Practice Math 102 Final Exam

1. Sketch the graph of $f(x) = \sqrt{x}$. What is the domain of f? $[0,\infty)$
   Use transformations to sketch the graph of $g(x) = \sqrt{-x} - 2$. What is the domain of g?

2. a. Given $f(x) = \dfrac{1}{3x-2}$ and $g(x) = \dfrac{1}{x}$

   What are the domains of f and g? Find $f\circ g(x)$ and the domain of $f\circ g$.

3. Find the formula for $f^{-1}(x)$ given $f(x) = \dfrac{x}{3x-2}$. State the domain and range of f.

4a. Analyze the function given by $f(x) = \dfrac{x-6}{x+2}$ and sketch the graph, labelling everything.

4b. Analyze the function given by $f(x) = \dfrac{x^2+4}{x^2-4}$ and sketch the graph, labelling everything.

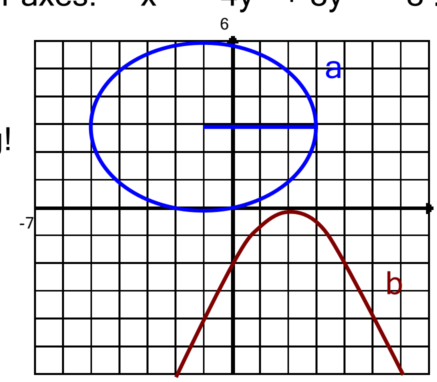
5. Solve $\dfrac{x}{x+4} \leq 2$. Express the solution set using interval notation.

6. For $f(x) = 2x^2 - 5x + 3$, find & simplify completely the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

7a. Find the center and radius of the circle with equation $x^2 + y^2 - 4x - 6y - 3 = 0$

7b. Identify the curve and show any vertices, foci, asymptotes or axes: $x^2 - 4y^2 + 8y = 8$.

8. a,b. Write an equation that would have the graph shown ⇨



9. Sketch the graph of $P(x) = -x^3 + 4x^2 - 4x$. Label everything!

10. For $P(x) = 2x^3 + 8x^2 + 3x - 10$,
    a. List all the possible rational zeros of P(x)
    b. Show that -2 is a zero of P(x).
    c. Find all the zeroes of P.

11. Find a fourth degree polynomial P(x) with 1+2i a zero, 1 a zero of multiplicity 2, and a constant term of 10. Express your answer in the form $P(x) = ax^3 + bx^3 + cx^2 + dx + e$, where a,b,c,d,e, are integers.

12. Simplify completely: $4\log_4(2x) - \log_4 x^5 + \log_4(4x)$.

13. Sketch the graph of $f(x) = e^{x-2}$ and the graph of $g(x) = \ln x + 2$.
    Describe the relationship between these graphs, and the relationship between f & g.

14. Solve for x: $\log_6(x\text{-}3) + \log_6(x+2) = 1$

15. Solve for x (exact answer.): $e^{x-2} = 3^x$

16. Find the maximum for $P(x,y) = 2x + 3y$, subject to conditions:
    $x \geq 0$; $y \geq 0$; $y \geq x + 1$; $x + y \leq 12$; $y \leq x + 6$. Complete justification required.

17. Find all points on the line $x - y = 7$ that lie exactly 5 units from the origin.
    (Hint: the points of the plane that lie 5 units from (0,0) form a conic section known as....)

18. \$2000 is invested in an account that pays 3% (annual percentage rate) compounded continuously. How long will it take for the balance to reach \$3000?

19. A colony contained 2000 bacteria at 6AM, 3000 at 8AM. How many will there be at noon? When will there be 10,000 bacteria?

20. Find the quadratic function passing through the points (−1,8) & (1,−4) & (2,−1). Hint: $f(x) = ax^2 + bx + c$.

21. Find the dimensions of a rectangle whose perimeter is 105 meters and area is 441 square meters.

Additional word problems are recommended:, for instance:

Text §4.1 page 308 #69

Text §6.3 page 512 #73

Text §7.1 page 545 # 59, 61

Text §7.2 page 562 #77 (at least set it up)

Text Chapter 7 Review page 628 #100, 102

Practice Math 102 Final Exam

PRACTICE MECHANICS- THESE ARE NOT TYPICAL FINAL QUESTIONS
These problems are included as a reference for those who need a brush-up on such details.

