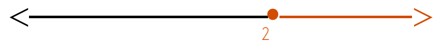
1. Express the following sets using interval notation and as number line graphs:

$A = \{ x \mid x \geq 2 \}$ = $[2, \infty)$ = 

$B = \{ x \mid -5 \leq x < 2 \}$ = $[-5, 2)$ = 

$A \cap B = \{x \mid x < 2 \text{ and } x \geq 2\}$ = the empty set = no interval at all, not even a point.

$A \cup B = \{x \mid -5 \leq x \}$ = $[-5, \infty)$ = 

2. $$\frac{\frac{1}{x+2} - \frac{1}{2}}{x} \quad \frac{(x+2)\cdot 2}{(x+2)\cdot 2}$$

We first clear the sub-fractions by multiplying numerator & denominator by (x+2) 2

$$\frac{2 - (x+2)}{x\,(x+2)\cdot 2}$$

The (x+2) factors reduce out in the first product, and the factor "2" reduces out in the second product.
Now it is time to simplify the numerator, combine the 2s.

$$\frac{-x}{x\,(x+2)\cdot 2}$$

The "2s" are gone because $2 - (x+2) = 2 - x - 2$.
Now reduce out the common factor x.

$$\frac{-1}{2\,(x+2)}$$

Provided $x \neq 0$ and $x \neq -2$

3. $$\left(\frac{2^{-1} x^{1/4}}{y^{1/3} x^{1/6}} \right)^3$$

Parentheses say the cubing exponent belongs to the entire fraction– both numerator and denominator ("N&D"), and, thus to each factor of the N&D.

$$\frac{(2^{-1})^3 (x^{1/4})^3}{(y^{1/3})^3 (x^{1/6})^3}$$

This is a step you should not need to write down, showing the distribution of the exponent.

$$\frac{2^{-3} \; x^{3/4}}{y^{3/3} \; x^{1/2}}$$

Now multiply N&D by 2^3, which replaces moves the 2^{-3} in the N to 2^3 in the D. Similarly, multiply N&D by $x^{-1/2}$....

$x^{3/4} \cdot x^{-1/2}$

$$\frac{x^{1/4}}{2^3 \; y}$$

$y^{3/3} = y$

And simplify those exponents.
It might also be nice to replace 2^3 by 8 ....

4. $$\frac{-2ax}{x+3} + 6 = \frac{-4x+1}{x+3}$$

To solve this equation for x, it would be nice to eliminate the fractions, and we can safely multiply both sides by x+3, stating the provision that $x+3 \neq 0$ (i.e. $x \neq -3$).

$$(x+3)\left(\frac{-2ax}{x+3} + 6 \right) = \frac{-4x+1}{x+3}(x+3)$$

Notice the necessary parentheses.

$$-2ax + 6(x+3) = -4x + 1$$
$$-2ax + 6x + 18 = -4x + 1$$
$$(10 - 2a)\,x = -17 \quad \Rightarrow \quad x = \frac{17}{2a - 10}$$

5. $$\frac{(y+2)^{\frac{4}{5}}}{(y+2)^{\frac{4}{5}}}\left((y+2)^{\frac{1}{5}} - (y+2)^{-\frac{4}{5}}\right)$$ No negative exponents!! ...so multiply by $(y+2)^{4/5} / (y+2)^{4/5}$

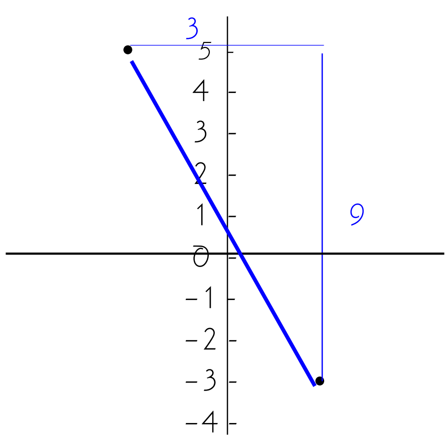
$$\frac{(y+2)^{\frac{5}{5}} - 1}{(y+2)^{4/5}} = \frac{y+1}{(y+2)^{4/5}}$$

