 parts = 5·$15 = $75

The biggest share is $75.

3. Ratio 12 to 16 →
Ratio 6: 8 →
Ratio 3:4 →



The above is shown in the style of the text. However,



← this also illustrates that 12 : 16 is 3 : 4

4. (Re PW 5A pp 82-90) text instructs us to not write the answers. Answers to check your work:

Exercise 34 (pp 82-83)

#1a.	3 : 4	#1b.	3 : 4
#2a.	5 : 3	#2b.	3 : 5
#3a.	2 : 3	#3b.	3 : 2
#4a.	5 : 3	#4b.	3 : 5
#5a.	3 : 7	#5b.	7 : 3
#6a.	6 : 5	#6b.	5 : 6

Exercise 35 (pp 84-85)

#1a. 12 : 30 = 2 : 5

#1b. 9 : 15 = 3 : 5

#2.	2 : 3	3 : 1	#3a.	5	#3g.	1
	1 : 4	3 : 5	#3b.	32	#3h.	12
	5 : 3	2 : 1	#3c.	36	#3i.	4
	5 : 6	4 : 5	#3d.	28	#3j.	15
	1 : 2	5 : 4	#3e.	1	#3k.	1
			#3f.	2	#3l.	4

#4. Length : width = 60" : 48" = 5 : 4

#5. 24 : 16 = 3 : 2

#6. 52 : 72 = 13 : 18

4 cont'd.

Exercise 36 (pp 86-87)

#1. 4 parts = 60; 7 parts = 105
There are 105 apples.

#2. 3 parts = 42cm; 8 parts = 112cm
The ribbon was 112 cm long.

#3. 3 parts = $24; 5 parts = $40 The blouse cost $40.

#4. 10 parts = $280. 4 parts = 4·$28 = $112. John received $112 more than Peter.

Exercise 37 (pp 88-89)

#1. 4 : 8 : 6 = 2 : 4 : 3
#2. 9 : 6 : 12 = 3 : 2 : 4
#3. 6 : 2 : 8 = 3 : 1 : 4
#4. 12 : 10 : 8 = 6 : 5 : 4
#5. 24 : 18 :30 = 4 : 3 : 5
#6. 12 : 12 : 8 = 3 : 3 : 2

Exercise 38 (p90)

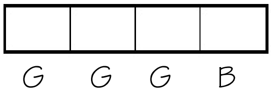
#1 Ratio of B to G to W is 5 : 2 : 3.
There are 90 B....

·2 5 parts = 90 beads
10 parts = 180 beads There are 180 beads total.

#2. 45 cm total is divided into sides in the ratio 3 : 2 : 4
Find the length of the longest side.
9 parts = 45 cm
4 parts = 20 cm is the longest side's length.

5. [Study the textbook! Hey! #@$%! What's #4 about?] (P.S. #@$% is zrklian for "I love it.")
(Re PT 6A Teacher's Solutions for Practice 3A#6,7 & Practice 3B #6-9)
Practice 3A #6:
There are 3 times as many girls as boys in a school choir.

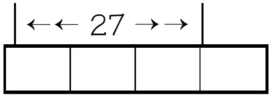
#6a.



What is the ratio of Girls to total in Choir? 3 : 4

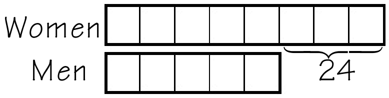
#6b. What fraction of the children [in the choir] are boys? One fourth.

#6c. If there are 27 girls,



how many children are there altogether?
3 parts = 27 so 1 part = 9 & 4 parts = 36. There are 36 children [in the choir].

#7. "The number of men is $\frac{5}{8}$ the number of women working in a factory.
If there are 24 more women than men, how many workers are there altogether?"



Notice everything in this problem is about comparison!
←The comparative bar diagram reveals all!

÷3 3 parts = 24 workers
1 part = 8 workers
·13 13 parts = 104 workers There are 104 workers altogether.

But wait! There's more.......

5 cont'd.

Practice 3B #6-9:

#6. "A sum of money was shared between Susan and Nancy in the ratio of 2 : 5.
Nancy received $36 more than Susan. How much money did Susan receive?"

The setup on this problem is very similar to that of problem #7 above.

Difference: 3 parts = $36.
÷ 3
1 part = $12.
· 2
Susan: 2 parts = $24
Answer: Susan received $24.

#7. "The ratio of Peter's money to Paul's money is 5 : 3.
If Peter has $25, how much do they have altogether?"

