♡ Math 102 ◇ Test §§5.1-6.2 ♧ April 10, 2008 ♤ ANSWERS

1. $P(x) = 2x^6 + 4x^5 + x^4 - 2x^3 - 4x^2 - 6x - 3$

Questions have been reworded to contain explanations.

a. P is a polynomial of degree 6 .

b. A polynomial of degree n must have n zeros (counting multiplicity). So P must have 6 zeros.

c. A polynomial of degree n has at most $n - 1$ turning points, so P may have a maximum of 5.... "turning points" (local max/min points).

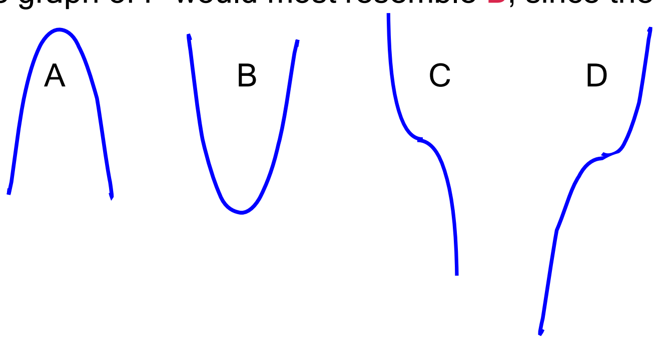
d. The y-intercept of the graph of P is $P(0) = -3$.

e. Polynomial functions have NO vertical asymptotes. So P has 0 V.A.

f. According to Descartes' Rule of Signs, P can have 1 positive real zero.

g. Similarly, since P(-x) has + −++−+ − 5 sign changes...P can have 5, 3, or 1 negative real zeros.

h. Viewed from afar, the graph of P would most resemble B, since the graph of $y = x^6$ looks like B.



NOTE: $2x^6 + 4x^5 + x^4 - 2x^3 - 4x^2 - 6x - 3$ is $(x+1)^2 (x^2+1)(2x^2 - 3)$ So there are actually three real negative zeros $(-1, -1, -\sqrt{3/2})$ and one positive zero $(\sqrt{3/2})$, and two complex zeros $(0 \pm i)$

2. LIST all the theoretically possible* rational zeroes of $P(x) = 9x^3 + 44x^2 + 31x - 4$.
* According to the rational zeroes theorem, only rational numbers of the form p/q, where p divides a_0 and $q|a_n$, may be zeroes of $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$:

This gives us: $\pm \dfrac{4,\ 2,\ 1}{9,\ 3,\ 1} = \pm\ 1,\ 2,\ 4,\ \dfrac{1}{3}, \dfrac{2}{3}, \dfrac{4}{3}, \dfrac{1}{9}, \dfrac{2}{9}, \dfrac{4}{9}$

NOTE The three roots of P are all in that list. <>

(12) 3. Write a polynomial function P with REAL coefficients having zeroes −2, with multiplicity 2, and 3i, such that $P(1) = 30$. Show that your polynomial has real coefficients.

−2 a zero of multiplicity two ⇨ $(x+2)^2$ is a factor of P.

3i a zero, and coefficients real ⇨ −3i is also a zero, and $(x - 3i)$ and $(x + 3i)$ are factors of P. Thus:

$P(x) = A\,(x-2)^2 (x - 3i)\ (x + 3i)$

$= A\,(x-2)^2 (x^2 + 9)$ (Showing the coefficients are real.)

And $P(1) = A\,(1+2)^2 (1^2 + 9) = 90A$ ...which must be 30, so A is ⅓.

So $P(x) = \frac{1}{3}(x-2)^2 (x^2 + 9) = \frac{1}{3}(x^2 - 4x + 4))(x^2 + 9) = \frac{1}{3}(x^4 - 4x^3 + 13x^2 - 36x + 36)$

$P(x) = \frac{1}{3}(x^4 - 4x^3 + 13x^2 - 36x + 36)$

is a polynomial function satisfying all the requirements.

(10) 4. Use the process of SYNTHETIC DIVISION to test the numbers 2 and –2 for possible zeroes of the polynomial $x^3 - 2x^2 + 2x + 20$. State whether each is a zero or NOT.

2	1	−2	2	20
		2	0	4
	1	0	2	24

... telling us that P(2) = 24

−2	1	−2	2	20
		−2	8	−20
	1	−4	10	0

...and P(−2) = 0

So −2 is a zero of P. And $P(x) = (x+2)(x^2 - 4x + 10)$ ← This helps with the next problem, #5.

(10) 5. Now finish the job: Find ALL the zeroes of the polynomial function given in #4.

By the work done in #4 we know: $P(x) = (x+2)(x^2 - 4x + 10)$ from finding that 2 is a zero.

The remaining two zeroes of this polynomial function can be found in $Q(x) = x^2 - 4x + 10$.

We solve $x^2 - 4x + 10 = 0$

...using the quadratic formula, $x = \dfrac{4 \pm \sqrt{-24}}{2} = \dfrac{4 \pm \sqrt{24}\, i}{2} = \dfrac{4 \pm 2\sqrt{6}\, i}{2} \quad 2 \pm \sqrt{6}\, i$

The zeroes of $P(x) = x^3 - 2x^2 + 2x + 20$ are -2 and $2 - \sqrt{6}\, i$ and $2 + \sqrt{6}\, i$

(10) 6. Given the function expressed as f(x) below, find its inverse: that is, find $f^{-1}(x)$.

