Rearranging:
\[
8 + 9 + 11 + 3 + 2 = 8 + 3 + 9 + 2 + 11 = 11 + 11 + 11 = 33
\]
\[
8 + 2 + 9 + 11 + 3 + 10 = 8 + 3 + 9 + 2 + 11 + 10 = 11 + 11 + 11 = 33
\]
\[
164 + 72 + 36 + 5 = 164 + 36 + 72 + 5 = 200 + 77 = 277
\]
\[
8 + 5 + 15 + 32 + 12 = 4 \times 3 \times 11 \times 5 = \quad \text{(Showing two different views)}
\]
\[
8+32 + 5 + 15 + 12 = 4 \times 5 \times 3 \times 11 =
\]

Splitting numbers:
\[
9 \times 70 = 9 \times 7 \times 10 = 63 \times 10 = 630 \quad \text{(You probably do this all the time, automatically.)}
\]
\[
5 \times 311 = 5 \times (300 + 11) = 1500 + 55 =
\]
\[
5 \times 220 =
\]
\[
5 \times 2 \times 110 =
\]
\[
10 \times 110 = 1100
\]

Compatible numbers:
\[
\ldots \text{we've been doing this all along, looking for splits & rearrangements that give us “nice” numbers.}
\]
\[
6 \times 55 = 2 \times 3 \times 5 \times 11 = 2 \times 5 \times 3 \times 11
\]
\[
5 + 37 + 35 + 18 = 5 + 35 + 37 + 18
\]
\[
14 \times 16 + 7 \times 8 = 7 \times 32 + 7 \times 8
\]
\[
8 \times 35 \times 55 =
\]
\[
2 \times 2 \times 35 \times 2 \times 55 = 2 \times 15 = 30, \quad 2 \times 25 = 50, \quad 2 \times 35 = 70, \quad \text{etc}
\]
\[
2 \times 70 \times 110 =
\]
\[
2 \times 7700 =
\]
\[
15400
\]
\[
4^3 + 6 \cdot 4^2 =
\]
\[
4 \cdot 4^2 + 6 \cdot 4^2 =
\]
\[
(4 + 6) \cdot 4^2 =
\]
\[
10 \cdot 16 = 160
\]
\[
1 + 2 + 3 + 4 + \cdots + 96 + 97 + 98 + 99 + 100 =
\]
\[
1 + 100 + 2 + 99 + 3 + 98 + 4 + 97 + \cdots + 50 + 51 =
\]
\[
101 + 101 + 101 + 101 + \cdots + 101 =
\]
\[
\left[ 50 \text{ pairs, each sums to 101} \right] = 50 \cdot 101 = 5050.
\]

Compensation:

Addition & Multiplication— one number gives to the other.
Subtraction & Division— do the same thing to each number (so that the second one is nice).

Addition: 96 + 46 = 96 + 4 + 42 = (Steal part of 46, to boost 96 up to 100)
Subtraction: 96 – 37 = 99 – 40 = (Shift both numbers up or down, to make the subtrahend “nice.”)
Multiplication: 55 \times 72 = 55 \times 2 \times 36 = 110 \times 36 = (Steal a factor 2 from 72 to boost 55.)
Division: 840 \div 14 = 420 \div 7 = (Reduce both dividend & divisor by common factor, or

\[
730 \div 50 = 73 + 5 = 146 + 10 = 14.6
\]
\[
140 \div 5 = 280 \div 10 = 28
\]

Left-to-right calculation:
\[
239 + 647 = 800 + 70 + 16 = 600 + 300 = 900, \text{ but a quick glance at the next column shows that we would have carried (a 1) from that column (since } 5 + 6 \times 9), \text{ so...}
\]
\[
508 + 407 = 500 + 8 + 400 + 7 + 900 + 15
\]

(with anticipation):
\[
239 + 647 = 5 + 3 + 6 + 4 + 10 = 6 + 5 + 3
\]
The fact that our numeration system is place value base TEN means that we can use certain “tricks” for numbers that are close to powers of ten. For example:

\[ 9 \times 34 = (10 - 1) \times 34 = 340 - 34 = \]
\[ 11 \times 34 = (10 + 1) \times 34 = 340 + 34 = \]
\[ 99 \times 12 = (100 - 1) \times 12 = 1200 - 12 = 1188 \]

Notice we are relying heavily on the distributive property in the above examples. More:

\[ 165 \div 15 = 11 \quad \text{...because 150 \div 15 = 10, so (150 + 15) \div 15 = 10 + 1} \]
\[ 286 \div 13 = 22 \quad \text{...because I wish the problem were 260 \div 13, which is 20, and 286 is 260+26.} \]
\[ 135 \div 15 = 9 \quad \text{...because 150 \div 15 would be 10, and 135 is just 15 less than 150. (150 -15) \div 15 = 10-1} \]
\[ 121 \div 11 = 11 \quad \text{...because we know 11\times11 = 121. Or because 121 = 110 + 11,} \]

More tricks of the trade:

\[ 2 \times 5 = 10 \quad \text{so 5 = 10 \div 2. \quad \text{To multiply by 5, multiply by 10, then halve that result.}} \]
\[ \text{To divide by 5, double the number, then divide by 10. (the opposite of above)} \]

Doubling and halving is relatively easy. Since 4 = 2 \times 2, to multiply by 4, double the number twice; to divide by 4, halve the number twice.