Peter: 5 parts = $25
÷5
1 part = $5
·8
Together: 8 parts = $40

Peter and Paul have $40 altogether.

Check: 5 parts = $25, 3 parts = $15. Total is $40, ✔
And ratio is 25 : 15 = 5 : 3 ✔

#8 "The ratio of the number of Chinese books to the number of English books in a library is 4 : 7.
There are 2200 Chinese books and English books altogether.
How many English books are there?"

There are altogether (4+7=) 11 parts. Those 11 parts = 2200 books.
÷11
1 part = 200 books
· 7
English books make up 7 parts. Those 7 parts = 1400 books
There are 1400 English books.

Check: 7 parts = 1400; 4 parts = 800.
Ratio of E to C is 1400 : 800 = 7·200 : 4·200 = 7 : 4 ✔
Total of books = 1400 + 800 = 2200 ✔

#9. "The sides of a triangle are in the ratio 4 : 5 : 6.
If the perimeter of the triangle is 60 cm, find the length of the shortest side."
Perimeter of triangle is the sum of the lengths of the three sides.
There are 15 parts altogether. 15 parts = 60 cm
1 part = 4 cm
4 parts = 16 cm
The shortest side has length 16 cm.

Check: 4 parts = 16 cm; 5 parts = 20 cm; 6 parts = 24 cm.
16 cm + 20 cm + 24 cm = 60 cm ✔
16 : 20 : 24 = 4·4 : 5·4 : 6·4 = 4 : 5 : 6 ✔

1a. $4\cdot 10 + 3\cdot 1 + 2\cdot\frac{1}{10} + 8\cdot\frac{1}{100} = 43.28$

1b. $3 + 7\cdot\left(\frac{1}{10}\right)^2 + 5\cdot\left(\frac{1}{10}\right)^7 = 3.0700005$

1c. $18\cdot\left(\frac{1}{10}\right)^5 = 0.00018$

2g. $32 \div 16000 = (32 \div 16) \div 1000 = 2 \div 1000 = 0.002$

This is probably easier to visualize in fraction form: $\frac{32}{16000} = \frac{16\cdot 2}{16\cdot 1000} = \frac{2}{1000} = 0.002$

2h. $52{,}000{,}000 \div 130{,}000 = \frac{52{,}000{,}000}{130{,}000} = \frac{52\cdot 10^6}{13\cdot 10^4} = \frac{13\cdot 4\cdot 10^6}{13\cdot 10^4} = 4\cdot 10^2 = 400$

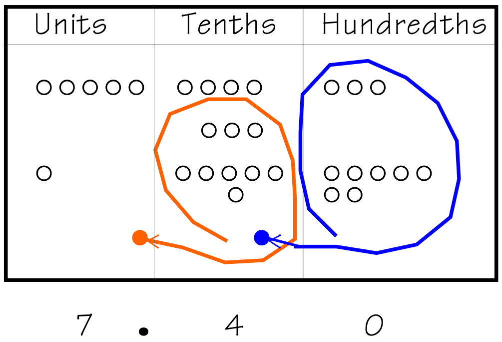
2i. $0.032\cdot 0.0010001 = 32\cdot\left(\frac{1}{10}\right)^3\cdot 10001\cdot\left(\frac{1}{10}\right)^7 = 320032\cdot\left(\frac{1}{10}\right)^{10} = 0.0000320032$

4a. $0.01\underline{7}39$ to the nearest thousandth: 0.017 $0.017\underline{3}9$ to the nearest ten thousandth: 0.0174

4b. $0.0\underline{4}95$ to the nearest hundredth: 0.05 $0.04\underline{9}5$ to the nearest thousandth: 0.050

(Rounded down this would be 0.049
Rounded up must be 0.049 + .001 = 0.050)

6a. Chip model for 5.73 + 1.67:

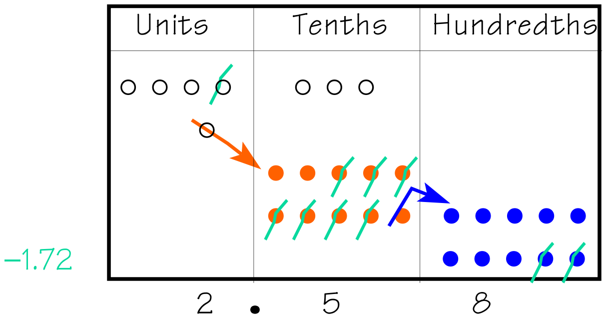


$$\begin{array}{r} \overset{1}{5}.\overset{1}{7}3 \\ +\ 1.67 \\ \hline 7.40 \end{array}$$

