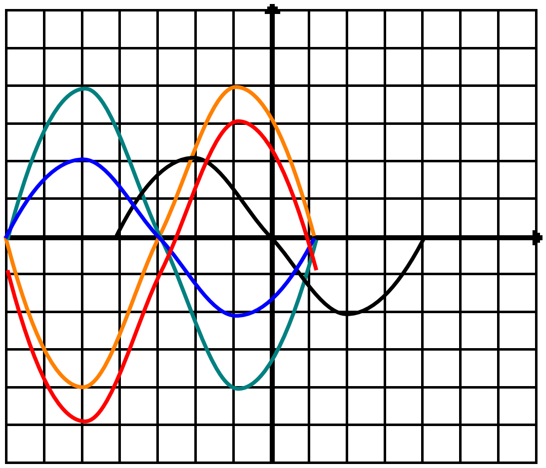
Math 102 ♢ 50-pt-quiz # 3 ♧ April 6, 2006 ♡ SOLUTIONS Note there are two versions!!!

(6) 1. Below is a sketch of the graph of the function g: y = g(x) .
On the same coordinate system, graph the function F given by F(x) = –2g(x+3) – 1 .



Given y = g(x)

y = g(x+3) is shifted 3 units left (of y = g(x))

y = 2g(x+3) is stretched vertically (by factor 2)

y = –2g(x+3) is reflected through the x-axis

y = – 2g(x+3) – 1 is shifted 1 unit down

(10) 2. Use the process of completing the square to analyze the function given by $f(x) = -2x^2 - 8x + 1$.
Use that work to identify any extreme values this function may have. (note: $f(x) = a(x - h)^2 + k$)
Sketch the graph of the function, marking any extreme points, and all intercepts.

$f(x) = -2x^2 - 8x + 1$

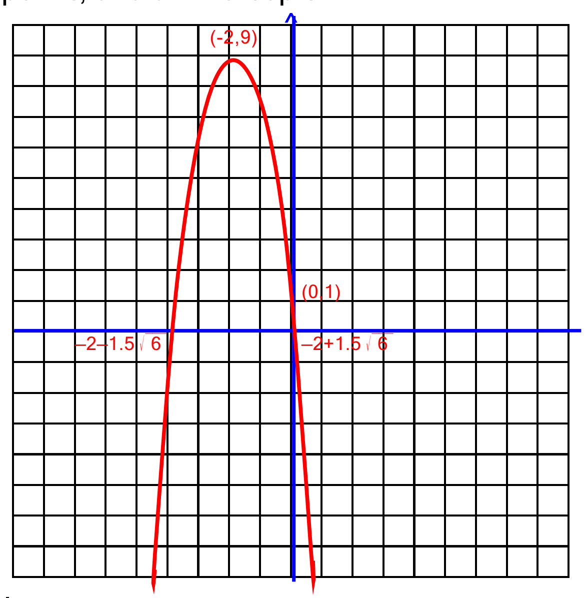
$f(x) = -2x^2 - 8x \qquad + 1$

$f(x) = -2 (x^2 + 4x \qquad) \qquad + 1$

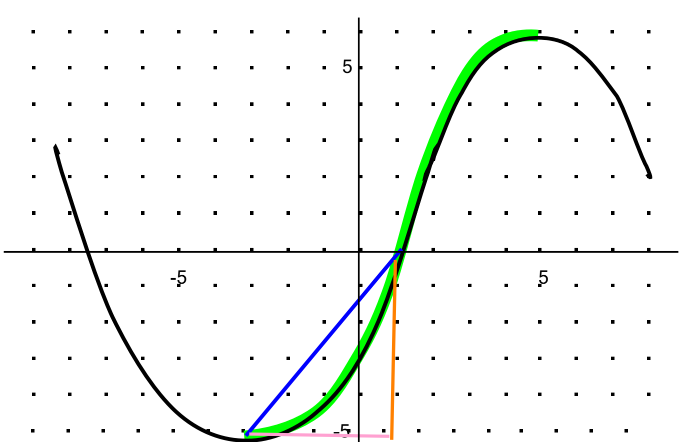
$f(x) = -2 (x^2 + 4x + 4) \qquad + 1 + 8$

$f(x) = -2 (x + 2)^2 \qquad + 9$

$y = (x + 2)^2$ is parabola shifted 2 units left
$y = -2 (x + 2)^2$ opens down, stretch factor 2
$y = -2 (x + 2)^2 + 9$ is shifted up 9 units
Vertex (–2,9). Intercepts: (0,1) $(-2 \pm 3/\sqrt{2}, 0)$



(10) 3. Given function F has the graph y = F(x) illustrated below,
a. find the average rate of change of F on the interval [-3, 1].
b. On what interval(s) does F appear to be increasing?

