Math 102 TEST CHAPTERS 3 & 4 Solutions & Comments Spring 2008

1. For $f(x) = 2x^2 - 6x$, find $\frac{f(x+h) - f(x)}{h}$ and simplify completely.

NOTE:
f(x+h) is NOT f(x) (x+h) !

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 6(x+h) - (2x^2 - 6x)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 6x - 6h - (2x^2 - 6x)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 6x - 6h - (2x^2 - 6x)}{h}$$

Many terms "cancel".
Notice, eg $-6x - (-6x) = 0$

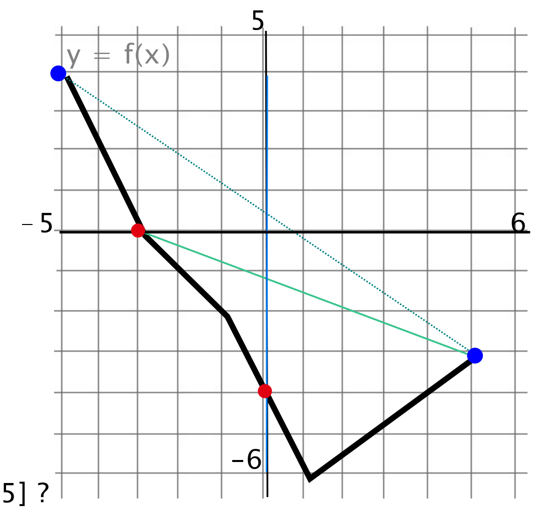
$$= \frac{4xh + 2h^2 - 6h}{h}$$

$$= \frac{h(4x + 2h - 6)}{h}$$

A factor must be a factor of the entire numerator and of the entire denominator in order to reduce (in order to "cancel").

$$= \boxed{4x + 2h - 6}$$

2. Use the graph of the function f at right to complete & answer the following.



f(-5) = 4

The domain of f is the interval [-5,5]

The range of f is the interval [-6,4]

The x-intercept is -3 or (-3,0).

The y-intercept is -4 or (0,-4).

On what interval is $f(x) \geq 0$? [-5,-3]

On what interval is f increasing ? [1, 5]

What is the average rate of change of f on interval [-5,5] ?

From (-5,4) to (5,-3):

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(-5)}{5 - (-5)} = \frac{-3 - 4}{5 - (-5)} = \boxed{\frac{-7}{10}}$$

→ Note slope of dotted line above ↑

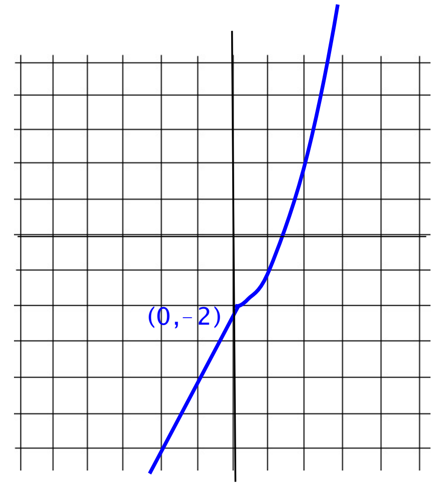
What is the average rate of change of f on interval [-3,5] ?

From (-3,0) to (5,-3):

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(-3)}{5 - (-3)} = \frac{-3 - 0}{5 - (-3)} = \boxed{\frac{-3}{8}}$$

→ Note slope of line above ↑

3. Let f be the function given by $f(x) = \begin{cases} 2(x-1) & \text{if } x < 0 \\ x^2 - 2 & \text{if } x \geq 0 \end{cases}$



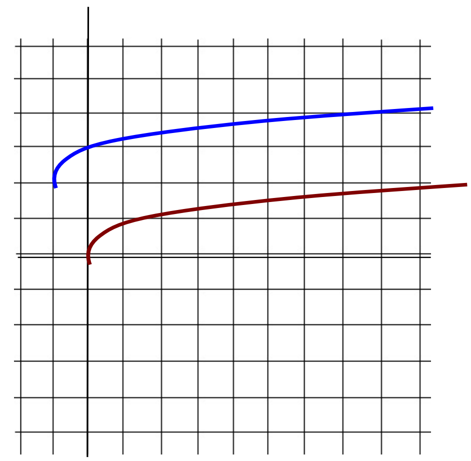
a. Find f(1)

b. Sketch the graph of f.