A. Simplify completely. Express your answer without negative exponents.
$(2x^4y^{-4/5})^5(16y^{-4})^{-1/4}$

First we give ourselves a little space to think in, then distribute the exponents, 5 and -1/4:

$$(2\,x^4\;y^{-4/5})^5\;\;(16\,y^{-4})^{-1/4}$$
$$= 2^5\,x^{20}\;y^{-4}\;\;(16\,y^{-4})^{-1/4}$$

e.g. $(y^{-4/5})^5 = y^{(-4/5)5} = y^{-4}$

$$= 2^5\,x^{20}\;y^{-4}\;\;16^{-1/4}\,y^{-4\,(-1/4)}$$
$$= 2^5\,x^{20}\;y^{-4}\;\;16^{-1/4}\,y$$ Regroup to facilitate combining like factors...
$$= 2^5\,16^{-1/4}\;x^{20}\;y^{-4}\,y$$
$$= 2^5\,(2^4)^{-1/4}\;x^{20}\;y^{-4}\,y$$
$$= 2^5\,2^{-1}\;x^{20}\;y^{-4}\,y$$
$$= 2^4\;x^{20}\;y^{-3}$$
$$= \frac{2^4\;x^{20}}{y^3}$$

This simplification requires only one or two steps,
... which were drawn out to reveal all details

B. Factor completely. $x^{3/2} + 6x^{1/2} + 9x^{-1/2}$

Observation: Multiplication by $x^{1/2}$ would make that a lot nicer!

$$\frac{x^{1/2}}{x^{1/2}}(x^{3/2} + 6x^{1/2} + 9x^{-1/2})$$
$$\frac{(x^{3/2+1/2} + 6x^{1/2+1/2} + 9x^{-1/2+1/2})}{x^{1/2}}$$
$$\frac{(x^2 + 6x^1 + 9)}{x^{1/2}}$$
$$\frac{(x+3)^2}{x^{1/2}}$$

C. Multiply as indicated. Express the result in the form "a + bi".

$$\frac{6i+9}{1-3i} = \frac{(6i+9)(1+3i)}{(1-3i)(1+3i)} = \frac{9\cdot 1 - 6\cdot 3 + 9\cdot 3i + 6i}{1+9} = -0.9 + 3.3i$$

D. Multiply: $(x-(2+3i))(x-(2-3i))$

$$= xx - (2-3i)x - (2+3i)x + (2+3i)(2-3i)$$
$$= x^2 - 2x + 3ix - 2x - 3ix + 4 - 9i^2$$
$$= x^2 - 2x \quad - 2x \quad + 4 - 9(-1)^2 = x^2 - 4x + 13$$

E. Divide $x^7 - 128$ by $x - 2$ and express $x^7 - 128$ as a product....

2	1	0	0	0	0	0	0	-128
		2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

$$x^7 - 128 = (x-2)(x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x)$$

PRACTICE MECHANICS- THESE ARE NOT TYPICAL FINAL QUESTIONS
These problems are included as a reference for those who need a brush-up on such details.

F. Let A = (2,-5) and B = (-1,3) be points in the xy-plane.

a. Find the length of segment AB. (simplify the answer)

Length of AB = $\sqrt{(\Delta x)^2 + (\Delta y)^2} = (3^2 + 8^2)^{1/2} = 73^{1/2}$

b. Find the midpoint of segment AB.

Midpoint of AB = (halfway between 2 & -1, halfway between -5 & 3)

$= \left({}^{(2+-1)}/_{2}, {}^{(-5+3)}/_{2} \right)$

$= (\frac{1}{2}, -1)$

G. Solve. Express your answer using interval notation.

$2|(\frac{1}{2})x + 3| + 3 \leq 15$ I like to see the x with coefficient 1, so I go ahead and multiply $2|(\frac{1}{2})x + 3|$

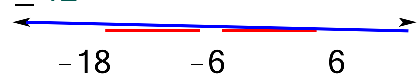
$|x + 6| + 3 \leq 15$

$|x + 6| \leq 12$ Here I LOOK at it and solve intuitively:

$|x - -6| \leq 12$ ... says "distance between x & -6" $\leq$ 12

[-6 - 12, -6 + 12]

[-18 , 6]



But this works, too:

$|x + 6| \leq 12$ if and only if

$-12 \leq x + 6 \leq 12$

$-18 \leq x \leq 6$ so x is in [-18 , 6]

H. Simple word problem: The manager of a cheap plastic furniture factory finds that it costs $2400 to produce 200 chairs in a day, and $6200 to produce 400 chairs in a day. Assuming that the relationship between cost (y) and the number of chairs produced (x) is linear, find an equation that relates x and y.