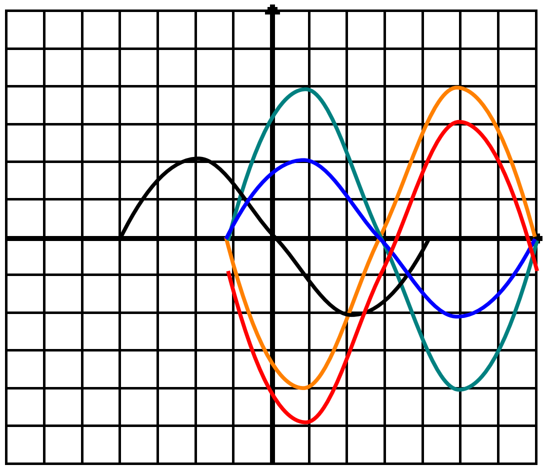


a. This is merely the slope of the line segment connecting (-3,-5) & (1,0)... (see the graph).

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - -5}{1 - -3} = \frac{5}{4}$$

b. The graph is rising for x in [-3,5]

(6) 1. Below is a sketch of the graph of the function g: $y = g(x)$.
On the same coordinate system, graph the function F given by $F(x) = -2g(x-3) - 1$.



Given $y = g(x)$

$y = g(x-3)$ is shifted 3 units right (of $y = g(x)$)

$y = 2g(x+3)$ is stretched vertically (by factor 2)

$y = -2g(x+3)$ is reflected through the x-axis

$y = -2g(x+3) - 1$ is shifted 1 unit down

(10) 2. Use the process of completing the square to analyze the function given by $f(x) = -2x^2 - 8x + 1$.
Use that work to identify any extreme values this function may have. (note: $f(x) = a(x - h)^2 + k$)
Sketch the graph of the function, marking any extreme points, and all intercepts.

$f(x) = -2x^2 - 8x + 1$

$f(x) = -2x^2 - 8x \qquad + 1$

$f(x) = -2 (x^2 + 4x \qquad) \qquad + 1$

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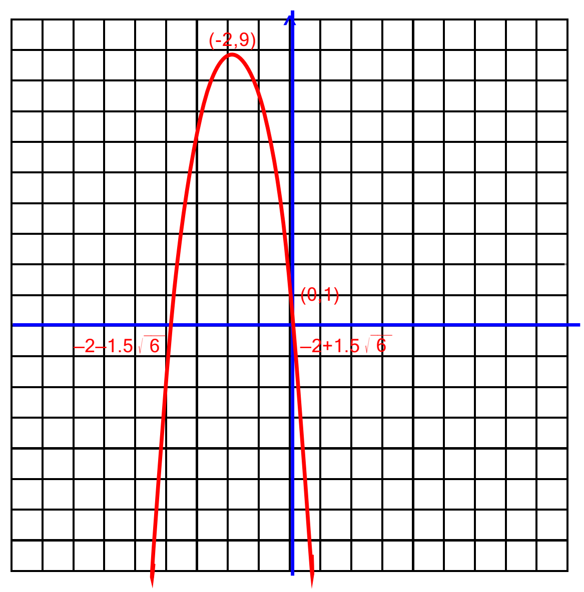
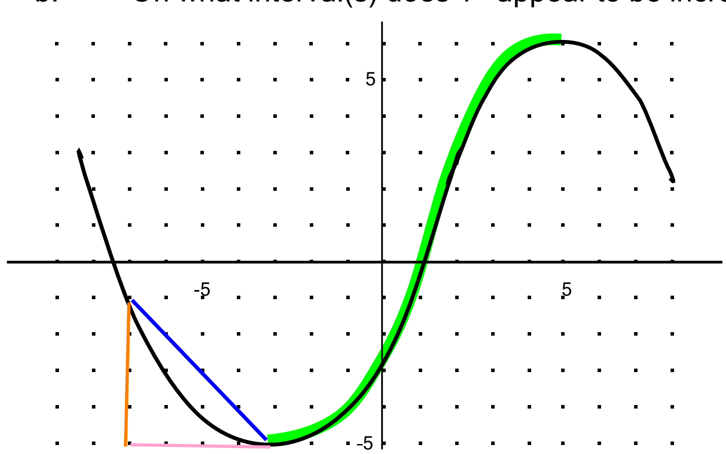
Vertex (–2,9). Intercepts: (0,1) $(-2 \pm 3/\sqrt{2}, 0)$



(10) 3. Given function F has the graph $y = F(x)$ illustrated below,
a. find the average rate of change of F on the interval [-7, -3].
b. On what interval(s) does F appear to be increasing?



