ANSWERS TO "QUESTIONS THAT ARE ANSWERED IN CHAPTERS R & 1 & 2"

1. What is the least number in the interval $[-1,4)$? -1 The greatest? NONE. There IS no greatest number in this interval. All numbers in this interval are less than 4. But 4 is not in the interval. Any number you name that is less than 4 can be shown to be less than some other number in the interval (e.g. 3.9998 cannot be the greatest number, because 3.99995 is greater, and still within the interval.

2. Which of the numbers listed below

$3 \quad -4 \quad 1/2 \quad 3.14 \quad \sqrt{-9} \quad \sqrt{3} \quad .\overline{3} \quad 1/7$

a. is an integer ? 3 -4
b. is rational ? 3 -4 ½ 3.14 $.\overline{3}$ 1/7
c. is real? All except for $\sqrt{-9}$, which is $3i$

The set of INTEGERS is { ··· -4, -3, -2, -1, 0, 1, 2, 3, 4, ··· },
i.e. the set of counting numbers and their additive inverses, and 0.

The set of RATIONALS is all numbers that can be expressed as a ratio of integers. That is, a rational is any number that can be written as $^p/_q$, where p & q are both integers and q is not 0. 3 qualifies since it is $^3/_1$, and -4, $^1/_2$, and $^1/_7$ likewise also clearly qualify.

Any terminating decimal is rational, as exemplified by 3.14 since it can be written as a ratio: $^{314}/_{100}$. This is apparent when the number is read, e.g. 5.207 is "five and 207 THOUSANDTHS". If it is a terminating decimal, then it is an integer divided by some power of 10.

Any repeating decimal is rational as illustrated by this example: Consider the number "x" represented by the repeating decimal: .015151515···. We can multiply this by 100:

$$100x = 1.51515151\cdots$$

and subtract x:

$$-\quad x = .01515151\cdots$$

to find:

$$99x = 1.5$$

So we know $990x = 15$ and thus we show that $x = {}^{15}/_{990} = {}^{1}/_{66}$

Just to complete the discussion of rationals:
We have shown above that any decimal number that terminates or repeats must be a rational number. The converse is also true: any rational number must be a terminating or repeating decimal. This is obvious by the division algorithm. To convert a rational to decimal form, perform the division. Either a remainder of 0 will result at some point (so the decimal terminates) or the remainder will repeat at some point (since we are dividing by a finite number "q", there are only q-1 possible values for the remainders). Conclusion:

THE SET OF RATIONAL NUMBERS IS PRECISELY THE SET OF TERMINATING AND REPEATING DECIMALS.

We know $\sqrt{3}$ is not rational because the square root of any integer that is not a perfect square is irrational. How do we know that? See the proof for $\sqrt{2}$ the at link below.
http://www.csun.edu/~cas24771/m102/pyth.pdf (Link also on the Math 102 Notes page.)

The set of REALS is all numbers that represent some position on the number line. We also can think of them as all decimal numbers. Thus $\sqrt{3}$ (the length* of the long arm of a right triangle with hypotenuse 2 and short arm 1) represents the point on the number line that far* from 0. But the square root of -9 is not real, because the square of any real number is 0 or greater.

3. Multiply the binomials: $(3x + 4)(2x - 1)$

$$
\begin{aligned}
&(3x + 4)(2x - 1) = \\
&3x\,(2x - 1) + 4\,(2x - 1) = \\
&3x \bullet 2x - 3x \bullet 1 + 4 \bullet 2x - 4 \bullet 1 = \\
&6x^2 - 3x + 8x - 4 = \\
&6x^2 + 5x - 4
\end{aligned}
$$

4. Divide $x^3 + 3x^2 + 9x + 27$ by $x + 3$. What is the Quotient? The Remainder?

"Long division":

$$
\begin{array}{r l}
 & x^2 + 0x + 9 \quad \leftarrow \textit{Quotient} \\
x + 3 \,) & x^3 + 3x^2 + 9x + 27 \\
 & x^3 + 3x^2 \\
\hline
 & 9x + 27 \\
 & 9x + 27 \\
\hline
 & 0 \quad \leftarrow \textit{Remainder}
\end{array}
$$

