CHAPTER 4 QUIZ QUESTIONS due March 27, 2007 NAME

Show your work in a neat and organized manner, write final answers on THIS page. Attach your work.

1. Which of the following statements are true?
   (In each statement, assume any function called "P" or "Q" is a polynomial function.)

a. T F If $P(4) = 7$ and $P(5) = -3$, then $P(r)$ must be 0 for some number r between 4 and 5.

b. T F If $P(x) = (x - 6)Q(x) + 2$, for some polynomial Q, then $P(6) = 2$.

c. T F If 6 is a zero of P, then $P(x) = (x - 6)Q(x)$ for some polynomial Q.

d. T F If $P(x) = x^3 - 2x^2 - 8x$ then the roots of P are 4 and –2.

e. T F If $(x - 6)^2$ divides P(x), but $(x - 6)^3$ does not, then we say: "6 is a zero of multiplicity 2 for P", and we count 6 as two of the zeroes (roots) of P.

2. On the Quiz, you used Descartes' Rule of Signs to predict the number of positive and negative real zeros of $P(x) = 2x^5 - 2x^4 - 3x^3 - 6x^2 - 2x + 4$. These are too hard to find.
   Alternate: Find all the zeros of the new $P(x) = 2x^5 - 4x^4 - 3x^3 + 6x^2 - 2x + 4$.

3. On the Quiz, you made a list of the possible rational zeros of $P(x) = 4x^4 + 4x^3 + 9x^2 + 12x - 9$.
   Find all the zeros. List them here:

4. On the Quiz, you investigated this statement: $P(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$ might have a zero at $x = ½$ or at $x = 2$. Find all the zeros of P. List them here:

5. How can we know that $P(x) = x^4 + 5x^2 + 7$ has no real roots, without a lot of work?

Look out below!

6. List ALL the theoretically possible rational roots of $P(x) = 2x^3 - ½ x^2 - 32 x + 8$:

7. Find all the roots of the polynomial given in #6. They are:
   Any surprises?

8. List all the theoretically possible rational roots of $P(x) = 2x^3 - 5 x^2 - 3x$:

9. Find all the roots of $P(x) = 2x^3 - 5 x^2 - 3x$. List them here:

10. Find all the roots of $P(x) = x^4 - x^2 - 20$. List them here:

11. Why does the function: $f(x) = \dfrac{x^2 - 4x + 4}{2x^2 - 8}$ NOT have a vertical asymptote at $x = 2$?

12. Find ALL the asymptotes of the function: $f(x) = \dfrac{4x^2}{2x - 1}$ Hint: divide and conquer!
    Sketch the graph on the reverse of this page.

1. In each statement, we assume any function called "P" or "Q" is a polynomial function.

a. If P(4) = 7 and P(5) = −3, then P (r) must be 0 for some number r between 4 and 5.

True. P is a polynomial function, therefore continuous, and so must pass through every value between −3 and 7 at least once on the interval between 4 and 5.

b. If P(x) = (x − 6)Q(x) + 2, for some polynomial Q, then P(6) = 2.

True. This is the remainder theorem. But it is also obvious: P(6) = (6 − 6)Q(x) + 2 = 0 + 2 .

c. If 6 is a zero of P, then P(x) = (x − 6)Q(x) for some polynomial Q.

True. This follows from the remainder theorem.

d. If $P(x) = x^3 - 2x^2 - 8x$ then the roots of P are 4 and –2.

False. P(x) = x(x − 4)(x + 2) so the roots are 0 and 4 and −2. PS cubic polynomial ⇨ 3 roots.

e. If $(x-6)^2$ divides P(x), but $(x-6)^3$ does not, then we say: "6 is a zero of multiplicity 2 for P", and we count 6 as two of the zeroes (roots) of P.

True.

2. On the Quiz, you used Descartes' Rule of Signs to predict the number of positive and negative real zeros of $P(x) = 2x^5 - 2x^4 - 3x^3 - 6x^2 - 2x + 4$.