Reduced to its essence, this problem asks us to find the equation of a line through (x,y) = (200,2400) and (400,6200).

SLOPE = $\Delta y/\Delta x$ = 3800/200 = 19 Here I use the point-slope form:

the equation is $y - y_0 = 19(x - x_0)$ where (x_0,y_0) is any point on the line

$y - 2400 = 19(x - 200)$ ←THIS IS SUFFICIENT

$y = 19x - 1400$ ←But you may also write this.

Here I use the slope-intercept form: $y = 19x + b$

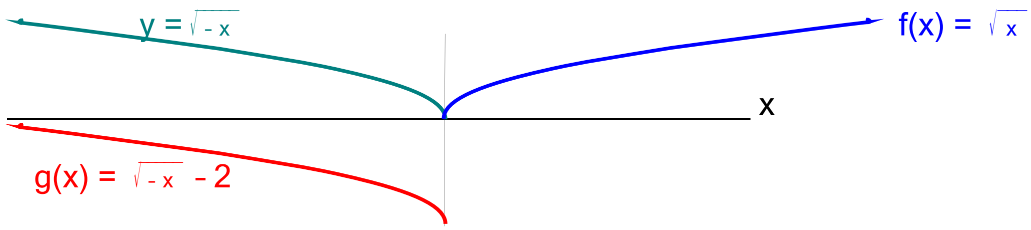
to find b, I take advantage of the fact that when x = 200, y = 2400:

$2400 = 19(200) + b$

So $b = 2400 - 19(200) = -1400$.

1. Sketch the graph of $f(x) = \sqrt{x}$. What is the domain of f? $[0,\infty)$
Use transformations to sketch the graph of $g(x) = \sqrt{-x} - 2$. What is the domain of g?

We used the intermediate function $y = \sqrt{-x}$



2. Given $f(x) = \dfrac{1}{3x-2}$ & $g(x) = \dfrac{1}{x}$

a. What are the domains of f and g?
b. Find f∘g(x) and its domain.

a. Domain f is $(-\infty, \tfrac{2}{3}) \cup (\tfrac{2}{3}, \infty)$ ...because 3x−2 must not be 0, thus x may not be ⅔.
Domain g is all reals except 0 ...aka $(-\infty, 0) \cup (0, \infty)$

b. $f \circ g(x) = f(g(x)) = \dfrac{1}{3g(x)-2} = \dfrac{1}{3\frac{1}{x}-2}$ ← Here we see that $3\frac{1}{x} - 2$ must not be 0.
...in addition to the restriction x not be 0.

So the domain of f∘g is all reals except for 0 and 3/2.

3. Find the formula for $f^{-1}(x)$ given f(x) below:

$f(x) = \dfrac{x}{3x-2}$ or $y = \dfrac{x}{3x-2}$

We could solve this for x in terms of y, but instead we switch x & y, then solve for y in terms of x.

$x = \dfrac{y}{3y-2}$

Keep in mind you are solving for y !

$3xy - 2x = y$ y !

$3xy = y + 2x$ y !

$3xy - y = 2x$ y !

$y(3x - 1) = 2x$

$y = \dfrac{2x}{3x-1}$ ...so: $f^{-1}(x) = \dfrac{2x}{3x-1}$

4a. Analyze the function given by $f(x) = \dfrac{x-6}{x+2}$ and sketch the graph, labelling everything.

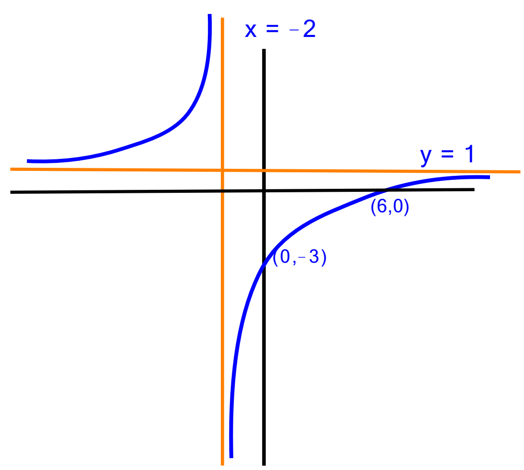
A typical linear rational function.

Vertical asymptote where denominator = 0 ... at x = −2.

Horizontal asymptote at y= 1 because as $x \to \infty$, $f(x) \to 1$.

x-intercept when x − 6 = 0 ... at (x=6,y=0)

y-intercept f(0) = (0−6)/(0+2) = −3 ...i.e. at (x=0, y=−3)



4b. Analyze the function given by $f(x) = \dfrac{x^2+4}{x^2-4}$ and sketch the graph, labelling everything.