6. $ax^2 + bx + c = 0$ has exactly one real solution when $b^2 - 4ac = 0$
$3x^2 - 4x + K = 0$ has exactly one real solution when $16 - 4 \cdot 3K = 0$ ... For $K = 4/3$

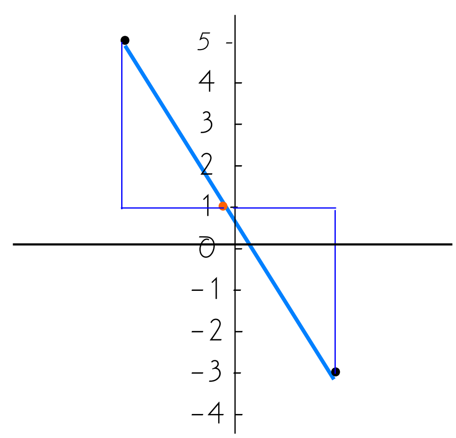
7. From (-2,5) to (1,-3): $\Delta x = 1 - (-2) = 3$ $\Delta y = -3 - 5 = -8$

Distance = $((\Delta x)^2 + (\Delta y)^2)^{1/2}$
$((3)^2 + (8)^2)^{1/2} = 73^{1/2}$

Midpoint = (average of the xs, average of the ys)
= ((-2 + 1)/2 , (5 + -3)/2) = (-½, 1)



Sketch it !



8. If P=(-2, 5) & Q = (1, -3) are the endpoints of a diameter of a circle,
a. the center is the MIDPOINT (see #7, above): (-½, 1)

b. The diameter is $73^{1/2}$, so the radius is half that. (➙ Since $D = 73^{½}$, $r = \frac{73^{1/2}}{2}$)

c. An equation for the circle:

$$(x - -½)^2 + (y - 1)^2 = 73/4$$

$$(x + ½)^2 + (y - 1)^2 = 73/4$$

9. The revenue per share for eBay Inc. was \$.20 in 1998 and \$.91 in 1999. "Assuming that this trend will continue **linearly**", we assume revenue per share will rise at the same rate. What rate?
Well it rose from .20 to .91 (+.71) in one year, a rate of \$.71/year.
If "t" is the number of years elapsed since 1998, then in 1998, t was 0.
So the revenue at time t = 0 was .20.

That gives us the slope (\$.71/year) and the "y-intercept" : revenue at (t=0) is .20,
so we should be able to write that equation:

Revenue per share (t) = (\$.71/year) (t) + \$.20 where t = time elapsed since 1998.

10. $\left(1 + \frac{1}{y} \right) + 3 \left(1 + \frac{1}{y} \right) = 40$ To solve for Y, we first solve for X, where $X = 1 + \frac{1}{y}$

$X^2 + 3X - 40 = 0$

$(x + 8)(x - 5) = 0$

$X = -8$ or $X = 5$

$1 + \frac{1}{y} = -8$ or 5

$\frac{1}{y} = -9$ or 4

$Y = -1/9$ or $+ 1/4$

11. $x^4 - 3x^3 - 9x^2 + 27x = 0$ To solve this, we factor. To factor, we look at groups.

$x^3(x - 3) - 9x(x - 3) = 0$ Here it may be important to recognize that $(-x + 3)$ Is just $-(x - 3)$

$(x - 3)(x^3 - 9x) = 0$

$(x - 3)\ x\ (x^2 - 9) = 0$

$(x - 3)\ x\ (x - 3)(x + 3) = 0$ If a product of factors is 0 then one of the factors is 0....

$x = 3$ or 0 or 3 or -3 (3 is called a repeated root.)

12. $x^2 + 2x + y^2 - 14y + 48 = 0$ To show this is an eqn. of a circle, complete the squares.

$x^2 + 2x + 1 + y^2 - 14y + 49 = -48 + 50$ (For more details, see chapter 2 self-test.)

