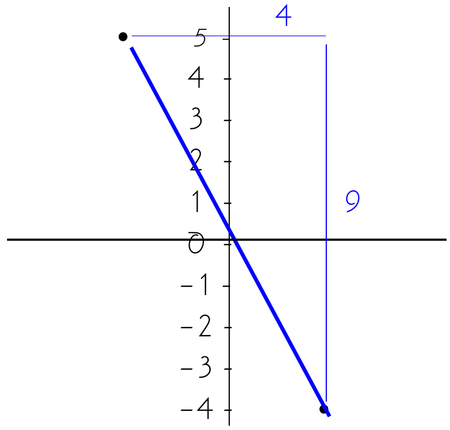
HOW to do what you need to be able to do, from Chapter 2

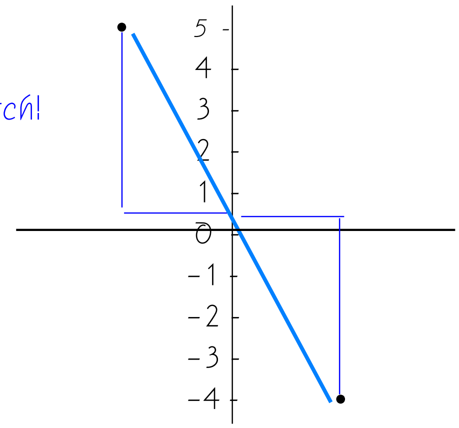
1. From $(-2,5)$ to $(2,-4)$: $\Delta x = 2 - (-2) = 4$ $\Delta y = -4 - 5 = -9$

Distance $= ((\Delta x)^2 + (\Delta y)^2)^{1/2}$
$((4)^2 + (9)^2)^{1/2} = 97^{1/2}$

Midpoint = (average of the xs, average of the ys)
$= ((-2 + 2)/2 , (5 + -4)/2) = (0, ½)$



Be safe: Make a sketch!

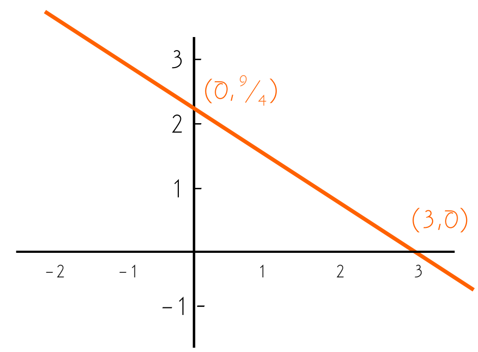


2. $3x + 4y = 9$ Slope is $-3/4$

$3 \cdot 0 + 4y = 9$ ⇨ when $x=0$, $y = 9/4$ (y-intercept)

$3x + 4 \cdot 0 = 9$ ⇨ when $y=0$, x is 3 (x-intercept)

these two intercepts are sufficient to draw the graph ↗



3. $x^2 + y^2 - 2x - 10y + 17 = 0$

$x^2 - 2x + y^2 - 10y = -17$

$x^2 - 2x + 1 + y^2 + 10y + 25 = -17 + 1 + 25$

$(x - 1)^2 + (y + 5)^2 = 9$

$(x - A)^2 + (y - B)^2 = r^2$

Seeing " $x^2 - 2x$ " tells us we want $(x - 1)^2$. But $(x-1)^2$ is $x^2 - 2x + 1$, not $x^2 - 2x$, so we need $+1$. Similarly, $y^2 + 10y$ is almost $(y + 5)^2$, but that requires adding $+25$ to both sides. Comparing to the general form we derived in class, we see that...

This is the equation of a circle with center at $(1,-5)$, and radius 3

With center $(-1,1)$, a circle passing through $(4,-11)$ must have radius $= ((-1 - 4)^2 + (1 - -11)^2)^{1/2}$
which is $(5^2 + 12^2)^{1/2} = 169^{1/2} = 13$

So an equation for this circle is: $(x - -1)^2 + (y - 1)^2 = 13^2$
or $(x + 1)^2 + (y - 1)^2 = 13^2$
In standard form: $x^2 + y^2 + 2x - 2y - 167 = 0$

4. a. parallel to the line $6x + 4y = 3$, passing through the point $(6,1)$.
Any line of the form $6x + 4y = K$ is parallel to the line above (in fact $3x + 2y = K$ is too).
When x is 6, y is 1, so: $6 \cdot 6 + 4 \cdot 1 = 40$ (finds K for us...)
$6x + 4y = 40$

Of course you can also work strictly within the "safety" of the slope-intercept form:

$6x + 4y = 3$ ⇨ $y = (-3/2)x + 3/4$ ⇨

our new line is $y = (-3/2)x + b$ ...where $1 = (-3/2)\cdot 6 + b$ ⇨ $b = 10$

So the equation is: $y = (-3/2)x + 10$

b. perpendicular to the line $4x + 3y = 1$, passing through the point (5,0).

Any line of the form $3x - 4y = K$ is perpendicular to the given line.

Passing through (5,0) says $3\cdot 5 - 4\cdot 0 = K$ .... so K = 15.

$3x - 4y = 15$

Again, you can work with the slope-intercept form of the line:

$4x + 3y = 1$ ⇨ $y = (-4/3)x + 1/3$ ⇨

our new line is $y = (3/4)x + b$ where $0 = (3/4)5 + b$ ⇨ $b = 15/4$

So the equation is: $y = (3/4)x + 15/4$

c. having x-intercept 6 and y-intercept 5.

When y is 0, x is 6; when x=0, y is 5 so I see: $5x + 6y = 30$.

Picture the interepts, (0,5) and (6,0)... Clearly the slope is $-5/6$,

So $y = (-5/6)x + b$

and since b is the y-intercept, $Y = (-5/6)x + 5$

5. The manager of a furniture factory finds that it costs \$2000 to manufacture 100 chairs in a day, and \$5400 to produce 300 chairs in a day.

a. Assuming that the relationship between cost and the number of chairs produced is linear, we merely need find the equation of a line through (n=100, C=\$2000) and (n=300, C = \$5400). Equation is shown at part c.

Slope = $\Delta C / \Delta n$ = (\$5400 – \$2000)/ (300-100) = \$3400/200 = \$17

b. What is the slope of the line in part (a) and what does it represent?

\$17/chair ... the units tell the story.

Slope here is the marginal cost of producing each additional chair.

c. What is the y-intercept of this line, and what does it represent?

To find the y-intercept we substitute (as in above problems): \$2000 + \$17(100) + b and see that b must be \$300.

The equation is : Cost = \$17/chair (n chairs) + \$300.

\$300 is the cost to produce 0 chairs., the "fixed cost" of being in this particular business.