Notice this function is EVEN: $f(-a) = f(a)$ for every a.

(Graph should be symmetric about the y-axis.)

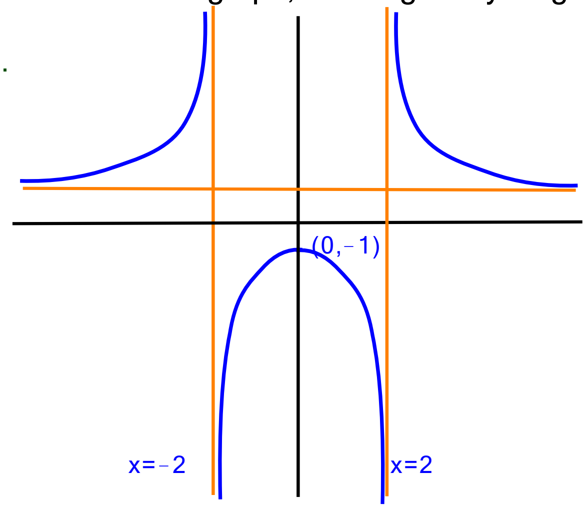
Horizontal asymptote at y= 1 because

$$f(x) = \frac{x^2+4}{x^2-4} = \frac{1+{}^4/x^2}{1-{}^4/x^2} \to \frac{1+0}{1-0} \text{ as } x \to \infty$$

Vertical asymptotes where $x^2 - 4 = 0$... $x = \pm 2$

y-intercept: $f(0) = -1$

x-intercept: $x^2 + 4$ is never 0. (None)



5. Solve. Express your answer in interval notation.

$$\frac{x}{x+4} \leq 2$$

Resist the temptation to multiply both sides. compare to 0, not 2 !! and SIMPLIFY

$$\frac{x}{x+4} - 2 \leq 0$$

$$\frac{x}{x+4} - \frac{2(x+4)}{(x+4)} \leq 0$$

$$\frac{x - 2(x+4)}{x+4} \leq 0$$

$$\frac{-x-8}{x+4} \leq 0$$

	−	−8	+	−4	−
$-x-8$	+		-		-
$x+4$	-		-		+
quotient	-		+		-

check endpoints: at -8: $0 \leq 0$ (inequality is true)
at -4: inequality is NOT true

State your conclusion

Statement is true for x in $(-\infty,-8] \cup (-4,\infty)$

6. For $f(x) = 2x^2 - 5x + 3$, find & simplify completely:

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 5(x+h) + 3 - (2x^2 - 5x + 3)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 5(x+h) + 3 - (2x^2 - 5x + 3)}{h}$$

Distribute that − !!

$$= \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 3 - 2x^2 + 5x - 3}{h}$$

Notice many terms "cancel" !

$$= \frac{4xh + 2h^2 - 5h}{h}$$

$$= 4x + 2h - 5$$

... And there you have it,
in all the excruciating detail....

7a. Find the center and radius of the circle with equation ...

$$x^2 + y^2 - 4x - 6y - 3 = 0$$

This is most easily done by completing the square.

$x^2 - 4x \quad + y^2 - 6y \quad = 3$ Preparing to complete the square

$(x - \text{what})^2 + (y - \text{what})^2 = 3$ + whatever I need....

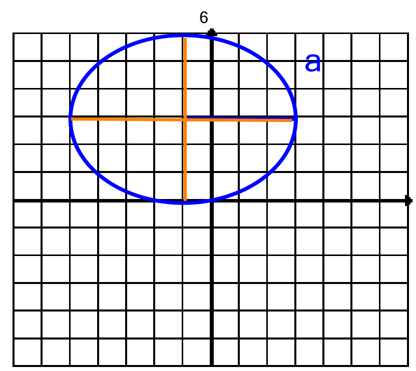
$x^2 - 4x + 4 \quad + y^2 - 6y + 9 \quad = 3 + 4 + 9$

$(x - 2)^2 + (y - 3)^2 = 3 + 13$ Fill in the gaps... and

$X^2 + Y^2 = 16$ we compare to the basic equation of a circle...

So we see this is the equation of a circle with center at (2,3) and radius 4.

