

Form A2

Score:

Name: **ANSWERS form A2**Show all your work on these papers (you may use the reverse) and **BOX** your final answers.

1. Express the number set



a) as an inequality

$$-2 \leq x < 3$$

b) using interval notation

$$[-2, 3)$$

2. Let
- $A = \{x \mid -3 < x < 3\}$
- and
- $B = \{x \mid -2 \leq x\}$
- .

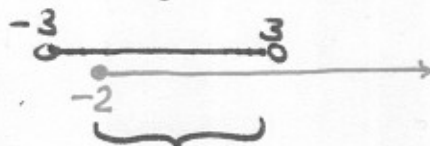
Find each of the following and express your answers using interval notation.

a) $A \cap B$

$$= [-2, 3)$$

INTERSECTION

= numbers common to both A and B.



Numbers from -2 to 3 (not including 3) belong to both A and B.

b) $A \cup B$

$$(-3, \infty)$$

UNION

= set of all numbers in A or in B.

3. Simplify as much as possible (eliminate any negative exponents):

$$\begin{aligned} & \left(\frac{4x^{-1}y^{1/3}}{x^{-2}y^{-1/3}} \right)^{3/2} \\ &= (4x^{-1+2}y^{1/3+1/3})^{3/2} \\ &= (4x^1y^{2/3})^{3/2} \\ &= 4^{3/2}x^{3/2}(y^{2/3})^{3/2} \\ &= (\sqrt{4})^3x^{3/2}y \\ &= 8x^{3/2}y \end{aligned}$$

$$\begin{aligned} &= \frac{4^{3/2}x^{-3/2}y^{1/2}}{x^{-2 \cdot 3/2}y^{-1/3 \cdot 3/2}} \\ &= \frac{4^{3/2}x^{-3/2}y^{1/2}}{x^{-3}y^{-1/2}} \\ &= 4^{3/2}x^{-3/2+3}y^{1/2+1/2} \\ &= 4^{3/2}x^{3/2}y^1 \\ &= 8x^{3/2}y \end{aligned}$$

- 7 4. Simplify (Factor completely): $4x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

$$\begin{aligned} & \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} \frac{4x^{-\frac{1}{2}} - x^{\frac{3}{2}}}{1} \\ &= \frac{4 - x^{\frac{4}{2}}}{x^{\frac{1}{2}}} \\ &= \frac{(2+x)(2-x)}{x^{\frac{1}{2}}} \end{aligned}$$

STANDARD OPERATING PROCEDURE -
see $x^{-1/2}$, multiply by $\frac{x^{1/2}}{x^{1/2}}$

OR WRITE

$$\begin{aligned} 4x^{-1/2} - x^{3/2} &= \frac{4}{x^{1/2}} - x^{3/2} \\ &= \frac{4}{x^{1/2}} - x^{3/2} \cdot \frac{x^{1/2}}{x^{1/2}} \\ &\text{et cetera...} \end{aligned}$$

same thing but
MORE
work!

- 7 5. Solve, and simplify your answer: $2x^2 - 6x + 3 = 0$

$$\begin{aligned} x &= \frac{6 \pm \sqrt{36 - 24}}{2 \cdot 2} \\ &= \frac{6 \pm \sqrt{12}}{4} \\ &= \frac{6 \pm 2\sqrt{3}}{4} \\ &= \frac{2(3 \pm \sqrt{3})}{4} \\ &= \frac{3 \pm \sqrt{3}}{2} \end{aligned}$$

- 7 6. Simplify completely:

$$\begin{aligned} & \frac{((x+1)^{-1} - 1)(x+1)}{((x+1)^{-1} + 1)(x+1)} \\ &= \frac{1 - (x+1)}{1 + (x+1)} \\ &= \frac{1 - x - 1}{1 + x + 1} \\ &= \frac{-x}{2 + x} \end{aligned}$$