[6c & 6d not shown]

6b. Chip model for 5.3 – 1.72

$$\begin{array}{r} \overset{4}{\cancel{5}}.\overset{2}{\cancel{3}}\overset{+10}{10} \\ -1.72 \\ \hline 3.58 \end{array}$$



10. $1.78 \div 0.0047 = \frac{1.78}{.0047} = \frac{17800}{47}$ ←largest

$17.8 \div 0.47 = \frac{17.8}{.47} = \frac{1780}{47}$ ← middle

$17.8 \div 4.7 = \frac{17.8}{4.7} = \frac{178}{47}$ ← smallest

11.

$$\begin{array}{r} .0053 \\ 2.6.\overline{)\,0.0.1378} \\ \underline{130} \\ 78 \\ \underline{78} \end{array} \qquad \begin{array}{r} .304 \\ .24.\overline{)\,0.07.296} \\ \underline{72} \\ 9 \\ \underline{0} \\ 96 \\ \underline{96} \end{array}$$

9.2 Text HW Rationals & Decimals

Math 210 pbf8

1. Express as fractions in simplest form:

a. $0.125 = \dfrac{125}{1000} = \dfrac{25\cdot5}{25\cdot40} = \dfrac{1}{8}$

b. $0.0875 = \dfrac{875}{10000} = \dfrac{25\cdot35}{25\cdot400} = \dfrac{7}{80}$

c. $0.525 = = \dfrac{525}{1000} = \dfrac{25\cdot21}{25\cdot40} = \dfrac{21}{40}$

d. $0.28125 = \dfrac{28125}{100000} = \dfrac{625\cdot45}{625\cdot160} = \dfrac{9}{32}$

2. Can the fraction be written as a finite decimal?

[Solutions for d & f are not given.]

a. $\dfrac{137}{625} = \dfrac{137\cdot16}{10000} = 0.2192$

b. $\dfrac{221}{1500} = \dfrac{221}{500\cdot3}$ and 3 does not divide 221. So this decimal does not terminate.

c. $\dfrac{27}{180} = \dfrac{9\cdot3}{9\cdot20} = \dfrac{3}{20} = \dfrac{3\cdot5}{20\cdot5} = 0.15$

e. $\dfrac{44}{260} = \dfrac{4\cdot11}{4\cdot65} = \dfrac{11}{13\cdot5}$ Does not terminate. The 13 will cause it to continue.

3. Write as finite or repeating decimals. [Parts f g h not shown. Part h takes the maximum to repeat!]

a. $\dfrac{3}{8} = \dfrac{3\cdot125}{8\cdot125} = 0.375$

b. $\dfrac{23}{20} = \dfrac{115}{100} = 1.15$

c. $\dfrac{11}{21} =$

$$\begin{array}{r}
.\overline{523809}\\
21\,\overline{)\,11.000000}\\
\underline{105}\\
50\\
\underline{42}\\
80\\
\underline{63}\\
170\\
\underline{168}\\
200\\
\underline{189}\\
11
\end{array}$$

repeats now

d. $\dfrac{24}{9} =$

$$\begin{array}{r}
2.\overline{6}\\
9\,\overline{)\,24.0}\\
\underline{18}\\
60\\
\underline{54}\\
6
\end{array}$$

now it repeats...
this part only.

e. $\dfrac{3}{13} =$

$$\begin{array}{r}
0.\overline{230769}\\
13\,\overline{)\,3.000000}\\
\underline{26}\\
40\\
\underline{39}\\
100\\
\underline{91}\\
90\\
\underline{78}\\
120\\
\underline{117}\\
3
\end{array}$$

3 now repeats everything from the first 3

4. Write the following fractions in simplest form. [Parts c, d, g & i not shown.]

a.
$$\begin{array}{rcl} 10\,x & = & 1.1111111\cdots \\ x & = & 0.1111111\cdots \\ \hline 9\,x & = & 1.0000000\cdots \end{array}$$
$$.\overline{1} = x = \frac{1}{9}$$

b.
$$\begin{array}{rcl} 100\,x & = & 1.010101010101\cdots \\ x & = & .010101010101\cdots \\ \hline 99\,x & = & 1 \end{array}$$
$$.\overline{01} = x = \frac{1}{99}$$

Multiply by 100 because the repetend is 2 digits long.