A polynomial of degree 5? We hope there are some rational zeroes.... Possible rational zeroes are ±1, 2, 4, ½ . We test those here:

½	2	−2	−3	−6	−2	4
		1	−½	$-\frac{7}{4}$	$\frac{31}{8}$	$-\frac{47}{16}$
	2	−1	$-\frac{7}{2}$	$-\frac{31}{4}$	$-\frac{47}{8}$	$\frac{17}{16} = 1\frac{1}{16}$

−½	2	−2	−3	−6	−2	4
		−1	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{21}{8}$	$-\frac{5}{16}$
	2	−3	$-\frac{3}{2}$	$-\frac{21}{4}$	$\frac{5}{8}$	$3\frac{11}{16}$

1	2	−2	−3	−6	−2	4
		2	0	−3	−9	−11
	2	0	−3	−9	−11	−7

−1	2	−2	−3	−6	−2	4
		−2	4	−1	7	−5
	2	−4	1	−7	5	−1

2	2	−2	−3	−6	−2	4
		4	4	2	−8	−20
	2	2	1	−4	−10	−16

−2	2	−2	−3	−6	−2	4
		−4	12	18	−24	52
	2	−6	9	12	−26	56

4	2	−2	−3	−6	−2	4
		8	24	84	312	1240
	2	6	21	78	310	1244

−4	2	−2	−3	−6	−2	4
		−8	40	−148	568	−2280
	2	−10	37	−154	570	−2276

We have struck out looking for rational zeroes; there are none.

From the work above, we do know that there are five real zeroes, in the following locations:
between −4 and −2 ...since P(−4) = −2276 and P(−2) = 56
between −2 and −1 ...since P(−2) = 56 and P(−1) = −1
between −1 and −½ ...since P(−1) = −1 and P(−½) > 0
between ½ and 1 ...since P(½) > 0 and P(1) < 0
between 2 and 4 ...since P(2) < 0 and P(4)>0

Finding very specific information about the roots of P would be outside the scope of this course.

3. Find the zeroes of $P(x) = 4x^4 + 4x^3 + 9x^2 + 12x - 9$.

Possible Rational Roots: $\pm (9,3,1)/(4,2,1) \Rightarrow \pm 1, 3, 9, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}$
A quick application of Descartes' Rule indicates 1 positive zero, and 1 or 3 negative zeroes.

½	4	4	9	12	−9
		2	3	6	9
	4	6	12	18	☺ !
	2	3	6	9	

4 6 12 18 represents $4x^3 + 6x^2 + 12x + 18$
which can be reduced to $2(2x^3 + 3x^2 + 6x + 9)$
Further, note all the signs are positive, so we need not check for any more positive roots!

−½	2	3	6	9
		−1	−1	$-\frac{5}{2}$
	2	2	5	$\frac{13}{2}$

−3	2	3	6	9
		−6	9	−45
	2	−3	15	☹

$-\frac{3}{2}$	2	3	6	9
		−3	0	−9
	2	0	6	0 !

From above we know that $P(x) = 4x^4 + 4x^3 + 9x^2 + 12x - 9$ factors to
$= (x - \frac{1}{2})\, 2\, (x + \frac{3}{2})\, 2\, (x^2 + 3)$

The zeroes of P are: $\frac{1}{2}$ $-\frac{3}{2}$ and $\pm\sqrt{3}\,i$

4. Find all the zeroes of $P(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

Possible Rational Roots: $\pm(6,3,2,1)/(2,1)$

½	2	−3	−7	−8	6
		1	−1	−4	−6
	2	−2	−8	−12	0 !

2 • (1 −1 −4 −6)

Notice that the resulting polynomial quotient has factor 2. Factoring out the 2, we have 1 −1 −4 −6, representing $x^3 - x^2 - 4x - 6$ so PRRs are only ± 6, 3, 2, 1. We guess 3 might be a zero.

3	1	−1	−4	−6
		3	6	6
	1	2	2	0 !

From all this work, we know that
$P(x) = (x - \frac{1}{2})2(x - 3)(x^2 + 2x + 2)$
So the last two roots will be found solving $x^2 + 2x + 2 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$

5. $P(x) = x^4 + 5x^2 + 7$ has no (0) sign changes.
Therefore, by Descartes' Rule of Signs, P has at most no (0) real positive zeroes.
$P(-x) = x^4 + 5x^2 + 7$ also has no sign changes.
Therefore, by Descartes' Rule of Signs, P has at most no real negative zeroes.
Also note that $x = 0$ is not a zero of P, since $P(0) = 7$.
Since P has no positive or negative or 0 real roots, P has no real roots.