$(x + 1)^2 + (y - 7)^2 = 2$

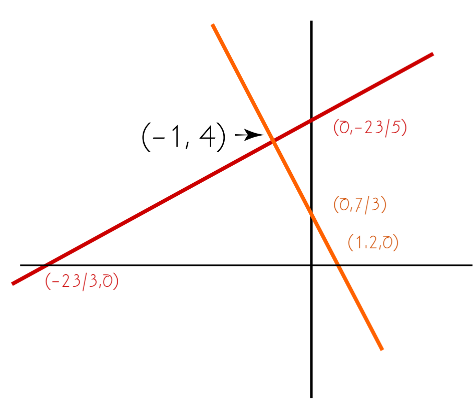
$X^2 + Y^2 = (\sqrt{2})^2$ ...'where $X = (x+1)$ and $Y = (y-7)$

The equation of a circle with radius $\sqrt{2}$,
and center ($X = 0$, $Y=0$)
($x = -1$, $y = 7$)

If $X = 0$ then $(x+1) = 0$ and $x = -1$
and if $Y = 0$, then $(y-7) = 0$ and $y = 7$.

13. a. parallel to: $3x - 5y = 10 \Rightarrow 3x - 5y = K$
through $(-1, 4) \Rightarrow 3(-1) -5(4) = K$, so K must be -23
Equation is $3x - 5y = -23$
So x-intercapt is $-23/3$ & y-intercept is $23/5$

b. perpendicular to: $3x - 5y = 10$
$\Rightarrow 5x + 3y = K$
through $(-1, 4) \Rightarrow 5(-1) + 3(4) = K$, so K must be 7
Equation is $5x + 3y = 7$
So x-intercapt is $7/5$ & y-intercept is $7/3$



(Knowing the x- and y-intercepts makes for fast graphing.)

14. $\left| \frac{1}{2}x - \frac{1}{4} \right| \geq \frac{1}{4}$

This is an inequality involving absolute value. Be careful.
We begin by multiplying by 2 or by 4.

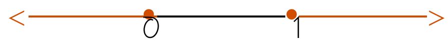
$|2x - 1| \geq 1$

We know if $|Q| \geq 1$, then $Q \leq -1$ or $Q \geq 1$

$2x - 1 \leq -1$ or $2x - 1 \geq 1$

Add 1 both sides, then divide by 2

$x \leq 0$ or $x \geq 1$



$(-\infty, 0] \cup [1, \infty)$

15. $\frac{2+x}{3-x} \leq 1$

DO NOT MULTIPLY BY (3 – x). DO NOT SOLVE LIKE AN EQUATION.

$\frac{2+x}{3-x} - 1 \leq 0$

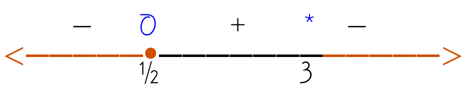
S. O. P. !!!

$\frac{2+x}{3-x} - \frac{3-x}{3-x} \leq 0$

S. O. P. !!!

$\frac{2x-1}{3-x} \leq 0$

S. O. P. !!!



Solution Set is: $(-\infty, \tfrac{1}{2}] \cup (3, \infty)$

16. $\frac{3+i}{2-4i}$

$\frac{3+i}{2-4i} \cdot \frac{2+4i}{2+4i}$

$\frac{3 \cdot 2 + i \cdot 4i + i \cdot 2 + 3 \cdot 4i}{2 \cdot 2 - 4i \cdot 4i}$

$\frac{3 \cdot 2 + i \cdot 4i + i \cdot 2 + 3 \cdot 4i}{2 \cdot 2 - 4i \cdot 4i}$

$\frac{6 - 4 + 2i + 12i}{4 + 16}$

$\frac{2 + 14i}{20}$

$\frac{1}{10} + \frac{7}{10}i$

17. $\sqrt{x+3} + 3 = x$

$\sqrt{x+3} = x - 3$

$x + 3 = x^2 - 6x + 9$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x = 6$ or $x = 1$

CHECK!! $\sqrt{6+3} + 3 = 6$

$3 + 3 = 6$ ✓

$\sqrt{1+3} + 3 = 1$

$2 + 3 = 1$ ✗

THE solution: $x = 6$