$f(1) = 1^2 - 2 = \boxed{-1}$

The graph of f behaves ...
as the line $y = 2x - 2$ for $x < 0$,
and as $x^2 - 2$ for $x \geq 0$.

4. a. Sketch the graph of $f(x) = \sqrt{x}$.

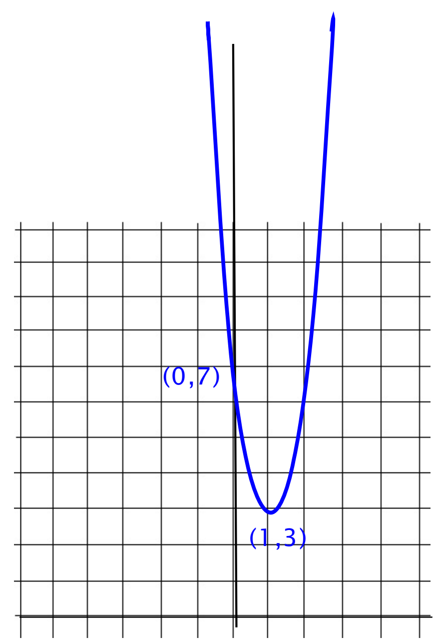


b. Write an expression for the function g, whose graph can be obtained by shifting the graph of f left 1 unit, then up two units.

$$\boxed{g(x) = f(x+1) + 2 = \sqrt{x+1} + 2}$$

c. Sketch the graph of g, and show its minimum & y-intercept.

5. Sketch the graph of $f(x) = 4x^2 - 8x + 7$.



b. Does f have a maximum or minimum value? What is it?

Minimum at (1,3)

c. Write the equation of the axis of symmetry.

Axis of symmetry is $\boxed{x = 1}$.

d. Sketch the graph of f. Label the vertex and y-intercept.

Vertex (0,0) moves to (1,3) by transformation shown below.

y-intercept: $f(0) = 4 \cdot 0^2 - 8 \cdot 0 + 7 = 7$

a. Express f(x) in the form $f(x) = a(x-h)^2 + k$

$$\begin{aligned} f(x) &= 4x^2 - 8x + 7 \\ &= 4(x^2 - 2x \quad) + 7 \\ &= 4(x^2 - 2x + 1) + 7 - 4 \end{aligned}$$

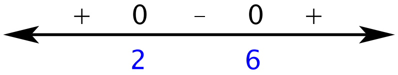
$$\boxed{f(x) = 4(x-1)^2 + 3}$$

6. Solve the inequality and express the result in interval form.

$x^2 - 8x \leq -12$

$x^2 - 8x + 12 \leq 0$

$(x - 2)(x - 6) \leq 0$



meaning $x^2 - 8x + 12$ is less than 0 on the interval (2,6),

and $x^2 - 8x + 12$ is equal to 0 at the endpoints,

so the solution to the given inequality is the closed interval [2,6] .

7. The revenue, in megadollars, produced for the Egglin Company by selling x Reciprocating Filangers, is $R(x) = 600x - 0.2x^2$. Find the number to sell to produce maximum revenue.

"Find the number to sell to produce maximum revenue."

Find x so that R(x) will be maximum.

We can easily see that R(x), being a quadratic function, is parabolic, opening down since a= −.2.
Thus $R(x) = 600x - 0.2x^2$ is maximum at $-b/2a = -600/(-.4) = 6000/4 =$ 1500 .

