❦ 50-pt QUIZ Chapter 5 March 29, 2007 ❧ Answers

1. Show that the functions f and g are inverses of each other, by calculating $f \circ g(x)$ and simplifying.
   Given: $f(x) = (2x - 3)^3$ and $g(x) = \dfrac{\sqrt[3]{x} + 3}{2}$

$f \circ g(x) = f(g(x)) = (2\, g(x) - 3)^3$

$$= \left(2 \, \frac{\sqrt[3]{x} + 3}{2} - 3\right)^3$$

$$= \left(\not{2} \, \frac{\sqrt[3]{x} + 3}{\not{2}} - 3\right)^3$$

$$= \left(\sqrt[3]{x} + \not{3} - \not{3}\right)^3$$

$$= \left(\sqrt[3]{x}\right)^3$$

$$= x$$

(10) 2. Showing your work, find $f^{-1}(x)$ for the function given by $f(x) = \dfrac{2x + 1}{x - 3}$

State the following:

Domain f: $\{x | x \neq 3\}$ Domain f^{-1}: $\{x | x \neq 2\}$

Range f: $\{x | x \neq 2\}$ Range f^{-1}: $\{x | x \neq 3\}$

$$y = \frac{2x + 1}{x - 3}$$

$$x = \frac{2y + 1}{y - 3}$$ First we interchange x & y then we will solve for y....

$xy - 3x = 2y + 1$ starting by multiplying both sides by $(y - 3)$

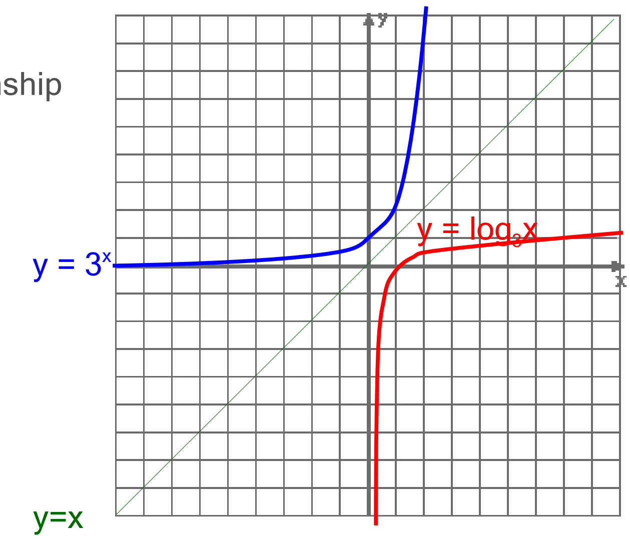
$xy - 2y = 3x + 1$ add 3x to both sides, subtract 2y....

$y(x - 2) = 3x + 1$ isolate y

$$y = \frac{3x + 1}{x - 2}$$

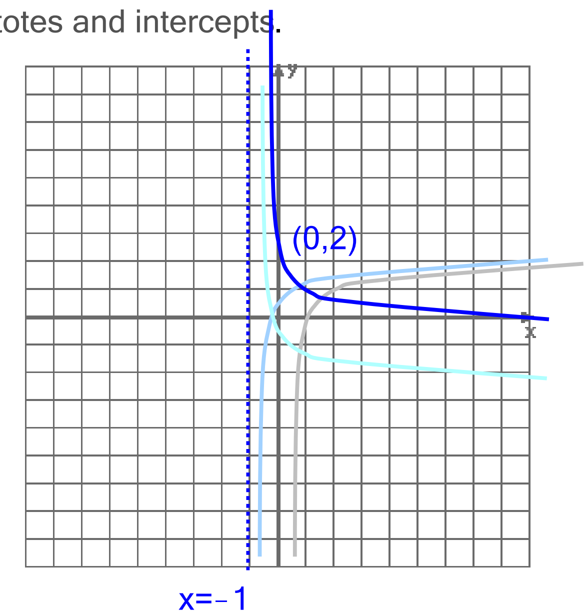
$$f^{-1}(x) = \frac{3x + 1}{x - 2}$$

(8) 3. Sketch the graph of $y = 3^x$ at right.
Then sketch the graph of $y = \log_3 x$ on the same set of coordinate axes. What special geometric relationship exists between these two graphs?



These functions are reflections of each other across the line y=x. This relationship exists between all pairs of functions which are inverses of each other.

(8) 4. Sketch the graph of $f(x) = 2 - \ln(x + 1)$, showing all asymptotes and intercepts.



The graph of $y = \ln x$ is shown in light gray.

The graph of $y = \ln(x+1)$ is pale blue, shifted one unit left.