e.
$$\begin{array}{rcl} 1000\,x & = & 189.189189189\cdots \\ x & = & .189189189\cdots \\ \hline 999\,x & = & 189 \end{array}$$
$$.\overline{189} = x = \frac{189}{999} = \frac{7}{37}$$

f.
$$\begin{array}{rcl} 100\,x & = & 50.505050\cdots \\ x & = & .505050\cdots \\ \hline 99\,x & = & 50 \end{array}$$
$$.5\overline{05} = x = \frac{50}{99}$$

We know this is reduced because $3 \nmid 50$ & $11 \nmid 50$.

h.
$$\begin{array}{rcl} 100\,x & = & 231.56565656\cdots \\ x & = & 2.56565656\cdots \\ \hline 99\,x & = & 229 \end{array}$$
$$2.31\overline{56} = x = \frac{229}{99}$$

Reduced?
Does $3|229$?
Does $11|229$?

j.
$$\begin{array}{rcl} 10000\,x & = & 12345.13451345\cdots \\ x & = & 1.23451345\cdots \\ \hline 9999\,x & = & 12343.9 \end{array}$$
$$x = \frac{12343.9}{9999} = \frac{123439}{99990}$$

that decimal needs to go!

So multiply numerator & denominator by 10.

This one's just a little bit different. No worries, we'll just eliminate the decimal at the end!

5a. [Work not shown here.]
$.\overline{9} = 1$ ... which is usually a shock, because we are not accustomed to the fact that numbers can have two different decimal representations.
In fact, every terminating decimal number has two different representations in decimal form.

5b. As an example, in part b... you should find that $4329.\overline{9}$ is 4330.

PS. You have long been accustomed to the fact that ⅓ = .333333 ⋯ & ⅔ = .666666 ⋯
The logical extension of these observations is that 3(⅓) must be .999999 ⋯ ! But 3(⅓) = 1.

PPS. If you add any tiny decimal number, say .00000000001, to .99999999 ⋯
you get a result that is clearly greater than 1. What does that tell you?

6. This is an instructive problem but time does not permit its inclusion.
(1/19 is another of those decimal expansions that takes the maximum possible time to repeat)
For the decimal expansion of 3/19, you'd start with the remainder 3 in the division for 1/19.

7. Mental math. Using the facts that $.\overline{01} = 1/99$, and $.\overline{001} = 1/999$, find:

a. $.\overline{03} = \frac{3}{99} = \frac{1}{33}$

b. $.\overline{324} = \frac{324}{999} = \frac{36}{111} = \frac{12}{37}$

[7c and 7d are not shown.]

6. "A truck contains 1000 pounds of sand and concrete in the ratio 2 : 3.
After x pounds of sand is added, the ratio of the sand [to] concrete becomes 4 : 5. Find x."

2 parts + 3 parts = 1000 pounds, so initial amounts of sand and concrete are 400 & 600 pounds.
400 + x : 600 = 4 : 5
Written in fraction form: $\frac{400 + x}{600}$ — $\frac{4}{5}$ (We multiply both sides by 600)
400 + x = 4·120
x = 480 – 400 = 80

7. (Re PT 6A pp47-53 Practice 4A)

#1. a. $8\% = \frac{8}{100} = \frac{2}{25}$ b. $25\% = \frac{1}{4}$ c. $50\% = \frac{1}{2}$ d. $66\% = \frac{33}{50}$

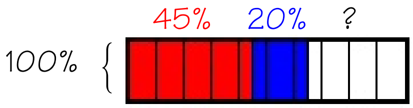
#2. a. 9% = 9/100 = .09 b. 90% = .90 c. 15% = .15 d. 62% = .62

#3. a. $\frac{2}{5}$ = .40 = 40% b. $\frac{7}{8}$ = .875 = 87.5% c. $\frac{9}{20} = \frac{45}{100}$ = 45% d. $\frac{1}{20}$ = 5%

e. 0.5 = .50 = 50% f. 0.08 = 8% g. 0.15 = 15% h. .245 = 24.5%

#4. 100% – 45% = 55%

#5.



100% – (45% + 20%) = 100% – 65% = 35%
35% of the pole is painted white.

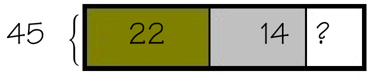
#6. "36 out of 400 seats are vacant. What percentage <u>of the seats</u> are vacant?"

The whole unit is the 400 seats; the giveaway is underlined above. $\frac{36}{400}$ = 9% are vacant.

#7. $\frac{2}{5}$ of the students in a school wear glasses. ↙The whole unit is given here.
What percentage of <u>the students in the school</u> wear glasses?
$\frac{2}{5}$ = .40 = 40% 40% of the students in the school wear glasses.

