1. Express the following sets using interval notation and as number line graphs:

\[ A = \{ x \mid x \geq 2 \} = [2, \infty) = \]

\[ B = \{ x \mid 5 \leq x < 2 \} = [-5, 2) = \]

\[ A \cap B = \{ x \mid x < 2 \text{ and } x \geq 2 \} = \text{the empty set} = \text{no interval at all, not even a point}. \]

\[ A \cup B = \{ x \mid -5 \leq x \} = [-5, \infty) = \]

\[
\frac{1}{x+2} - \frac{1}{2} \cdot \frac{(x+2)}{2} = \frac{2 - (x+2)}{x(x+2) \cdot 2} = \frac{-x}{x(x+2) \cdot 2} \]

We first clear the sub-fractions by multiplying numerator & denominator by \((x+2)\cdot 2\).

The \((x+2)\) factors reduce out in the first product, and the factor "2" reduces out in the second product.

Now it is time to simplify the numerator, combine the 2s.

The "2s" are gone because \(2 - (x+2) = 2 - x - 2\).

Now reduce out the common factor \(x\).

Provided \(x \neq 0\) and \(x \neq -2\)

\[
\left(\frac{2^{-1/4}}{y^{1/3} \cdot x^{1/6}}\right)^3 \]

Parentesises say the cubing exponent belongs to the entire fraction- both numerator and denominator ("N&D"), and, thus to each factor of the N&D.

\[
\frac{2^{-1/3} \cdot (x^{1/4})^3}{(y^{1/3})^3 \cdot (x^{1/6})^3} \]

This is a step you should not need to write down, showing the distribution of the exponent.

\[
\frac{2^{-3} \cdot x^{3/4}}{y^{3/3} \cdot x^{1/2}} \]

Now multiply N&D by \(2^3\), which replaces moves the \(2^{-3}\) in the N to \(2^3\) in the D. Similarly, multiply N&D by \(x^{-1/2}\) ....

And simplify those exponents. It might also be nice to replace \(2^3\) by \(8\) ....

\[
\frac{-2ax}{x+3} + 6 = \frac{-4x + 1}{x+3} \]

To solve this equation for \(x\), it would be nice to eliminate the fractions, and we can safely multiply both sides by \(x+3\), stating the provision that \(x+3 \neq 0\) (i.e. \(x \neq -3\)).

\[
(x+3) \left(\frac{-2ax}{x+3} + 6\right) = \frac{-4x + 1}{x+3} (x+3) \]

Notice the necessary parentheses.

\[
-2ax + 6(x+3) = -4x + 1 \]

\[
-2ax + 6x + 18 = -4x + 1 \]

\[
(10 - 2a)x = -17 \implies x = \frac{17}{2a - 10} \]

5. \[ \frac{(y + 2)^{\frac{4}{5}}}{(y + 2)^{\frac{1}{5}}} - \frac{1}{(y + 2)^{\frac{4}{5}}} \] No negative exponents! ...so multiply by \((y+2)^{\frac{4}{5}}/(y+2)^{\frac{4}{5}}\)

\[ \frac{(y + 2)^{\frac{5}{5}} - 1}{(y + 2)^{\frac{4}{5}}} = \frac{y + 1}{(y + 2)^{\frac{4}{5}}} \]

6. \[ ax^2 + bx + c = 0 \text{ has exactly one real solution when } b^2 - 4ac = 0 \]

\[ 3x^2 - 4x + K = 0 \text{ has exactly one real solution when } 16 - 4 \cdot 3K = 0 \]

... For \( K = 4/3 \)

7. From \((-2,5)\) to \((1,-3)\):

\[ \Delta x = 1 - (-2) = 3 \quad \Delta y = -3 - 5 = -8 \]

Distance = \( ((\Delta x)^2 + (\Delta y)^2)^{\frac{1}{2}} \)

\[ (3)^2 + (8)^2 = 73^{\frac{1}{2}} \]

Midpoint = (average of the xs, average of the ys)

\[ = \left( \frac{-2 + 1}{2}, \frac{5 + (-3)}{2} \right) = \left( -\frac{1}{2}, 1 \right) \]

Sketch it!

8. If \( P=(-2, 5) \) & \( Q = (1, -3) \) are the endpoints of a diameter of a circle,

a. The center is the MIDPOINT (see #7, above): \((-\frac{1}{2}, 1\))

b. The diameter is \(73^{\frac{1}{2}}\), so the radius is half that.

c. An equation for the circle:

\[(x - - \frac{1}{2})^2 + (y - 1)^2 = 73/4 \]

\[(x + \frac{1}{2})^2 + (y - 1)^2 = 73/4 \]

9. The revenue per share for eBay Inc. was \$0.20 in 1998 and \$0.91 in 1999. "Assuming that this trend will continue linearly", we assume revenue per share will rise at the same rate. What rate?

Well it rose from \$0.20 to \$0.91 (+.71) in one year, a rate of \$0.71/year.

If \( t \) is the number of years elapsed since 1998, then in 1998, \( t \) was 0.

So the revenue at time \( t = 0 \) was .20.