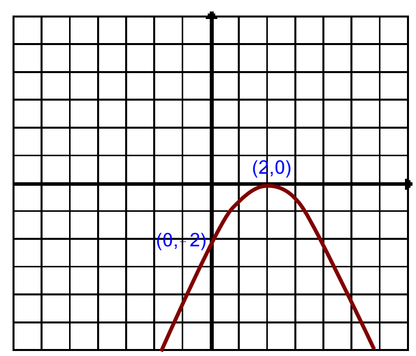
8. a. Write an equation that would have the graph shown ⇨



Make note of the major and minor axes of the ellipse!
These tell us a lot... enough to write the equation...
We see the center is at (−1,3) &
the distance to the major vertices, at (−5,3 & (3,3), is a = 4.
The minor axis (from ((−1,0) to (−1,6)) is 6 units long, so b=3.
This provides sufficient infomation to write the equation:

$$\frac{(x+1)^2}{4^2} + \frac{(y-3)^2}{3^2} = 1$$

b. Assuming this is the graph of a parabola (in x), the equation and seeing the vertex at (2,0), the equation could be:



$$(y - 0) = A(x - 2)^2$$
$$y = A(x-2)^2$$

But what is A? Well, we have not used all the information available in the illustration... for instance, the y-intercept– substituting x=0, y= −2:

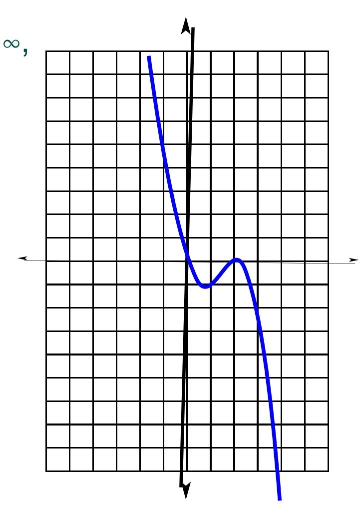
$$-2 = A(0-2)^2$$
$$A = -½$$

Thus: $y = -(½)(x-2)^2$

9. Sketch the graph of $P(x) = -x^3 + 4x^2 - 4x$. Label everything!

As $x \to \infty$, $P(x) \to -\infty$
since as x grows larger, the leading term dominates; that is, as $x \to \infty$, $-x^3 \to -\infty$ faster than $4x^2 - 4x \to +\infty$, so their sum $\to -\infty$.
Similarly, as $x \to -\infty$, $P(x) \to +\infty$.
P(0) = 0, so that's the y=intercept, at (0,0)
x-intercepts:

$$-x^3 + 4x^2 - 4x = 0$$
$$-x(x^2 - 4x + 4) = 0 \quad .$$
$$-x(x-2)(x-2) = 0$$



... we see x-intercepts at 0, 2, and 2.
Repeated root at 2 → brush with x-axis.
(make allowances for the graphing...)

10. For $P(x) = 2x^3 + 8x^2 + 3x - 10$,

a. List all the possible rational zeros of P(x)

Because the coefficients of P(x) are all integers, any rational zero of P must be of the form: $\pm \frac{p}{q}$ where p divides (is a factor of) the constant term -10, and q divides the leading coefficient 2.

That gives us: $\pm \frac{10 \text{ or } 5 \text{ or } 2 \text{ or } 1}{2 \text{ or } 1} = \pm\ 1,\ 2,\ 5,\ 10,\ \frac{1}{2},\ \frac{5}{2}$

b. Show that -2 is a zero of P(x).

-2	2	8	3	-10
		-4	-8	10
	2	4	-5	0

This synthetic division shows, with the zero remainder, that -2 is a zero of P(x).

It also shows us that $P(x) = (x+2)(2x^2 + 4x - 5)$

c. Find all the zeros of P(x). Simplify your answers.

The remaining roots of P must be found in $(2x^2 + 4x - 5)$
Using the quadratic formula to solve $2x^2 + 4x - 5 = 0$... we find:

$$x = \frac{-4 \pm \sqrt{16+40}}{4} = -1 \pm \frac{\sqrt{14}}{2}$$

The third root of this cubic polynomial is -2, the root we found above.

11. Find a fourth degree polynomial P(x) with 1+2i a zero, 1 a zero of multiplicity 2, and a constant term of 10. Express your answer in the form $P(x) = ax^3 + bx^3 + cx^2 + dx + e$, where a,b,c,d,e, are integers.

If a,b,c,d and e are all integers. they must be real numbers.
Since P has only real coefficients, any complex roots must occur in conjugate pairs (that is, for every root a+bi there must also be a root a - bi).