More examples, mixed:

\[ 120 \div 5 = (100 + 20) \div 5 = 20 + 4 \quad \text{[split]} \quad \text{or} \quad 120 \div 5 = 240 \div 10 = 24 \quad \text{[compensate +]} \]
\[ 4 \times 13 = 2 \times 2 \times 13 = 2 \times 26 = 52 \quad \text{[split 4 = 2\cdot 2]} \]
\[ 80 \div 5 = (50 + 30) \div 5 = 10 + 6 \quad \text{or} \quad 80 \div 5 = (100 - 20) \div 5 = 20 - 4 \quad \text{or} \quad 80 \div 5 = 160 \div 10 \]
\[ 184 \div 8 = ((184 +2) \div 2) \div 2 = 92 +4 = 46 +2 = 23 \quad \text{or} \quad 184 = 2\cdot 92 = 2\cdot 2\cdot 46 = 2\cdot 2\cdot 2\cdot 23 = 8\cdot 23, \text{so...} \]
\[ \text{(This looks a lot more complicated than it thinks! 184 \rightarrow 92 \rightarrow 46 \rightarrow 23} \]
\[ \text{Multiplying by 8 can be done by multiplying by 2 repeatedly — multiplying by } 2^3, \text{ to be precise...} \]
\[ \text{So dividing by 8 is the same as dividing by } 2^3... \text{ That is dividing by 2, and again, and again.} \]
\[ 900 + 120 = 900 + 100 + 20 = 1000 + 20 = 1020 \quad \text{[splitting & rearranging]} \]
\[ 90 \times 400 = 36000 \quad \text{because } 90 \times 400 = 9 \times 10 \times 4 \times 100 = 36 \times 1000 \quad \text{[splitting & rearranging]} \]
\[ 37 \times 101 = 37 \times (100 + 1) = 3700 + 37 = 3737 \quad \text{[splitting; also used distributive property]} \]
\[ 48 \times 1002 = 48 \times (1000 + 2) = 4800 + 96 = 4896 \quad \text{[" ]} \]
\[ 34 \times 98 = 34 \times (100 - 2) = 3400 - 68 = 3332 \quad \text{[" ]} \]
\[ 108 \div 9 = (90 + 18) \div 9 = 10 + 2 = 12 \quad \text{(If you know that 9 \times 12 = 108, you don't need this.)} \]
\[ 47 + 25 + 87 + 53 + 75 = 47 + 53 + 25 + 75 + 87 = 287 \]
\[ 2 \times 3 \times 25 \times 66 = 2 \times 25 \times 2 \times 33 \times 3 = 100 \times 99 = 9900 \quad \text{[compensate x; also rearrange]} \]
The hints shown here are some suggestions.... Other methods may occur to you!

1a. \((26+83) + 54 = 83 + 26 + 54 = 83 + 100\)

1b. \((4\times34)\times25 = 34 \times 4 \times 25\)

1c. \(256 \times 6 = (200 + 50 + 6) \times 6 = 1200 + 300 + 36\)
\(256 \times 6 = (250 + 6) \times 6 = 1500 + 36\)
(*because you know that \(6 \times 25\$ = \$1.50\))

1d. \(288 \div 24 = 72 \div 6 = 36 \div 3\)
\(288 \div 24 = (240 + 48) \div 24 = 10 + 2\)

1e. \(44 \times 56 + 56^2 = (44 + 56) \times 56\)

1f. \(402 \times 12 = (400 + 2) \times 12\)

2a. \(123 + 326 + 4 + 77 = 123 + 77 + 326 + 4\)

2b. \(2 \times 6 \times 7 \times 5 = 6 \times 7 \times 2 \times 5\)

2c. \((3200 \times 24) \div 16 = (3200 \div 16) \times 24\)
This is more obvious in fraction form: \(\frac{3200 \times 24}{16}\)

3a. \(197 + 568 = 200 + 566\)

3b. \(62 - 39 = 63 - 40\)

3c. \(48 \times 25 = 12 \times 4 \times 25\)

3d. \(500 \div 25 = 5 \times 100 \div 25\)
\(500 \div 25 = 100 \div 5\)

3e. \(71 - 42 = 69 - 40\)

3f. \(180 \div 15 = 360 \div 30\)
\(180 \div 15 = (150 + 30) \div 15\)
\(180 \div 15 = 36 \div 3\)

4. These are the same as the standard algorithms, but working from left to right, so you must glance to the right, to see if “carrying” would increment the current column.

