ANSWERS to Chapter P Quiz sample questions (revised 2/08/06)

1. Express number sets using interval notation, graphs, and inequalities.

a) The interval $[2,\infty)$ $= \{ x \mid 2 \le x \}$ 

b) $\{x \mid -2 < x \le 5\} = (-2, 5]$ 

c) This: is $(-\infty, 2]$ and is $\{ x \mid x \le 2 \}$

d) $[-4, 0) = \{ x \mid -4 \le x < 0) =$ 

e) $\{ x \mid x \le -1\} = (-\infty, -1] =$ 

2. Find the intersection & union of two sets of numbers. Express answers using interval notation.

Find i) $A \cap B$ ... and ii) $A \cup B$ ... for each of the following pairs of sets

a) $A = \{x \mid x < 3 \}$ and $B = \{ x \mid -2 < x \le 5 \}$ 

Putting these together (union!) ← we get this...

A: 

B: 

$A \cup B$: 

$= (-\infty, 5]$

And $A \cap B$ = only the overlap, what is in both: 

$= (-2, 3)$

b) $A = [-1, 6)$ and $B = [5, 8]$ $A \cap B = [5,6)$ $A \cup B = [-1,8]$

c) $A = \{ x \mid x \le 0 \}$ and $B = \{ x \mid x < -3 \}$ $A \cap B = B = (-\infty,-3)$ $A \cup B = (-\infty, 0]$

d) $A = (-\infty, 2]$ and $B = [2, \infty)$ $A \cap B = \{2\}$ $A \cup B = (-\infty, \infty)$

e) $A = [-1, \infty)$ and $B = (1, 3]$ $A \cap B = (1,3]$ $A \cup B = [-1,\infty)$

3. Simplify expressions, with rational exponents, as much as possible (no negative exponents).

a) $(2 x^4 y^{-4/5})^3 (8 y^2)^{2/3}$ The exponent 3 belongs to all factors in the parentheses ( )

$2^3 x^{3\cdot4} y^{-4/5\cdot3} 8^{2/3} y^{2(2/3)}$ Similarly, the exponent $^{2/3}$ applies to factors 8 & y^2 as well.

$8 x^{12} y^{-12/5} (4) y^{4/3}$ Combining like factors gives us...

$8\cdot4 \; x^{12} y^{-12/5 + 4/3}$ Which further simplifies to: $32 x^{12} y^{-16/15} = \dfrac{32 x^{12}}{y^{16/15}}$

b) $(27 x^9)^{-4/3} = 27^{-4/3} (x^9)^{-4/3} = (27^{1/3})^{-4} (x^9)^{-4/3} = 3^{-4} x^{-9\cdot4/3} = 3^{-4} x^{-12} = \dfrac{1}{81x^{12}}$

Once you get to this stage, there are many ways to think about simplifying...

c) $\dfrac{(y^9 z^{-3})^{1/3}}{(y^{-4} z^2)^{1/4}} = \dfrac{y^{9\cdot1/3} z^{-3\cdot1/3}}{y^{-4\cdot1/4} z^{2\cdot1/4}} = \dfrac{y^3 z^{-1}}{y^{-1} z^{1/2}} = \dfrac{y^3 z^{-1} \cdot z \cdot y}{y^{-1} z^{1/2} \cdot z \cdot y} = \dfrac{y^4}{z^{3/2}}$

$= \dfrac{y^3 z^{-1}}{y^{-1} z^{1/2}} = \dfrac{y^{3-(-1)}}{z^{1/2-(-1)}} = \dfrac{y^4}{z^{3/2}}$

Here's another, using the fact $\dfrac{a^n}{a^m} = a^{n-m}$

Parts (d) and (e) are shown using the most "reliable" method:

d) $\dfrac{(9st)^{3/2}}{(27s^3t^{-4})^{2/3}} = \dfrac{9^{3/2}s^{3/2}t^{3/2}}{27^{2/3}s^{3\cdot 2/3}t^{-4\cdot 2/3}} = \dfrac{27}{9}\dfrac{s^{3/2}}{s^2}\dfrac{t^{3/2}\cdot t^{8/3}}{t^{-8/3}\cdot t^{8/3}} = \dfrac{3t^{25/6}}{s^{1/2}}$

e) $\dfrac{3x^{1/2}y^3}{x^2y^{-1/2}} = \dfrac{3x^{1/2}x^{-1/2}y^3y^{1/2}}{x^2x^{-1/2}y^{-1/2}y^{1/2}} = \dfrac{3y^{3+1/2}}{x^{2-1/2}} = \dfrac{3y^{7/2}}{x^{3/2}}$

4. Factor an expression involving rational exponents. EG: Factor completely:

*USEFUL FACT: $x^p \cdot x^{-p} = x^0 = 1$ So, for instance, multiply $x^{-½}$ by $x^{½}$ & get 1.

BUT: you cannot just multiply by $x^{½}$, which would change the value.