Thus we know the four roots of P must be 1+2i, 1-2i, 1 and 1.
And P must be the product of these factors:

$P(x) = A\,(x - (1+2i))\,(x - (1-2i))\,(x - 1)\,(x - 1)$ where A is some non-zero constant.

$P(x) = A\,(x^2 - 2x + 5)\,(x^2 - 2x + 1)$ which has constant term 5A.
Since they want the constant term to be 10, A had better be 2. (5A = 10)

$P(x) = 2\,(x^2 - 2x + 5)\,(x^2 - 2x + 1)$

$P(x) = 2x^4 - 4x^3 + 20x^2 - 24x + 10$

12. Simplify completely: $4 \log_4(2x) - \log_4 x^5 + \log_4(4x)$.

$4 \log_4(2x) - \log_4 x^5 + \log_4(4x)$

$\log_4(2x)^4 - \log_4 x^5 + \log_4(4x)$

$\log_4 \frac{(2x)^4\,(4x)}{x^5}$

$\log_4\ 64$

One way:
Using the properties of logarithm functions, we can combine these terms into one log expression, and see if the result can be simplified. It can.

$4\log_4(2x) - \log_4 x^5 + \log_4(4x)$

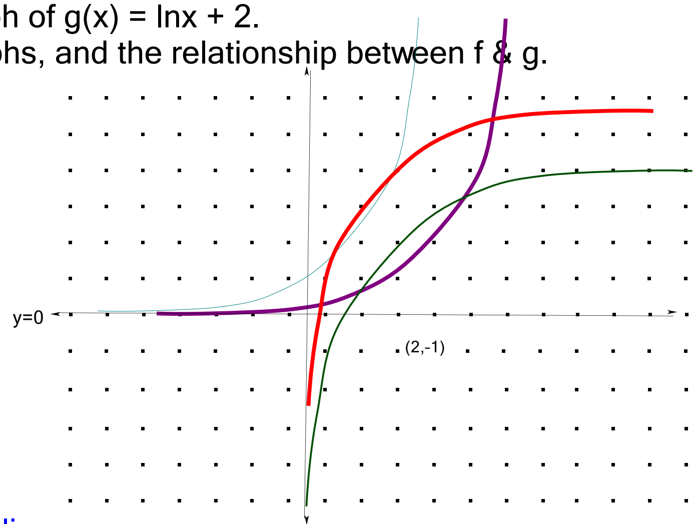
$4\log_4 2 + 4\log_4 x - 5\log_4 x + \log_4 4 + \log_4 x$

$\log_4 16 + \log_4 4 + (4 - 5 + 1)\log_4 x$

$2 \quad + \quad 1 \quad + \quad 0$

Alternate method:
Using the properties of logarithm functions, we can expand & separate these terms so the $\log_4 x$ terms are alone.

13. Sketch the graph of $f(x) = e^{x-2}$ and the graph of $g(x) = \ln x + 2$.
Describe the relationship between these graphs, and the relationship between f & g.



Working from the known graph of $y = e^x$,

$f(x) = e^{x-2}$ is the same graph,
shifted to the right two units

Again, working from a known graph,
we sketch $y=\ell n x$, then shift two units
up to obtain the graph of $g(x) = \ell n x + 2$

The relationship between the graphs
$f(x) = e^{x-2}$ and $g(x) = \ell n x + 2$:
They are reflections of each other across the line y=x .
That is appropriate, since f & g are inverses of each other.

14. Solve for x: $\log_6(x-3) + \log_6(x+2) = 1$

$\log_6(x-3) + \log_6(x+2) = 1$

$\log_6((x-3)(x+2)) = 1$

$((x-3)(x+2)) = 6^1$

$x^2 - x - 6 = 6$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or -3

Better to have one log expression than two,
Now we can "undo" the $\log_6$ using exponential base 6.

CHECK THOSE SOLUTIONS!!!!

Only x = 4 is a valid solution.

15. Solve for x (exact answer.): $e^{x-2} = 3^x$

$e^{x-2} = 3^x$

$\ln(e^{x-2}) = \ln(3^x)$

$(x-2)\ln(e) = x\ln(3)$

$x - 2 = x\ln 3$

$x - x\ln 3 = 2$

$(1 - \ln 3)x = 2$

$x = 2/(1-\ln 3)$ That's an exact answer.

Std. Operating Procedure:
x inside an exponential function, use ℓn....
and use those logarithmic properties!
... and keep in mind that $\ln 3$ is just a number.