"Synthetic Division"

$$
\begin{array}{r|rrrr}
-3 & 1 & 3 & 9 & 27 \\
 & & -3 & 0 & -27 \\
\hline
 & 1 & 0 & 9 & 0
\end{array}
$$

Quotient $x^2 + 0x + 9$ ↘*Remainder* $= 0$

PS: This means: $x^3 + 3x^2 + 9x + 27 = (x+3)(x^2 + 9) + 0$

5. Divide $2x^3 + x^2 - 3x + 7$ by $x^2 - 2$. What is the Quotient? The Remainder?

"Long division":

$$
\begin{array}{r l}
 & 2x + 1 \quad \leftarrow \textit{Quotient} \\
x^2 - 2 \,) & 2x^3 + x^2 - 3x + 7 \\
 & 2x^3 \qquad - 4x \\
\hline
 & x^2 + x + 7 \\
 & x^2 \qquad -2 \\
\hline
 & x + 9 \quad \leftarrow \textit{Remainder}
\end{array}
$$

(degree of remainder < degree of divisor)

PS: This means: $2x^3 + x^2 - 3x + 7 = (x^2 - 2)(2x + 1) + x + 9$

6. Factor $6x^2 + 5x - 6$.

If this has "nice" factors, they must be of the form $(ax+b)(cx+d)$ where $ac = 6$ and $bd = -6$.

Possibilities for					
a b are	2 3	2 -3	2 6	2 -6	6 2
c d	3 -2	3 2	3 -1	3 1	1 -3

. . . et cetera

Checking the coefficient of x, the first combination: $-4 + 9 = 5$ "works", so

$$6x^2 + 5x - 6 = (2x + 3)(3x - 2)$$

7. Evaluate $|2x - 3| + 3$ for $x = -1$.

$$
\begin{aligned}
|2(-1) - 3| + 3 &= \\
|-2 - 3| + 3 &= \\
5 + 3 &= 8
\end{aligned}
$$

Solve $|2x - 3| + 3 = 7$

$$
\begin{aligned}
|2x - 3| + 3 &= 7 \\
|2x - 3| &= 4
\end{aligned}
$$

$2x - 3 = -4$ OR $2x - 3 = 4$

$x = -\frac{1}{2}$ $\qquad$ $x = \frac{7}{2}$

8. Solve $-2 < 2x + 6 \le 2$

$-2 < 2x + 6 \le 2$ subtract 6 from all "sides"

$-8 < 2x \le -4$ divide all "sides" by 2

$-4 < x \le -2$

x is in the interval $(-4, 2]$

9. Find the equation of a line with slope 4 passing through the point (-2, 3).
Using SLOPE-INTERCEPT form: $y=mx+b$ and we know that $m=4$
so we know the equation is: $y = 4x + b$.
We substitute (-2, 3) and read the results:

$3 = 4(-2) + b$ and we can solve this for b: $b = 11$

So the equation must be $y = 4x + 11$

Using the POINT-SLOPE form: $(y - y_0) = m(x - x_0)$ for line with slope m through (x_0,y_0)

$y - 3 = 4 (x - -2)$

$y - 3 = 4(x + 2)$

10. Find the slope of the line given by $5x - 4y = 20$. What are the intercepts?

$$5x - 4y = 20$$
$$-4y = -5x + 20$$
$$y = 5x/4 + 5$$

SLOPE = 5/4

$5x - 4y = 20$

If $x = 0$: $0 - 4y = 20 \Rightarrow y = -5$

If $y = 0$: $5x - 0 = 20 \Rightarrow x = 4$

So intercepts are (0,-5) & (4,0).

11. Sketch the graph of $f(x) = x^2$ ... x^4 ... x^6 .... Sketch the graph of $f(x) = x^3$ ... x^5 ....

The graphs of $y= x^2$, x^4, x^6 all appear superficially similar to $y = x^2$.

The graphs of $y= x^3$, x^5, x^7 all appear superficially similar to $y = x^3$.

12. Sketch the graph of the function $f(x) = 1/x$ ... $f(x) = \sqrt{x}$ ... $f(x) = |x|$ ...

These graphs all appear in the text.