Math 102 ♢ Quiz# 1 ♡ March 13, 2008 ♡ ANSWERS (your questions might vary slightly)

1-3. For the function given by $f(x) = \frac{1}{x^2 - 4}$

1. The domain consists of all real numbers except for −4 and 4 (because division by 0 is undefined!).

2. Write EQUATIONS for any vertical asymptotes: $x = -4$ & $x = 4$
(because as x becomes close to either 4 or −4, the denominator shrinks toward 0 while numerator is 1.)

3. Write EQUATIONS for any horizontal asymptotes: $y = 0$
(Because the degree of the denominator exceeds the degree of the numerator, hence the denominator grows faster than the numerator. In fact the numerator in this case does not change. When the denominator grows towards infinity faster than the numerator, the fraction shrinks towards 0. )

4-7. Write AN EQUATION for the horizontal asymptote for each function. If none, write "none".

4. $f(x) = \frac{3x + 100}{x^2 - 4}$ has horizontal asymptote: $y = 0$ (Deg denominator > degree numerator)

5. $f(x) = \frac{3x^2 + x}{x^2 - 4}$ has horizontal asymptote: $y = 3$

Degrees of numerator and denominator are the same.
Other terms shrink in significance in the ratio, so as x grows large, the fraction behaves like $\frac{3x^2}{x^2}$

6. $f(x) = \frac{x^3 - 8}{x^2 - 4}$ has horizontal asymptote: NONE

7. $f(x) = \frac{x^3 - 8}{x - 1}$ has horizontal asymptote: NONE

Deg numerator > deg denominator

So numerator grows faster than denominator in both #6 & #7.

8-11. Find all the asymptotes for the functions above.

#4 & #5 have vertical asymptotes ("VA") at $x = -2$ and $x = 2$

#6 reduces to $\frac{x^2 + 2x + 4}{x+2}$ for all x except $x = 2$. Thus it has only one VA, $x = -2$.

There is a hole in the graph at $x = 2$.... And no asymptote at $x=2$ because both numerator & denominator of the reduced form approach reasonable numbers ( 12 / 4) as x approaches 2.

#7 has a VA at $x = 1$, because as x approaches 1, the numerator approaches −7, while the denominator approaches 0.

All horizontal asymptotes were found above.
For non-horizontal asymptotes, in #6 & #7, we divide:

#6. $f(x) = \frac{x^3 - 8}{x^2 - 4} = \frac{x^2 + 2x + 4}{x + 2} = x + \frac{4}{x + 2}$

$$\begin{array}{r|l} & x \\ x+2 & x^2 + 2x + 4 \\ & \underline{x^2 + 2x} \\ & \quad\quad\quad 4 \end{array}$$

as x grows towards ∞, $4/(x+2)$ approaches 0, so
f(x) approaches the line $y = x$ So #6 f has an oblique linear asymptote, $y = x$.

#7.
$$\begin{array}{r|l} & x^2 + x + 1 \\ x - 1 & x^3 \quad\quad - 8 \\ & \underline{x^3 - x^2} \\ & \quad x^2 \\ & \quad \underline{x^2 - x} \\ & \quad\quad x - 8 \\ & \quad\quad \underline{x - 1} \\ & \quad\quad\quad -7 \end{array}$$

So $f(x) = \frac{x^3 - 8}{x - 1} = x^2 + x + 1 - \frac{7}{x - 1}$

Rational function f has no linear asymptote other than $x = 1$.
However, this division reveals that the graph of f is asymptotic to the graph of the more familiar parabola,
$y = x^2 + x + 1$.