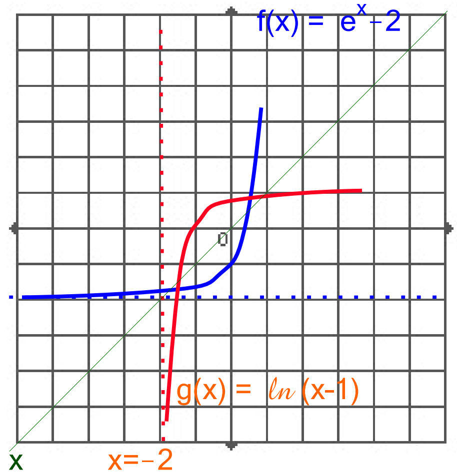
1. Sketch the graph of $f(x) = e^x - 2$. Label the intercept and asymptote.
Sketch the graph of $g(x) = \ln(x+2)$. Label the intercept and asymptote.



The graph of $f(x) = e^x - 2$ is the graph of $y = e^x$ shifted down 2 units.
$y = e^x$ has a horizontal asymptote at the x-axis, $y = 0$.
$f(x) = e^x - 2$ has a horizontal asymptote at $y = -2$.

The graph of $g(x) = \ln(x+2)$ is the graph of $y = \ln(x)$ shifted left 2 units.
$y = \ln(x)$ has a vertical asymptote at the y-axis, $x=0$, so
$g(x) = \ln(x+2)$ has a vertical asymptote at $x = -2$.

These functions are reflections of each other across the line $y = x$;
this is evidence of the fact that these two functions are inverses of each other.

2. Solve for x: $\ln(3x+2) + \ln(x) = 0$

To solve a logarithmic equation, we use exponential function. But first it would help to get one $\ln$ expression.

$\ln(x(3x+2)) = 0$

Now we are ready to take the exponential function on each side

$e^{\ln(x(3x+2))} = e^0$

and simplify....

$(x(3x+2)) = 1$

Resulting in a quadratic equation which we may solve by factoring, or by quadratic formula

$3x^2 + 2x - 1 = 0$

$(3x-1)(x+1) = 0$

$x = ⅓$ or -1

$x = ⅓$

And now, you must check these "solutions".
Since $\ln(-1)$ does not exist, -1 does not solve the equation.
⅓ does: $\ln(3) + \ln(⅓) = \ln(3) - \ln(3) = 0$

3. Solve for x: $5^{x-3} = 2^x$.

To solve an exponential equation, we use a logarithm. Taking $\ln$ of both sides...

$\ln 5^{x-3} = \ln 2^x$

Using the logarithmic property $\ln x^p = p \ln x$ ...

$(x-3)\ln 5 = x \ln 2$

$x \ln 5 - 3 \ln 5 = x \ln 2$

Keep in mind that you are solving for x

$x \ln 5 - x \ln 2 = 3 \ln 5$

$x(\ln 5 - \ln 2) = 3 \ln 5$

One division step away from the answer...

$$x = \frac{3 \ln 5}{\ln 5 - \ln 2}$$

Solve for x: $8^{x-6} = 2^x$

can be solved in the same manner as above, or we can take advantage of the fact that $2^3 = 8$

$(2^3)^{x-6} = 2^x$

to get

$2^{3x-18} = 2^x$

And since exponential functions are 1-1, we know this follows:

$3x - 18 = x$

$x = 9$

4. Find inverse function: $f(x) = \dfrac{1}{3x-1}$.

We can use the method shown at right, or...

observe that f multiplies by 3, subtracts 1, then takes reciprocal

so f^{-1} must take reciprocal, add 1, then divide by 3.

That is, $f^{-1}(x) = \dfrac{\frac{1}{x}+1}{3} = \dfrac{1+x}{3x}$

$f(x) = y = \dfrac{x+1}{3x-1}$ First we interchange x & y

then we solve for y....