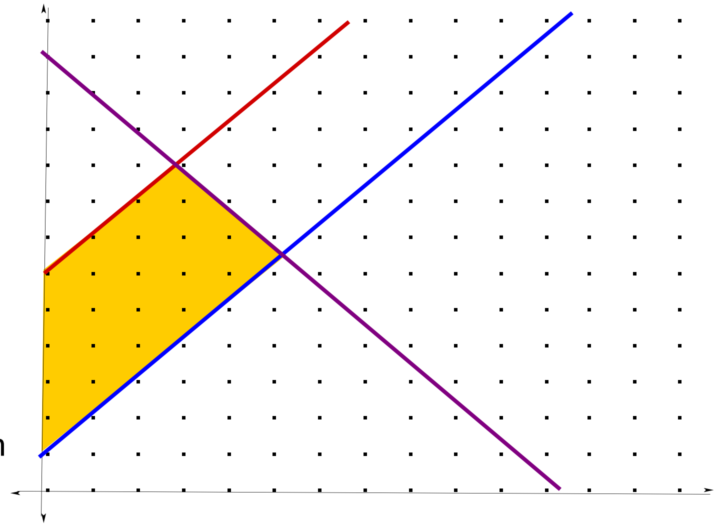
16. Find the maximum for P(x,y) = 2x + 3y, subject to conditions:
$x \geq 0$; $y \geq 0$; $y \geq x + 1$; $x + y \leq 12$; $y \leq x + 6$. Complete justification required.

We first graph the "region of feasibility"- that part of the plane where the conditions are met:
Conditions $x \geq 0$ & $y \geq 0$ restrict us to the first quadrant.

$y \geq x + 1$ ON & ABOVE the line $y = x+1$

$x + y \leq 12$ ON & BELOW the line $y = 12 - x$

$y \leq x + 6$ ON & BELOW the line $y = x + 6$



The vertices (corners) of this region are:

(0,1) (0,6) (3,9) and (6,6)

Then we are guaranteed that the linear function P(x,y) = 2x+3y has both a maximum value and a minimum value somewhere in this polygonal region, and that those extreme values must be seen at the vertices.
So evaluating P at all the vertices will provide sufficient information to locate them.
P(0,1) = 2·0 + 3·1 = 3
P(0,6) = 18
P(3,9) = 33 ← Maximum value
P(6,6) = 30

17. Find all points on the line x − y = 7 that lie exactly 5 units from the origin.
(Hint: the points of the plane that lie 5 units from (0,0) form a conic section known as....)

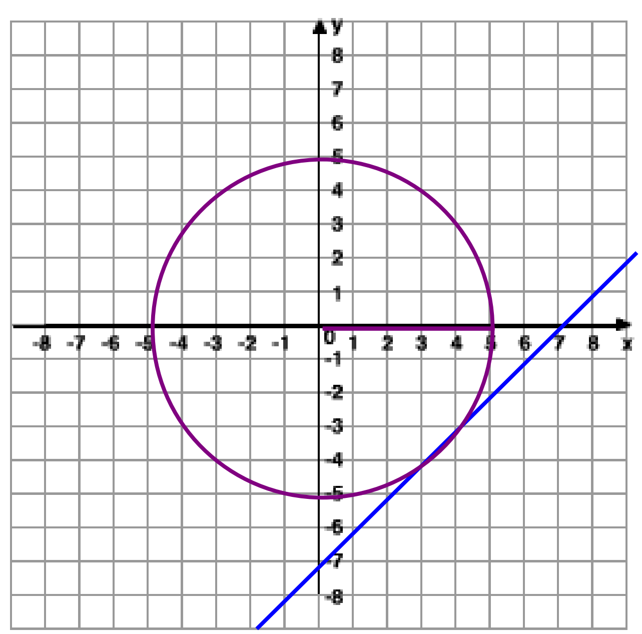
A point (x,y) lies 5 units from the origin if, and only if, $x^2 + y^2 = 5^2$.
If (x,y) lies on the line given, then (x,y) must satisfy $x - y = 7$. We must solve this system.

We solve $x - y = 7$ for either x or y, then substitute into the statement: $x^2 + y^2 = 5^2$.

Let's choose y: $y = x - 7$

Substituting:

$$x^2 + (x-7)^2 = 5^2$$
$$x^2 + x^2 - 14x + 49 = 25$$
$$2x^2 - 14x + 24 = 0$$
$$x^2 - 7x + 12 = 0$$
$$(x - 3)(x - 4) = 0$$
$$x = 3 \text{ or } 4$$



There are two points on the line
x − y = 7 that lie 5 units from (0,0):
(3,−4) and (4,−3).

