Solutions to Review Outline Chapter 3 Examples

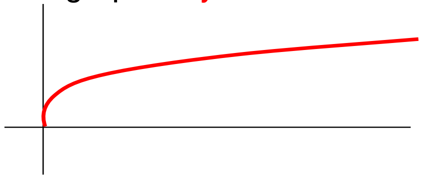
1. $$\frac{H(x+h) - H(x)}{h} = \frac{(3 - 4(x+h) - 4(x+h)^2) - (3 - 4x - 4x^2)}{h}$$

$$= \frac{3 - 4x - 4h - 4(x^2 + 2xh + h^2) - (3 - 4x - 4x^2)}{h}$$

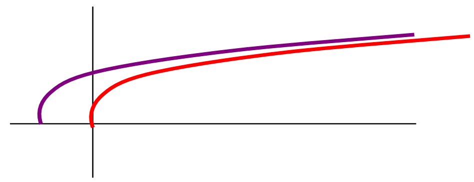
$$= \frac{\cancel{3} - \cancel{4x} - 4h - \cancel{4x^2} - 8xh - 4h^2 - (\cancel{3} - \cancel{4x} - \cancel{4x^2})}{h}$$

$$= \frac{-4h - 8xh - 4h^2}{h} = \frac{\cancel{h}(-4 - 8x - 4h)}{\cancel{h}} = -4 - 8x - 4h \quad \text{(provided } h \neq 0\text{)}$$

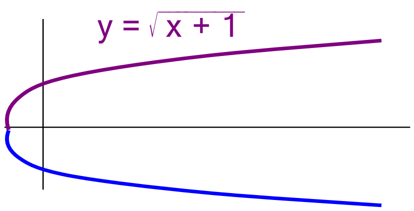
2a. The graph of $y = \sqrt{x}$ is



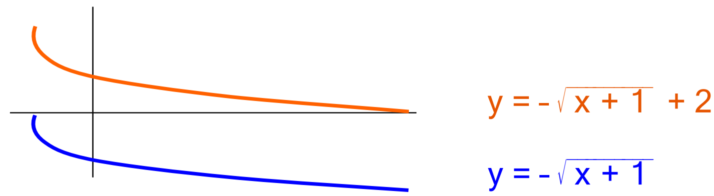
The graph of $y = \sqrt{x+1}$ is the graph $y = \sqrt{x}$ shifted 1 unit to the left.



PS: Consider the respective domains of $\sqrt{x}$ and $\sqrt{x+1}$

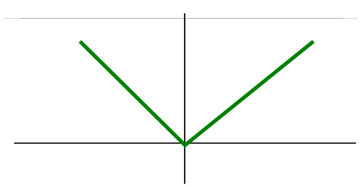


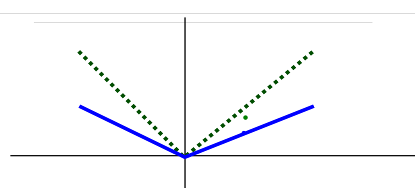
& the graph of $y = -\sqrt{x+1}$

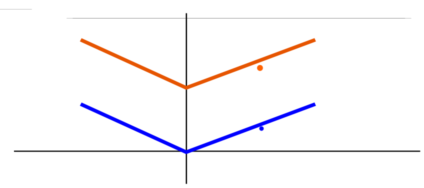


& finally the graph of $y = -\sqrt{x+1} + 2$

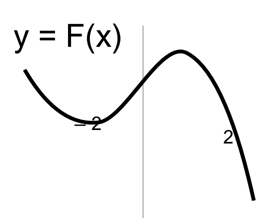
2b. The graph of $y = |-x|$ is the reflection (or "flip") of $y = |x|$ through the y-axis.