$x = \dfrac{y+1}{3y-1}$

$3xy - x = y + 1$ multiply both sides by $(3y-1)$

$3xy - y = 1 + x$ then add x, add $-y$

$y = \dfrac{1+x}{3x-1}$ $f^{-1}(x) = \dfrac{1+x}{3x-1} = f(x)$

(10) 5. Simplify as completely as possible:

$\log_2 8 - \log_2(3) + \log_2(6) + \log_2(¼) + \ln e^2$

$\log_2 8 + \log_2(6) - \log_2(3) + \log_2(¼) + \ln e^2$

$\log_2 2^3 + \log_2(6/3) + \log_2(2^{-1}) + \ln e^2$

$\log_2 2^3 + \log_2(2) + \log_2(2^{-2}) + \ln e^2$

$3 + 1 + -2 + 2 = 4$

Several approaches are possible. But, since we don't know the value of $\log_2 3$ or $\log_2 6$, it is essential to notice we can combine them ....

(12) 6. A population of beetles had 4 members on Monday, and 400 on Wednesday.
How many will there be on Friday? 40,000 When will the population reach 100000?

Intuitively: the population must be multiplying by 10 each day. That is:
Mon 4 Tues 40 Wed 400 Thurs 4000 Fri 40000 et cetera....

To answer the second question, we solve: $4 \cdot 10^t = 100000$... $10^t = 25000$... $t = \ln 25000/\ln 10$.

Using "SOP": $A(t) = A_0 e^{rt}$

$400 = 4e^{r2}$

$100 = e^{2r}$

$\ln 100 = 2r$

$r = (½)\ln 100 = \ln 10$

SO $A(t) = 4e^{t\ln 10}$

$4e^{t\ln 10} = 100000$

$e^{t\ln 10} = 25000$

$t\ln 10 = \ln 25000$

$$t = \frac{\ln 25000}{\ln 10}$$

7. According to the rational zeroes theorem, only certain rational numbers may be zeroes of the polynomial function: $P(x) = 8x^3 - 36x^2 - 2x + 9$. List all the theoretically possible rational zeroes of P(x).

Rational Zeroes Theorem says the only rational possible zeroes are p/q where p divides 9, q a factor of 8.

This gives us: $\pm \dfrac{9, 3, 1}{8, 4, 2, 1} = \pm\ 1, 3, 9, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}, \frac{1}{8}, \frac{3}{8}, \frac{9}{8}$

8. Use the process of synthetic division to test the numbers ½ and –½ for possible zeroes of the polynomial function $P(x) = 2x^3 - 5x^2 + 5x + 4$.

½	2	−5	5	4
		1	−2	3/2
	2	−4	3	5 ½

−½	2	−5	5	4
		−1	3	−4
	2	−6	8	0

... telling us that P(½) = 5 ½ and P(−½) = 0

So −½ is a zero of P. And $P(x) = (x+ \tfrac{1}{2})(2x^2 - 6x + 8)$ ← This helps finish #9.
$= (x+ \tfrac{1}{2})2(x^2 - 3x + 4)$

9. Now finish the job: Find ALL the zeroes of the polynomial function given in #8.

The remaining two zeroes of this polynomial function can be found in $Q(x) = x^2 - 3x + 4$.

Using the quadratic formula, Q(x) = 0 when $x = \dfrac{3 \pm \sqrt{-7}}{2} = \dfrac{3 \pm \sqrt{7}\,i}{2}$

The zeroes of $P(x) = 2x^3 - 5x^2 + 5x + 4$ are $-\frac{1}{2}$ and $\frac{3}{2} - \frac{\sqrt{7}\,i}{2}$ and $\frac{3}{2} + \frac{\sqrt{7}\,i}{2}$

10. Write a polynomial function with real coefficients having zeroes: 1 a zero of multiplicity two, and i a zero. The value of the polynomial at x=2 must be 15.

1 a zero of multiplicity two ⇨ $(x-1)^2$ is a factor of P.
i a zero, and coefficients real ⇨ −i is also a zero, and (x −i) and (x + i) are factors of P. Thus:

$P(x) = A(x-1)^2 (x-i)(x+i)$

$= A(x-1)^2 (x^2 + 1)$

And $P(2) = A(2-1)^2 (2^2 + 1) = 5A$ ...which must be 15, so A is 3.

So $P(x) = 3(x-1)^2 (x^2 + 1)$

$P(x) = 3x^4 - 6x^3 + 6x^2 - 6x + 3$ is a polynomial function satisfying all the requirements.