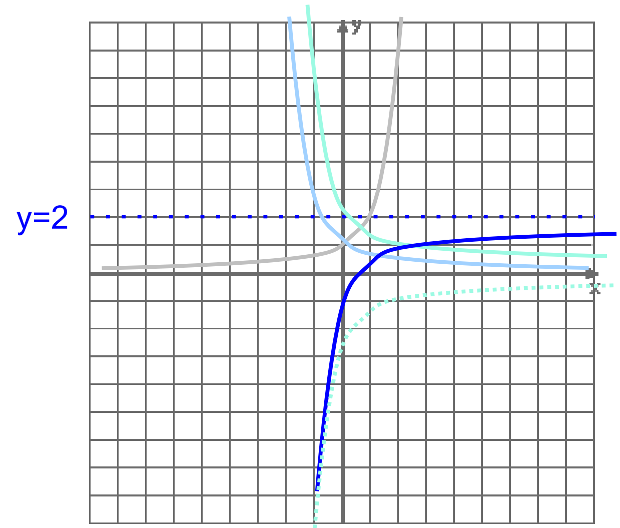
The graph of $y = -\ln(x+1)$ is pale cyan, and is the above flipped around the x-axis.

The final graph, $y = 2 - \ln(x+1)$ is the same, but shifted 2 units up.

y-intercept: $f(0) = 2 - \ln(0+1) = 2$
x-intercept: $2 - \ln(x+1) = 0$ when $\ln(x+1) = 2$

$$x+1 = e^2$$
$$x = e^2 - 1$$

(8) 5. Sketch the graph of $f(x) = 2 - e^{-x+1}$, showing all asymptotes and intercept.s.



The graph of $y = e^x$ is shown in light gray.

The graph of $y = e^{-x}$ reflects that through the y-axis, shown in pale blue.

The graph of $y = e^{-x+1}$, the above shifted one unit right, shown in pale green.

And $y = -e^{-x+1}$ reflects again, through the x-axis, shown in dotted pale green.

The graph of $y = 2 - e^{-x+1}$ is just two units higher.
It passes through the y-axis at $(0, 2-e)$

(5) 6. Simplify, showing your work:

$\log_3 27 + \log_3 81 + \log_3 \sqrt{3} + \log_3 1/9 + \log_3 1$

$3 + 4 + ½ + -2 + 0$

5 ½ or 5.5 or 11/2

(9) 7. Solve for x: $\ln x + \ln(2x+1) = 0$

Since $\ln A + \ln B = \ln AB$

$\ell n(x(2x+1)) = 0$

Taking "exp" of both sides.

$x(2x+1) = e^0 = 1$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$x = ½$ or $x = -1$

$\ell n(½) + \ell n(2•½ + 1) = 0$ ✓
$\ell n(-1)$ does not exist. ✗

But x = -1 is invalid, so only x = ½ solves the equation.

(8) 8. Solve for x: $2^{x+1} = 5^x$

Exponential equation without convenient numbers....
Standard Operating Procedure: take ℓn of both sides.

And use this fact: $\ell n\, x^p = p\, \ell n\, x$

$\ell n\, 2^{x+1} = \ell n\, 5^x$

$(x+1)\, \ell n\, 2 = x\, \ell n\, 5$

$x\, \ell n\, 2 + \ell n\, 2 = x\, \ell n\, 5$

$x\, \ell n\, 2 - x\, \ell n\, 5 = \ell n\, 2$

$x(\ell n\, 2 - \ell n\, 5) = \ell n\, 2$

$$x = \frac{\ell n\, 2}{\ell n\, 2 - \ell n\, 5}$$

(10) 9. A population of beetles multiplied from 1200 beetles on May 25 to 3600 beetles on May 27.
What is the rate of population growth per day?
On what day will there be over 100,000 beetles?

"1200 beetles on May 25 to 3600 beetles on May 27. " ⇒ every 2 days, population triples ($3^{t/2}$) and the population at day "t" is $A(t) = 1200 \cdot 3^{t/2}$. But here's the "SOP" approach:

$A(t) = A_0 e^{rt}$
$A(0) = A_0$ and this must be 1200,
so $A(t) = 1200\, e^{rt}$
To find r:

$1200\, e^{r2} = 3600$

so $e^{r2} = 3$

$r2 = \ell n 3$

$r = (½)\, \ell n 3$

$A(t) = 1200\, e^{(½)\, \ell n 3\, t}$

Then we use this to solve for t when A(t) will be 100,000.

$100{,}000 = 1200\, e^{(½)\, \ell n 3\, t}$

$1000/12 = e^{(½)\, \ell n 3\, t}$ We take ℓn...

$\ell n(1000/12) = (½)\, \ell n 3\, t$

$$t = \frac{2\, \ell n(1000/12)}{\ell n 3} \doteq 8.05$$

So the beetle pop. will surpass 100,000 on (during) the eighth day.