Or, again, we know R(x) is downward-opening parabola, with x-intercepts 0 and 600/.2 = 3000, since $R(x) = 600x - 0.2x^2 = x(600 - .2x)$....
And so maximum must occur halfway between 0 and 3000... at 1500.

Or we complete the square to see $R(x) = -.2(x-1500)^2 + .2 \cdot 1500^2$

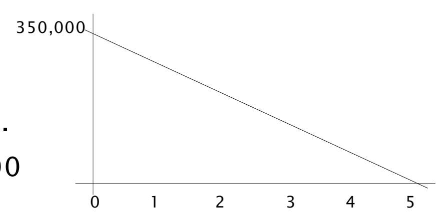
8. The Company just bought a new machine to manufacture its Rfs, at a cost of $350,000, which they are depreciating using the straight-line method over the next five years.

a. Write a linear function that expresses the value V of the machine in terms of its age x.

b. According to the model, what is the value of the machine after 2.5 years?

"depreciating using the straight-line method over the next five years"

Decreasing in value along a straight line so that all the value is used up over 5 years [V(5) = 0]



At right is an illustration of what the company will say is the value, where the y-coordinate is $-value, and the x-coordinate is # years.
... a line with y-intercept 350,000 and slope −350,000/5 = −70,000

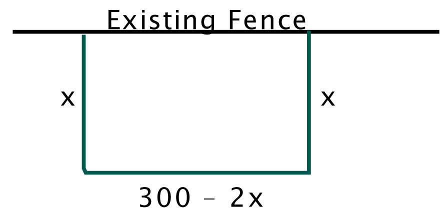
$V(x) = -70000x + 350000$ (With units: $V(x) = \$350000 - x \cdot \$70000/\text{year}$)

$V(2.5) = -70000(2.5) + 350000 = 350000 - 175000 = 175000$

Value at 2.5 years is $175,000 . (Notice this is half its value— exactly as it should be.)

9. You have 300 feet of fencing to enclose rectangular corral. You need to enclose only three sides, because you are building it next to a long existing fence.

a. Express the area of the corral in terms of the width, x.
b. Find the maximum area of such a corral.



a. Area $= \text{length} \cdot \text{width}$
$= (300 - 2x)\,x$

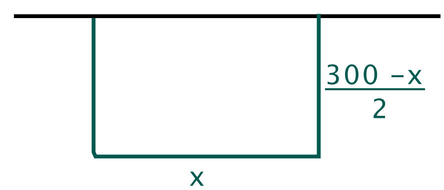
b. ... a down-opening parabolic function
with zeroes at $x = 0$ & $x = 150$, thus maximum at $x = 75$.

Maximum area = (75')(150') = 11250 ft^2

Who's to say the width is as pictured above?
If you took the opposite view, then you should find this:

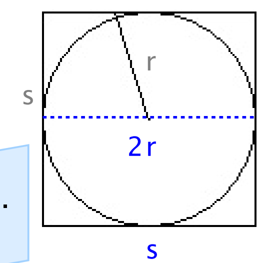
a. Area $= \text{width} \cdot \text{length}$
$= x\,(300 - x)/2$

b. ... a down-opening parabola with zeroes $x=0$ & 300
thus a maximum at $x = 150$.
Maximum = $150(300-150)/2 = 11250$, of course



10. A circle of radius r is inscribed in a square with side s, as shown.

a. Express the PERIMETER of the square as a function of r.
b. Express the CIRCUMFERENCE of the circle as a function of s.
(Recall $C = 2\pi r$.)



For both parts a, and b, we will need a connection between r and s.
The connection is that $s = 2r$.

a. Perimeter of square = distance around the square
$= 4s$
$= 4(2r)$
$= 8r$.

$P = 8r$

b. (We should know that) $C = 2\pi r$
since $s = 2r$ (or, equivalently, $r = s/2$)...

$C = \pi s$