18. \$2000 is invested in an account that pays 3% (annual percentage rate) compounded continuously. How long will it take for the balance to reach \$3000?

When amount P_0 is invested at r % interest compounded N times per year, the value of the account at time t years is

$$P(t) = P_0 \left(1 + r/N\right)^{Nt}$$

As $N \to \infty$, this approaches

$$P_0 e^{rt}$$

... the value achieved with continuous compounding.

So, the value of this account after t years is $\$2000\, e^{.03t}$ and we want to solve for t so that this value will be \$3000.

$$\$2000\, e^{.03t} = \$3000$$

$$e^{.03t} = 1.5$$

$$.03\,t = \ln 1.5$$

$$t = \frac{\ln 1.5}{.03} \text{ years} \quad (\doteq 13.52 \text{ years})$$

19. A colony contained 2000 bacteria at 6AM, 3000 at 8AM. How many will there be at noon? When will there be 10,000 bacteria?

Bacteria multiply just like continuous compounding, but usually a lot faster! So we can expect the amount (A) of bacteria at time t to be $A(t) = A_0 e^{rt}$.
Since $A(0) = A_0 e^{r \cdot 0} = A_0$ & $A(0) = 2000$ (considering 6AM to be time 0), we say $A_0 = 2000$. Next we find r:

At 8AM (t=2) there are 3000 bacteria, so: $A(2) = 2000\, e^{r2} = 3000$. Solve for r:

$$e^{r2} = 3000/2000 = 3/2 \text{ or } 1.5$$

Taking ln of both sides, we get:

$$2r = \ln 1.5$$

$$r = (½) \ln 1.5$$

So we now have a complete model for the bacteria population:

$$A(t) = 2000\, e^{(½)\, t \ln 1.5}$$

At noon (t = 6) there will be: $A(6) = 2000\, e^{(½)\, 6 \ln 1.5} = 2000\, (1.5)^3 = 6750$

There will be 10000 bacteria when A(t) = 10000.... When $2000\, e^{(½)\, t \ln 1.5} = 10000$

We solve: $e^{(½)\, t \ln 1.5} = 5$

by taking ln of both sides: $(½)\, t \ln 1.5 = \ln 5$

$$t = \frac{2 \ln 5}{\ln 1.5} \text{ (Hours after 6am)}$$

20. Find the quadratic function passing through the points (–1,8) & (1,–4) & (2,–1).
Hint: $f(x) = ax^2 + bx + c$.

$f(-1) = a(-1)^2 + b(-1) + c$ must = 8: $a - b + c = 8$
$f(1) = a(1)^2 + b(1) + c$ must = –4: $a + b + c = -4$
$f(2) = a(2)^2 + b(2) + c$ must = –1: $4a + 2b + c = -1$

This system is easily solved for a & b & c (they are 3 & –6 & –1), so $f(x) = 3x^2 - 6x - 1$.

21. Find the dimensions of a rectangle whose perimeter is 105 meters and area is 441 square meters.

A rectangle's perimeter and area are understood in terms of its length and width, so we will call those dimensions x and y.

Perimeter = 105 meters ⇒ $2x + 2y = 105$ m
Area = 441 m^2 ⇒ $xy = 441\ m^2$ Note this is a nonlinear system.

Solving the first equation for y in terms of x: $y = 52.5 - x$

Substituting in the second equation for y: $x(52.5 - x) = 441$

which yields the quadratic equation: $x^2 - 52.5x + 441 = 0$
using either the quadratic formula on this or factoring the equivalent: $2x^2 - 105x + 882 = 0$
$(2x - 21)(x - 42) = 0$
we solve for x, obtaining x = 10.5 and x = 42 as candidates...
If x = 10.5, then y = 42. If x = 42, then y = 10.5.
The dimensions of the rectangle are 10.5 m by 42 m.

Additional word problems:

Text §4.1 page 308 #69 Answer in text

Text §6.3 page 512 #73 Answer in text

Text §7.1 page 545 # 59, 61 Answer in text

Text §7.2 page 562 #77 (set-up at right) Answer in text

Setup for #77: (Using S B A)

protein: $30S + 15B + 3A = 78$
Carbs: $20S + 2B + 25A = 59$
Vitamin: $2S + 20B + 32A = 75$

Text Chapter 7 Review page 628 #100, 102
#100 answer: 466.25 acres in corn, 533.75 acres in soybeans.
#102 answer: $x \geq 0$; $y \geq 0$; $4x + 3y \leq 960$; $2x + 3y \leq 576$.