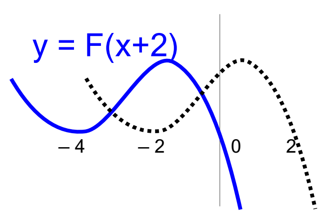


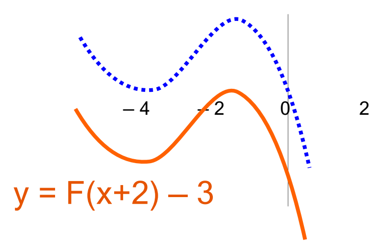


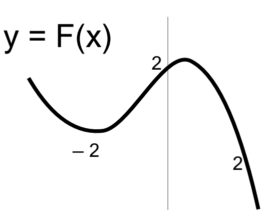


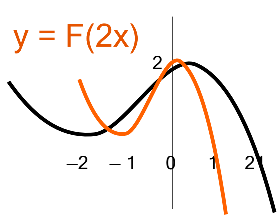
At each x, the graph of $y = (½)|-x|$ is half as high as that of $y = |-x|$
At each x, the graph of $y = (½)|-x| + 2$ is 2 units higher than that of $y = (½)|-x|$
(so we shift upwards by 2) .

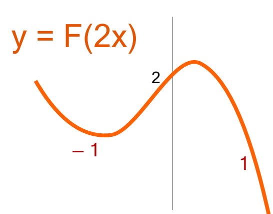












Another way to show the graph of F(2x)– Change the x-scale.

Solutions to Review Outline Chapter 3, cont'd.

3. Given: $f(x) = 2x^2 - 8x + 11$ we complete the square

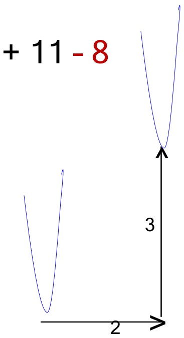
$f(x) = 2(x^2 - 4x + \ ? \ ) + 11 \ ...??$

$f(x) = 2(x - \text{what})^2 + 11$

$f(x) = 2x^2 - 8x + 11$

$f(x) = 2(x^2 - 4x + 4) + 11 - 8$

$f(x) = 2(x - 2)^2 + 3$



We identify this as a parabola whose <u>minimum</u> occurs at (2,3)– a parabola that is "stretched twice as tall" as the basic $y = x^2$, then shifted 2 units to the right, and 3 units upward.

4. At x = -3 the value of the function (y = f(-3)) appears to be 2.
At x = 2 the value of the function appears to be -1.
Therefore the average rate of change of f on the interval [-3, 2] is

$$\frac{\triangle f}{\triangle x} \text{ or } \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - -3} = \frac{-3}{5}$$

f appears to decrease on $(-\infty, -1]$ and on $[4, \infty)$. f appears to increase on [-1, 4] and has local minimum at (-1,-2), local maximum at (4,4).

5. Technically, $f \circ g(x) = \dfrac{1}{g(x) + 3} = \dfrac{1}{\frac{1}{x - 2} + 3} = \dfrac{x - 2}{1 + 3(x - 2)} = \dfrac{x - 2}{3x - 5}$ (*provided $x \neq 2$*)

$f \circ g(x)$ is defined for all x except 2 (see note above) and except for any x such that $1/(x - 2) = -3$... which occurs when $x = 5/3$.

6. If $f(x) = \dfrac{2 + 5x}{3 - 4x}$ What is $f^{-1}(x)$?

Since the inverse function reverses the ordered pairs of the original function,
for each pair (a,b) [where b = f(a)]...
the inverse function contains the reverse pair (b,a) [and $a = f^{-1}(b)$.]
Essentially the inverse function interchanges the roles of x & y.
The inverse is usually found by writing y = f(x), then solving for x in terms of y.

$y = \dfrac{2 + 5x}{3 - 4x} = \dfrac{5x + 2}{-4x + 3}$ ⇦ByTheWay: This makes clear the fact that y can never be - 5/4... since 5x/(-4x) is -5/4.

$y(3 - 4x) = 2 + 5x$

$3y - 4y = 2 + 5x$

$3y - 2 = 5x + 4yx$ Note that x IS $f^{-1}(y)$.

$3y - 2 = x(5 + 4y)$ So:

$x = \dfrac{3y - 2}{5 + 4y}$ $f^{-1}(y) = \dfrac{3y - 2}{5 + 4y}$

and, since it does not matter what we call the variable: $f^{-1}(x) = \dfrac{3x - 2}{5 + 4x}$

...which, ByTheWay, clearly cannot equal 3/4 !

NOTICE:
The domain of f is all x except 3/4, and the range consists of all numbers except -5/4
The domain of f^{-1} is all x except -5/4, and the range consists of all numbers except 3/4 !

 These facts noted above are, of course, consistent with the idea of an inverse function.