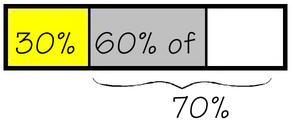
#8. "Eva had 3m of cloth. She used 75 cm of it to make a dress for her doll.
What percentage <u>of the cloth</u> did she use for the dress?" The whole ↓ unit
75 cm is what % of the cloth... 75 = ?? of 3m. First rewrite 3m as 3m $\frac{100\text{cm}}{1\text{ m}}$ = 300 cm .
75 = X · 300 So $X = \frac{75}{300} = \frac{25}{100}$ = 25% Eva used 25% of the cloth for the dress.

#9. "<u>45 medals</u> were given out[...]. There were 22 bronze medals and 14 silver medals. The rest were gold medals. What percentage <u>of the medals</u> were gold medals?"
The whole unit is the 45 medals that were given out.



45 – (22+14) = 9 $\frac{9}{45} = \frac{1}{5} = \frac{20}{100}$ 20% of the medals were gold medals.

#10. K spends 30%of savings & 60% of the remainder. What is left?
Whole unit = original savings.
She spent 30% + 60% of 70% = 30% + 42%, for a total of 72%.
100% – 72% = 28%. 28% of her savings is left.

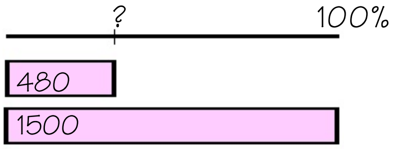


8. (Re PT 6A, pp 55-59, Practice C)

tells us the whole unit (100%) is the 1.5L, aka 1500mL.

#1. "Express 480 mL as a percentage of 1.5L."

First we convert L: $1.5\text{ L} = 1.5\text{L}\cdot\frac{1000\text{ mL}}{\text{L}} = 1500\text{ mL}.$



Our job is to compare 480mL to 1500mL.

1500mL → 100%
1 mL → 1/15 %
480 mL→ 480/15 % = 160/5 % = 32%

Above is the Primary Text's method.

Direct method $\frac{480}{1500} = \frac{160}{500} = \frac{32}{100} = 32\%$

480 mL is 32% of 1500 mL

#2. "What percentage of 2 hours [120 min] is 30 minutes?" 30 minutes is 25% of 2 hours.

#3. "Express the length of A [36m] as a percentage of the length of B [24m]."

Compare the length of A to the length of B and express as a percentage of B. $\frac{36}{24} = 150\%$

"How many percent longer is A than B?"

How much longer is A than B? Express result as percentage of B.

A is 12 m longer than B. 12 m is half of 24m. Half is 50%. A is 50% longer than B.

#4. "MB had 2.5 kg of sugar. She used 650g of it to make syrup.
What percentage of the sugar was used for making the syrup?"

The whole unit is 2.5 kg (which is 2500 g).

650 g is what part of 2500 g? $\frac{650\text{g}}{2500\text{g}} = \frac{26}{100} = 26\%$

26% of the 2.5 kg of sugar was used to make syrup.

#5. "The price of a [TV] was reduced from \$200 to \$150.
By what percentage was the price reduced?"

The Q asks us to compare the reduction to the original price. $\frac{50}{200} = \frac{25}{100} = 25\%$

The price was reduced 25%.

#6. "A club had 80 members last year. This year it has 96 members.
By what percentage was the membership increased? "

The Q asks us to compare the increase to the old membership.

The membership increased by 20%.

#7. "The price of beef increased from \$12 per kg to \$15 per kg. [Express as percent increase.]"

Percent increase or decrease always compares the CHANGE in value to the OLD value.

$\frac{\text{Change}}{\text{Old value}} = \frac{\$3}{\$12} = \frac{1}{4} = 25\%$ The price of beef increased 25%.

#8. "Kyle bought a pair of shoes for \$51. The usual price of the shoes was \$60.
How many percent discount was given to Kyle?"

$\frac{\text{Change}}{\text{Old value}} = \frac{\$9}{\$60} = \frac{3}{20} = \frac{15}{100} = 15\%$ Kyle was given a 15% discount.

#9. "A factory has 600 workers. 250 ... are men and the rest are women.
How many percent *more women* than men are there?"

350 are women. 100 more women than men. Compare the "more women" to the men.

$\frac{100}{250} = \frac{10}{25} = \frac{40}{100} = 40\%$ There are 40% more women than men.

[number 10 is not shown.]