OR
$$\frac{\left(\frac{1}{x+1} - 1\right)(x+1)}{\left(\frac{1}{x+1} + 1\right)(x+1)}$$

$$\begin{aligned} &= \frac{1 - (x+1)}{1 + (x+1)} \\ &= \frac{1 - x - 1}{1 + x + 1} \\ &= \frac{-x}{2 + x} \end{aligned}$$

7. Solve for x : $\frac{ax+c}{x+1} = b$

$$ax+c = b(x+1)$$

$$ax+c = bx+b$$

$$ax-bx = b-c$$

$$(a-b)x = b-c$$

$$x = \frac{b-c}{a-b}$$

we multiply both sides by $x+1$, making the assumption that $x+1 \neq 0$

we need to solve for x !

provided $a \neq b$ and $x \neq -1$.
($c \neq a$)

8. Divide. Express your answer in the form $a + bi$: $\frac{1 + \sqrt{-9}}{1 - \sqrt{-9}}$

$3i$ is easier to use than $\sqrt{-9}$

$$\frac{1+3i}{1-3i} \cdot \frac{1+3i}{1+3i}$$

$$= \frac{1^2 + (3i)^2 + 6i}{1^2 - (3i)^2}$$

$$= \frac{1-9+6i}{1-9i^2} = \frac{-8+6i}{1+9} = -\frac{8}{10} + \frac{6}{10}i = -\frac{4}{5} + \frac{3}{5}i$$

9. Solve: $\sqrt{x+10} + 2 = x$

$$\sqrt{x+10} = x-2$$

$$x+10 = x^2-4x+4$$

$$x^2-5x-6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6 \text{ or } -1$$

we need to isolate the $\sqrt{\quad}$ so that when squared, no $\sqrt{\quad}$ will remain.

put quadratic into standard form and solve...

Then CHECK solutions (REQUIRED!) because squaring can bring in extraneous solutions.

Check: $x=6$: $\sqrt{16} = 6-2$
 $4 = 4$

$x=-1$: $\sqrt{9} = -1-2$ NO

The only solution is $x=6$.

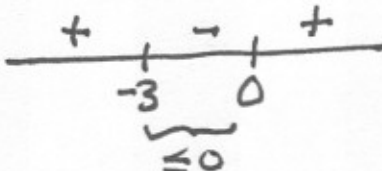
- 7 10. Solve for x : $\frac{3x+3}{x} \leq 2$ Express your answer using interval notation.

Compare to 0: $\frac{3x+3}{x} - 2 \leq 0$ Do NOT multiply by x ! (Inequality.)

simplify: $\frac{3x+3}{x} - \frac{2x}{x} \leq 0$

$$\frac{x+3}{x} \leq 0$$

CRITICAL VALUES ARE -3 & 0

$\frac{x+3}{x} \rightarrow$ 

At $x=0$, expression is undefined.

At $x=-3$,

$$\frac{3(-3)+3}{-3} = \frac{-9+3}{-3} = \frac{-6}{-3} = 2 \leq 2$$

so -3 is a solution

$$[-3, 0)$$

- 7 11. Solve $|4-3x| \geq 5$. Express your answer using interval notation.

$|Q| \geq 5$ if & only if $Q \leq -5$ or $Q \geq 5$

$$4-3x \leq -5 \quad \text{OR} \quad 4-3x \geq 5$$

$$9 \leq 3x \quad \text{OR} \quad -1 \geq 3x$$

$$3 \leq x \quad \text{OR} \quad -\frac{1}{3} \geq x$$

$$(-\infty, -\frac{1}{3}] \cup [3, \infty)$$

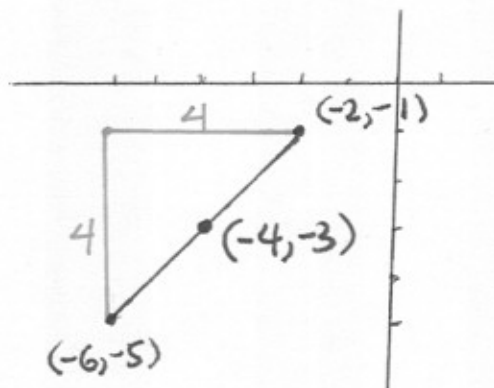
- 7 12. For $A = (-6, -5)$ and $B = (-2, -1)$, find