6. List ALL the theoretically possible rational roots of $P(x) = 2x^3 - \frac{1}{2}x^2 - 32x + 8$
$=(\frac{1}{2})(4x^3 - x^2 - 64x + 16)$ $\quad \frac{\pm 16, 8, 4, 2, 1}{4, 2, 1}$

(THE RATIONAL ZEROES THEOREM REQUIRES INTEGER COEFFICIENTS !)
Possible rational zeroes are: ± 1,2,4,8,16, ½, ¼

7. Find the zeroes of $P(x) = 2x^3 - \frac{1}{2}x^2 - 32x + 8 = (\frac{1}{2})(4x^3 - x^2 - 64x + 16)$
(Bet on ±16 or ¼.)

¼	4	−1	−64	16
		1	0	−16
	4	0	−64	0 !!!

so $P(x) = (\frac{1}{2})(x - \frac{1}{4})(4x^2 - 16)$
$= (\frac{1}{2})(x - \frac{1}{4})\,4\,(x^2 - 4)$
$= (\frac{1}{2})(x - \frac{1}{4})\,4\,(x-2)(x+2)$

.... The zeroes of P are **¼   2   −2**

8. List all the theoretically possible rational roots of $P(x) = 2x^3 - 5x^2 - 3x$: **0, ±1, 3, ½, $^3/_2$**
$= x(2x^2 - 5x - 3)$

(THE RATIONAL ZEROES THEOREM REQUIRES NON-ZERO CONSTANT TERM!)

9. Find all the roots of $P(x) = 2x^3 - 5x^2 - 3x$. The zeroes of P are: **0, −½, 3**
$P(x) = x(2x^2 - 5x - 3)$
$= x(2x+1)(x-3)$

10. Find all the roots of $P(x) = x^4 - x^2 - 20$. The zeroes of P are : **$-2i$, $2i$, $-\sqrt{5}$, $\sqrt{5}$**
$= (x^2 - 5)(x^2 + 4)$

11. Why does the function: $f(x) = \dfrac{x^2 - 4x + 4}{2x^2 - 8}$ NOT have a vertical asymptote at x = 2 ?

...BECAUSE... : $f(x) = \dfrac{(x-2)^2}{2(x+2)(x-2)} = \dfrac{x-2}{2(x+2)}$ ...a function that has no vertical asymptote at x = 2.

Provided x is NOT 2

These two functions are identical, except that f is undefined at x = 2.

12. Find ALL the asymptotes of: $f(x) = \dfrac{4x^2}{2x-1} = 2x + 1 + \dfrac{1}{2x-1}$

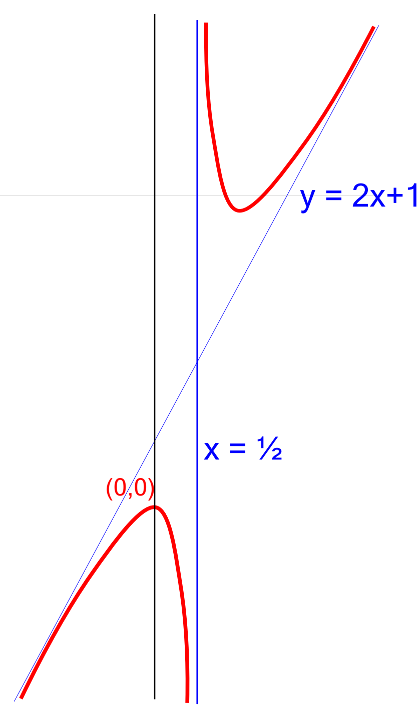
Vertical asymptote at x = ½ .

We divide to find the non-vertical asymptote:

y = 2x + 1

```
          2x  +  1
2x - 1 | 4x²
         4x² - 2x
               2x
               2x - 1
                    1
```

f	−		−		+
		0		½	



y-intercept = x-intercept = (0,0)
This is also a local maximum.

There is also a local minimum symmetrically located through the intersection of the asymptotes... at (1,4).