5a. \(78 \times 9 = 78 \times (10 - 1)\)

5b. \(37 \times 4 = 37 \times 2 \times 2\)

5c. \(136 \div 8 = ((136 \div 2) \div 2) \div 2\)

5d. \(1500 \div 25 = 15 \times (100 \div 25)\)

5e. \(1575 \div 25 = (1500 + 75) \div 25\)

5f. \(325 \div 5 = (300 + 25) \div 5\)

6a. \(37 + 99 = 36 + 100\) PV !

6b. \(20 \times 40 = 2 \times 4 \times 10 \times 10\) PV !

6c. \(7 \times 102 = 7 \times (100 + 2)\) PV !

6d. \(13 \times 28 = 13 \times (30 - 2)\) DP or X

6e. \(326 - 98 = 328 - 100\) PV !

6f. \(337 + 879 ?@#%?\) X

6g. \(119 \div 7 = (140 - 21) \div 7\) DP

6h. \(3 \times 32 = 3 \times (30 + 2)\) DP & PV !
Below is Practice 3B #9 from p 47, very similar to 3C#9 on p 48:
Harry weighs 36 kg.
He is 4 times as heavy as his brother.
How heavy is his brother?

Harry
Brother

\[ 4 \times ? = 36 \]
\[ 36 \div 4 = ? \]

So Harry's brother must weigh \( ? \) kg.

Let's check this and see if this makes sense:
- Harry's brother weighs 9 kg, and
- Harry is 4 times as heavy as his brother, so
- Harry would have to weigh 36 kg.

...and that agrees with the first statement.

2b. (Re 3A p48 Practice 3C #9):
Lynn poured 16 qt of syrup equally into 4 bottles.
How many quarts were in each bottle?

\[ 4 \times ? = 16 \]
\[ 16 \div 4 = ? \]
There are 4 quarts of syrup in each bottle.

2b. (Re WB5A p25 #9):
1. Elaine has 274 beads. 150 are blue, 70 red, and the rest white.
How many more red beads than white beads are there?

Blue 150
Red 70
White ??

Step 1: Find the number of white beads.
\[ 150 + 70 + ? = 274 \]
\[ 274 - (150 + 70) = ? \]
The number of white beads is 274 - 220 = 54

Step 2: Find how many more red than white.
\[ 54 + ?? = 70 \]
\[ 70 - 54 = \text{the difference, } 16 \]
There are 16 more red beads than white.
2.3 Text Exercises Continued

2b cont’d. 2. Tickets to a concert cost $15 per adult and $8 per child. Matthew bought tickets for 4 adults and 5 children. How much did he spend altogether?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult</td>
<td>$15</td>
<td></td>
</tr>
<tr>
<td>Child</td>
<td>$8</td>
<td></td>
</tr>
</tbody>
</table>

Matthew spent \(4 \times 15 = 60\) for adult tickets,
\(5 \times 8 = 40\) for child tickets,
So he spent \(60 + 40 = 100\) altogether.

3. At a carnival Ann sold 314 bottles of drinks a day. She sold 66 bottles more in the afternoon than in the morning. How many bottles did she sell in the morning?

\[
\begin{array}{c}
\text{PM} \\
? \\
66
\end{array} \quad \begin{array}{c}
\text{AM} \\
? \\
66
\end{array} \quad 314
\]

\[
314 - 66 = 248
\]
\[
248 \text{ is divided equally between the two boxes marked “?”}
\]
\[
248 \div 2 = 124
\]
Ann sold 124 bottles of drinks in the morning.
CHECK: 124 in the AM, 124 + 66 = 190 in the PM. 124+190 = 114+200 = 314 total.

4. Adam bought a pen. He also bought a book which cost 3 times as much as the pen. He spent $112 altogether. Find the cost of the book.

\[
\begin{array}{c}
\text{Pen} \\
\text{Book}
\end{array} \quad \begin{array}{c}
\$112 \\
4 \text{ UNITS} = 112
\end{array} \quad 1 \text{ UNIT} = 112 \div 4 = 28
\]

The book cost 3 UNITS = 3 \times 28 = 84.
The book cost $84.

Check: The book cost $84, the pen cost $28, altogether they cost 84 + 28 = 112.

STEPS IN PROBLEM SOLVING:

Understand the problem.
(\text{Put yourself into the picture; draw diagrams; make examples; use easy numbers.})

Calculate, and/or solve the equation(s).

State the answer to the question.

Check your answer.