a. the length of $\overline{AB}$ (simplify your answer)

$$\begin{aligned} |\overline{AB}| &= \sqrt{(-6-(-2))^2 + (-5-(-1))^2} \\ &= \sqrt{4^2 + 4^2} \\ &= \sqrt{32} \quad \text{or} \quad 4\sqrt{2} \end{aligned}$$

b. the midpoint of $\overline{AB}$.

$$\begin{aligned} \text{midpt} &= (\text{average } x, \text{average } y) \\ &= \left(\frac{-6-2}{2}, \frac{-5-1}{2} \right) = (-4, -3) \end{aligned}$$

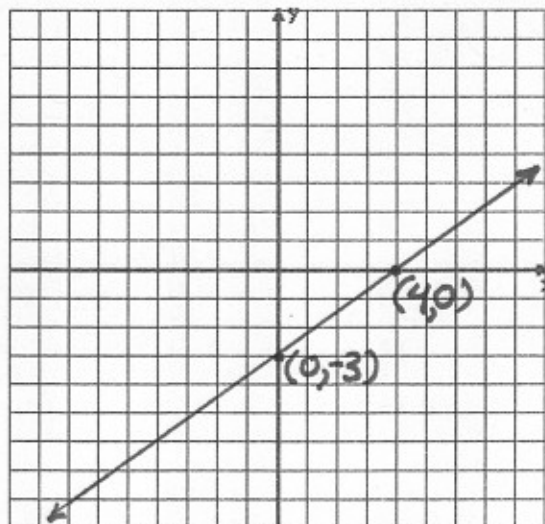


13. For the line given by the equation $3x - 4y = 12$:

find the slope; sketch the graph,
and label the x and y intercepts.

slope: $\frac{3}{4}$

$$\begin{aligned} 3x - 4y &= 12 & y=0 \Rightarrow x=4 \leftarrow \\ y &= \frac{3}{4}x - 3 & \rightarrow (3x - 4 \cdot 0 = 12) \\ x=0 &\Rightarrow y = -3 \end{aligned}$$



14. Find the center and radius of the circle with equation given.

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

$$\begin{aligned} x^2 - 6x + 9 + y^2 + 4y + 4 &= 3 + 9 + 4 \\ (x-3)^2 + (y+2)^2 &= 16 \\ (x-a)^2 + (y-b)^2 &= r^2 \leftarrow \text{Compare with...} \end{aligned}$$

$\Rightarrow$ this circle's center is at $(3, -2)$
and radius is 4.

15. Find the equation of a line passing through $(-3, 4)$ and perpendicular to the line given by $2x + 3y = 5$.

$$\Rightarrow y = -\frac{2}{3}x + \frac{5}{3}$$

$$\Rightarrow \text{slope} = -\frac{2}{3} \Rightarrow \perp \text{slope is } \frac{3}{2}$$

$$\text{so line is } y = \frac{3}{2}x + b$$

To find b , we know line must
contain $(-3, 4)$, so:

$$4 = \frac{3}{2}(-3) + b$$

$$\Rightarrow b = 4 + \frac{9}{2} = \frac{17}{2} \Rightarrow y = \frac{3}{2}x + \frac{17}{2}$$

ANOTHERWAY:

$$\perp \text{ to } 2x + 3y = 5$$

$$\text{is } 3x - 2y = k$$

$$\text{contains } (-3, 4) \Rightarrow$$

$$3(-3) - 2(4) = k \Rightarrow k = -17$$

so equation of line is

$$3x - 2y = -17$$