That gives us the slope (\$0.71/year) and the "y-intercept": revenue at \( t=0 \) is .20, so we should be able to write that equation:

\[ \text{Revenue per share (t)} = (0.71\text{year}) (t) + 0.20 \text{ where t = time elapsed since 1998.} \]
10. \[ \left(1 + \frac{1}{Y}\right) + 3 \left(1 + \frac{1}{Y}\right) = 40 \]

To solve for \(Y\), we first solve for \(X\), where \(X = 1 + \frac{1}{Y}\)

\[X^2 + 3X - 40 = 0\]
\[X = -8 \quad \text{or} \quad X = 5\]
\[1 + \frac{1}{Y} = -8 \quad \text{or} \quad 5\]
\[\frac{1}{Y} = -9 \quad \text{or} \quad 4\]
\[Y = -\frac{1}{9} \quad \text{or} \quad +\frac{1}{4}\]

11. \[x^4 - 3x^3 - 9x^2 + 27x = 0\]

To solve this, we factor. To factor, we look at groups.

\[x^3(x - 3) - 9x(x - 3) = 0\]
\[(x - 3)(x^3 - 9x) = 0\]
\[(x - 3)x(x^2 - 9) = 0\]
\[(x - 3)(x - 3)(x + 3) = 0\]

If a product of factors is 0 then one of the factors is 0...

\[x = 3 \quad \text{or} \quad 0 \quad \text{or} \quad 3 \quad \text{or} \quad -3\]

(3 is called a repeated root.)

12. \[x^2 + 2x + y^2 - 14y + 48 = 0\]

To show this is an eqn. of a circle, complete the squares.

\[x^2 + 2x + 1 + y^2 - 14y + 49 = -48 + 50\]

(For more details, see chapter 2 self-test.)

\[(x + 1)^2 + (y - 7)^2 = 2\]

The equation of a circle with radius \(\sqrt{2}\), and center \((x = 0, \ y = 0)\)

\[(x = -1, \ y = 7)\]

13. a. parallel to: \(3x - 5y = 10\) \(\Rightarrow \) \(3x - 5y = K\)

through \((-1, 4)\) \(\Rightarrow \) \(3(-1) - 5(4) = K, \) so \(K\) must be \(-23\)

Equation is \(3x - 5y = -23\)

So x-intercept is \(-23/3\) & y-intercept is \(23/5\)

b. perpendicular to: \(3x - 5y = 10\)

\(\Rightarrow \) \(5x + 3y = K\)

through \((-1, 4)\) \(\Rightarrow \) \(5(-1) + 3(4) = K, \) so \(K\) must be \(7\)

Equation is \(5x + 3y = 7\)

So x-intercept is \(7/5\) & y-intercept is \(7/3\)

(Knowing the x- and y-intercepts makes for fast graphing.)
14. \[ \left| \frac{1}{2}x - \frac{1}{4} \right| \geq \frac{1}{4} \] 
This is an inequality involving absolute value. Be careful.

We begin by multiplying by 2 or by 4.

We know if \( |Q| \geq 1 \), then \( Q \leq -1 \) or \( Q \geq 1 \).

\[ 2x - 1 \leq -1 \text{ or } 2x - 1 \geq 1 \]
Add 1 both sides, then divide by 2.

\[ x \leq 0 \text{ or } x \geq 1 \]

\[ (-\infty, 0] \cup [1, \infty) \]

15. \[ \frac{2 + x}{3 - x} \leq 1 \]
DO NOT MULTIPLY BY \((3 - x)\). DO NOT SOLVE LIKE AN EQUATION.

\[ \frac{2 + x}{3 - x} - 1 \leq 0 \]
S. O. P. !!!

\[ \frac{2 + x}{3 - x} - \frac{3 - x}{3 - x} \leq 0 \]
S. O. P. !!!

\[ \frac{2x - 1}{3 - x} \leq 0 \]
S. O. P. !!!

\[ (-\infty, \frac{1}{2}] \cup (3, \infty) \]

16. \[ \frac{3 + i}{2 - 4i} \]
\[ \frac{3 + i}{2 - 4i} \]
\[ \frac{2 + 4i}{2 - 4i} \]
\[ \frac{3 \cdot 2 + i \cdot 4i + i \cdot 2 + 3 \cdot 4i}{2 \cdot 2 - 4i \cdot 4i} \]
\[ \frac{2 \cdot 2 - 4i \cdot 4i}{3 \cdot 2 + i \cdot 4i + i \cdot 2 + 3 \cdot 4i} \]
\[ \frac{6 - 4 + 2i + 12i}{4 + 16} \]
\[ \frac{2 + 14i}{20} \]
\[ \frac{1}{10} + \frac{7}{10} \]

17. \[ i \sqrt{x + 3} + 3 = x \]

\[ i \sqrt{x + 3} = x - 3 \]

\[ x + 3 = x^2 - 6x + 9 \]

\[ x = 6 \text{ or } x = 1 \]

CHECK!!

\[ i \sqrt{6 + 3} + 3 = 6 \]

\[ 3 + 3 = 6 \]

\[ \sqrt{1 + 3} + 3 = 1 \]

\[ 2 + 3 = 1 \]

THE solution: \( x = 6 \)