$y = F(x)$

a. This is merely the slope of the line segment connecting (-7,0) [or is it (-7,-1)?] & (-3,-5)... (see the graph).

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - -1}{-3 - -7} = \frac{-4}{4}$$

b. The graph is rising ╱ for x in [-3,5]

Math 102 ♢ 50-pt-quiz # 3 ♡ NAME:

(8) 4. a. Find $f \circ g\,(x)$ given $g(x) = 2x + 4$ and $f(x) = \sqrt{x}$

b. What is the domain of $f \circ g$?

a. $f \circ g\,(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{2x + 4}$

b. For $\sqrt{2x + 4}$ to be real, $2x + 4$ must be non-negative.

$$2x + 4 \geq 0$$

$$2x \geq -4$$

$$x \geq -2$$

so the domain of $f \circ g$ is $[-2, \infty)$

(8) 5. Given $f(x) = \dfrac{2}{3x - 1}$, find $f^{-1}(x)$.

Method two:

$$y = \frac{2}{3x - 1}$$

$$y(3x - 1) = 2$$

$$3yx - y = 2$$

$$3yx = 2 + y$$

$$x = \frac{2 + y}{3y}$$

Noting that $x = f^{-1}(y)$ and exchanging the y-variable for x

$$f^{-1}(x) = \frac{2 + x}{3x}$$

Method one (which we can use here Because x appears once in f(x):

F does this:
Multiply by 3
Subtract 1
Take reciprocal
Multiply by 2

So f^{-1} must: Divide by 2
Take reciprocal
Add 1
Divide by 3.

So: $f^{-1}(x) = \dfrac{(x/2)^{-1} + 1}{3}$

$$= \frac{2 + x}{3x}$$

(8) 6. Find and SIMPLIFY the difference quotient, $\dfrac{f(x + h) - f(x)}{h}$

for $f(x) = x^2 + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 4 - (x^2 + 4)}{h}$$

$$= \frac{(x^2 + 2xh + h^2 + 4 - (x^2 + 4)}{h}$$

$$= \frac{(\cancel{x^2} + 2xh + h^2 + \cancel{4} - (\cancel{x^2} + \cancel{4})}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= 2x + h$$

Math 102 ♢ 50-pt-quiz # 3 ♡ NAME:

(8) 4. a. Find $f \circ g\,(x)$ given $g(x) = 1 - x$ and $f(x) = \sqrt{x}$

b. What is the domain of $f \circ g$?

a. $f \circ g\,(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{1-x}$

b. For $\sqrt{1-x}$ to be real, $1 - x$ must be non-negative.

$$1 - x \geq 0$$

$$1 \geq x$$

...so the domain of $f \circ g$ is $(-\infty, 1]$

(8) 5. Given $f(x) = \dfrac{2x}{3x-1}$, find $f^{-1}(x)$.

$f(x) = ⅔ + \dfrac{2}{9x-3}$

Method two:

$$y = \frac{2x}{3x-1}$$

$$y(3x-1) = 2x$$

$$3yx - y = 2x$$

$$3yx - 2x = y$$

$x = \dfrac{y}{3y-2}$ ...Noting that $x = f^{-1}(y)$ and restating for x

$$f^{-1}(x) = \frac{x}{3x-2}$$

Method one (requires extra work to get x to appear once in f(x):

f does this: Multiply by 9
Subtract 3
Take reciprocal
Multiply by 2, add ⅔

So f^{-1} must: Subtract ⅔ & divide by 2
Take reciprocal
Add 3
Divide by 9.

So: $f^{-1}(x) = \dfrac{(x - ⅔/2)^{-1} + 3}{9} = \dfrac{3x}{9x-6}$

(8) 6. Find and SIMPLIFY the difference quotient, $\dfrac{g(x+h) - g(x)}{h}$ for $g(x) = x^2 + 4x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 4(x+h) - (x^2 + 4x)}{h}$$

$$= \frac{(x^2 + 2xh + h^2 + 4x + 4h - (x^2 + 4x)}{h}$$

$$= \frac{2xh + h^2 + 4h}{h}$$

$$= \frac{2xh + h^2 + 4h}{h}$$

$$= 2x + h + 4$$