So you multiply by ONE in the form of $\dfrac{x^{½}}{x^{½}}$

a) $x^{-3/2} + 2x^{-1/2} + x^{1/2} = \dfrac{x^{3/2}}{x^{3/2}}\left(x^{-3/2} + 2x^{-1/2} + x^{1/2}\right)$

$= \dfrac{1}{x^{3/2}}\left(1 + 2x + x^2\right)$

$= \dfrac{(1+x)^2}{x^{3/2}}$

Observations: These powers are not descending, as is more obvious after being multiplied by $x^{3/2}$ to kill off* the negative exponents.

You may read advice to "factor out the largest negative power", but that is easier said, and this method eliminates the negative exponents right at the start!

Plus: This method avoids the confusion to figure out that factoring $x^{-3/2}$ "out of" $x^{1/2}$ "leaves" x^2.

Here are the two methods (try it yourself on part c):

b) $x^{-1/2}(x+1)^{1/2} + x^{1/2}(x+1)^{-1/2} = x^{-1/2}(x+1)^{-1/2}\left((x+1) + x\right) = \dfrac{2x+1}{x^{1/2}(x+1)^{1/2}}$

(Here we "factor out" the factors with negative exponents.)

(It's fun figuring out what's left when you factor out $x^{-½}$....)

b) $x^{-1/2}(x+1)^{1/2} + x^{1/2}(x+1)^{-1/2} = \dfrac{x^{1/2}(x+1)^{1/2}}{x^{1/2}(x+1)^{1/2}}\left(x^{-1/2}(x+1)^{1/2} + x^{1/2}(x+1)^{-1/2}\right) = \dfrac{x+1+x}{x^{1/2}(x+1)^{1/2}}$

Here we multiply to "kill" or "cancel" the factors with negative exponents....

c) $4x^{-1/2} + 5x^{1/2} + x^{3/2} = \dfrac{x^{1/2}}{x^{1/2}}(4x^{-1/2} + 5x^{1/2} + x^{3/2}) = \dfrac{4x^0 + 5x + x^2}{x^{1/2}} =$

$\dfrac{x^2 + 5x + 4}{x^{1/2}} = \dfrac{(x+4)(x+1)}{x^{1/2}}$

d) $3(1+x)^{1/3} - x(1+x)^{-2/3} = \dfrac{(1+x)^{2/3}}{(1+x)^{2/3}}\left(3(1+x)^{1/3} - x(1+x)^{-2/3}\right) =$

$$\frac{3(1+x)^{1} - x(1+x)^{0}}{(1+x)^{2/3}} = \frac{3+2x}{(1+x)^{2/3}}$$

e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = \dfrac{x^{1/2}}{x^{1/2}}\left(3x^{3/2} - 9x^{1/2} + 6x^{-1/2}\right) = \dfrac{3x^2 - 9x + 6}{x^{1/2}}$

f) $x^2 - 64 = (x-8)(x+8)$ $\qquad A^2 - B^2 = (A-B)(A+B)$

g) $x^3 - 64 = (x-4)(x^2+4x+16)$ $\qquad A^3 - B^3 = (A-B)(A^2+AB+B^2)$

5. Simplify a complex rational expression. EG: Simplify completely:

a) $$\frac{\frac{1}{t+h} - \frac{1}{t}}{h} = \frac{t - (t+h)}{h\,t\,(t+h)} = \frac{-h}{h\,t\,(t+h)} = \frac{-1}{t\,(t+h)}$$

b) $$\frac{1}{1+a^n} + \frac{1}{1+a^{-n}} = \frac{1}{1+a^n} + \frac{1}{(1+a^{-n})}\cdot\frac{a^n}{a^n} = \frac{1}{1+a^n} + \frac{a^n}{a^n+1} = 1$$

Provided $a^n \neq -1$

c) $$x^2 - \frac{y^2}{\left(\frac{1}{x^2}+\frac{1}{y^2}\right)}\cdot\frac{x^2y^2}{x^2y^2} = x^2 - \frac{x^2y^4}{x^2+y^2} = \frac{x^2(x^2+y^2)}{(x^2+y^2)} - \frac{x^2y^4}{x^2+y^2}$$

$$= \frac{x^4 + x^2y^2 - x^2y^4}{x^2+y^2} = \frac{x^2(x^2 + y^2 - y^4)}{x^2+y^2}$$

... provided $xy \neq 0$

d) $$\frac{1+\frac{2}{c-2}}{1-\frac{2}{c-2}}\cdot\frac{c-2}{c-2} = \frac{c-2+2}{c-2-2} = \frac{c}{c-4}$$

provided $c \neq 2$

e) $$\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}\cdot\frac{xy}{xy} = \frac{y^2 - x^2}{x - y} = \frac{(x+y)(y-x)}{x-y} = -(x+y)$$

...provided $x \neq 0$ and $y \neq 0$ and $x \neq y$!

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note: solution....

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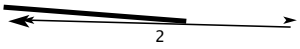
c) This: 

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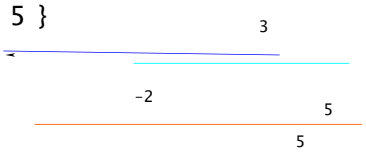
$A \cup B$: 

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