$f(x) = \dfrac{2x - 1}{x - 3}$ We note this says: $y = \dfrac{2x - 1}{x - 3}$

$x = \dfrac{2y - 1}{y - 3}$ Since the inverse function reverses the (x,y) pairs, we interchange x and y, then solve for y (the new y) to determine the inverse function, $f^{-1}(x)$

$x(y - 3) = 2y - 1$

$xy - 3x = 2y - 1$

$xy - 2y = 3x - 1$

$y(x - 2) = 3x - 1$

$y = \dfrac{3x - 1}{x - 2}$

which is the equivalent of:

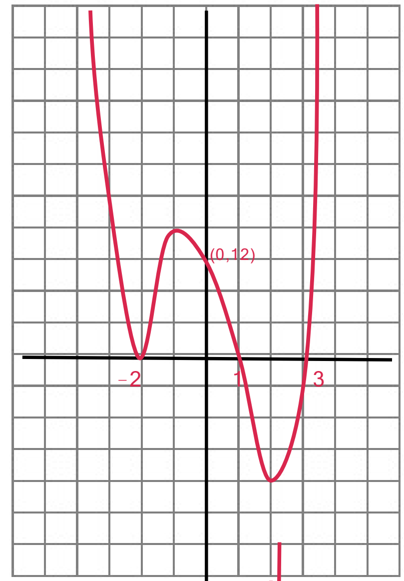
$f^{-1}(x) = \dfrac{3x - 1}{x - 2}$

(10) 7. Sketch the graph of the polynomial function given by

$$f(x) = (x + 2)^2 (x - 1)(x - 3)$$

Show the behavior of the function clearly.

Label all intercepts .



f is a polynomial function of degree 4.
So its behavior far from 0 is that of $y = x^4$.

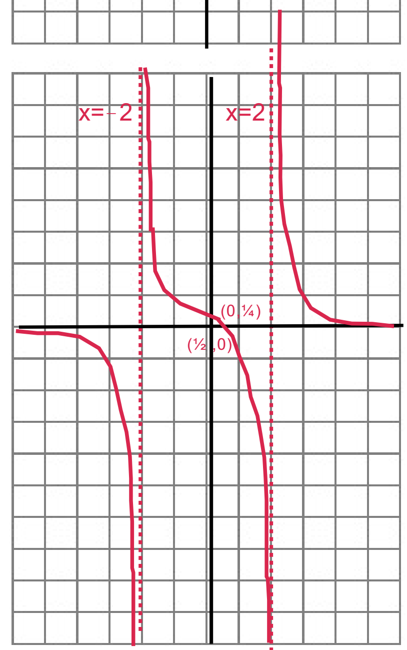
−2 is a double zero; 1 & 3 are single zeros.
So graph touches & rebounds at −2, passes though x-axis at 1 & 3.

$f(0) = (0+2)^2 (0 - 1) (0-3) = (4)(-1)(-3) = 12$

a few other points: (−3, 24) & (2,−16) & (4,108)

(10) 8. Sketch the graph of the function given. Show the behavior clearly.
Show and label on the graph all asymptotes and x- & y- intercepts.

$$f(x) = \frac{2x - 1}{x^2 - 4}$$



Domain all reals except ±2. Since 2x−1 is non-zero while $x^2 - 4$ is zero at x=2 and x= −2, Vertical Asymptotes occur there.

H.A. At y=0, since denominator grows faster than numerator.

Intercepts: y-intercept, when x=0, is f(0) = (−1)/(−4) = ¼
y=0 when 2x−1 = 0 .... when x = ½

Signs say: – | + 0 – | +
−2 ½ 2

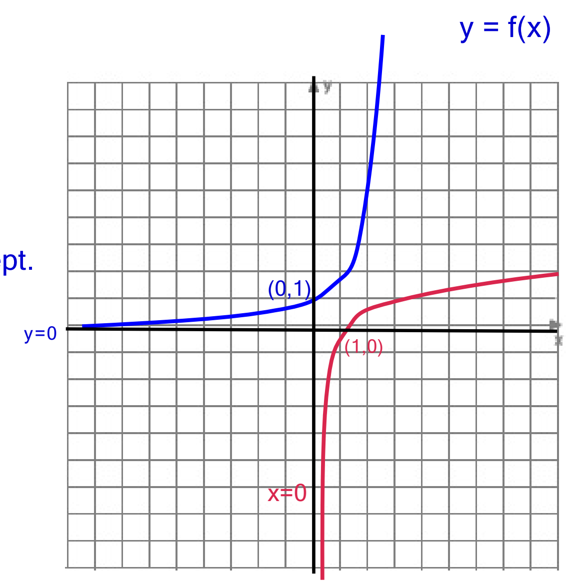
(10) 9. Given the graph of a function f, as shown...
sketch the graph of the inverse of the function given.

Show and label on the graph all asymptotes

and x- & y- intercepts for f^{-1} .

The function f has t-0 as an asymptote, and (0,1) as y-intercept.

So f^{-1} .has x=0 as asymptote, and (1,0) as x-intercept.



(2) 10. Show that $f(x) = \sqrt{2x - 1}$ and $g(x) = \frac{x^2 + 1}{2}$ are inverses.

$$f \circ g(x) = f(g(x)) = \sqrt{2g(x) - 1} = \sqrt{2\,\frac{x^2+1}{2} - 1}$$

$$= \sqrt{2\,\frac{x^2+1}{2} - 1} = \sqrt{x^2 + 1 - 1} = \sqrt{x^2} = x, \text{ provided } x \geq 0 .$$