CHAPTER 4 "Quiz" Answers (if you r questions aren't here, they are very similar)

1. $P(x) = 2x^5 - 2x^4 - 3x^3 - 6x^2 - 2x + 4$

a. The degree of polynomial P is _5_ and P must have _5_ zeros (roots).

b. The y-intercept of the graph of P is _(0,4)_. The number of vertical asymptotes of P is _0_.

c. According to Descartes' Rule of Signs, P can have _0 or 2_ positive real zeros.

There are two sign changes in $P(x) = 2x^5 - 2x^4 - 3x^3 - 6x^2 - 2x + 4$.
According to Descartes' Rule of Signs, there can be either 2 or 0 positive roots.

d. Similarly, P can have _1 or 3_ negative real zeros.

P cannot have 3 or 1 negative real roots because :
$P(-x) = 2(-x)^5 - 2(-x)^4 - 3(-x)^3 - 6(-x)^2 - 2(-x) + 4$
$= -2x^5 - 2x^4 + 3x^3 - 6x^2 + 2x + 4$
... three sign changes, so 3 or 1 positive roots for P(-x), so 3 or 1 negative roots for P(x).

2. List all theoretically possible rational roots of the polynomial: $P(x) = 4x^4 + 4x^3 + 9x^2 + 12x - 9$

Since this polynomial has all coefficients in the set of integers,
any rational roots must be of the form p/q
where p is a factor of the constant term (so p = ± 9, 3 or 1) and
q is a factor of the leading coefficient (so q = ± 4, 2, or 1).

Thus candidates for rational roots of this polynomial are:

± 9 ±3 ±1 ±9/2 ± 3/2 ±1/2 ± 9/4 ± 3/4 ±1/4

(There are 18 in all.)

3. Construct the smallest polynomial with: real coefficients, roots −2 and 2 and 3 + i, with leading coefficient 5. You may leave the polynomial in factored form.

Real coefficients + having root 3+i requires that it also have root 3−i.
If "r" is a root, then (x − r) must be a factor, so...
$P(x) = A\,(x - -2)\,(x - 2)\,(x - (3+i))\,(x - (3-i))$

4. The polynomial $P(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$ might have a zero at x= ½ or at x = 2.
Use synthetic division to demonstrate that one of these IS, indeed, a zero, and the other is NOT.
Identify which of these is a zero, and which is not.

½	2	−3	−7	−8	6
		1	−1	−4	−6
	2	−2	−8	−12	0

Telling us that
$2x^4 - 3x^3 - 7x^2 - 8x + 6$
$= (x - ½)(2x^3 - 2x^2 - 8x - 12$

(So ½ is a zero of P .)

2	2	−3	−7	−8	6
		4	2	−10	−9
	2	1	−5	−18	−3

Showing us that P(2) = −3

(So 2 is NOT a zero of P .)

5. For each function below, list the equation(s) of the vertical and horizontal asymptote(s), if any. If there are none, write "none".

Function	Vertical asymptote(s)	Horizontal asymptote(s)
$g(x) = \dfrac{x^2 - 9}{x^2 + 4}$	NONE	$y = 1$
$f(x) = \dfrac{3x^4 + x}{x^2 - 9}$	$x = 3$ and $x = -3$	NONE
$h(x) = \dfrac{6x + 3}{x^2 + 2x + 1}$	$x = -1$	$y = 0$

Vertical asymptotes of rational functions occur where the function grows unboundedly large (↑ or ↓) because the denominator shrinks toward 0 while the numerator does not shrink.
This can only happen where the denominator of the rational function is 0.

For g, the denominator, $x^2 + 4$, is never 0, so g cannot have a vertical asymptote.

For f, the denominator, $x^2 - 9 = (x+3)(x-3)$, is 0 at -3 and 3. The numerator is not 0 at $x = -3$ or 3. Therefore, it is clear that f grows unboundedly large as x approaches -3 and as x approaches 3.

For h, the denominator is $x^2 + 2x + 1 = (x+1)^2$. This is 0 when $x = -1$. $6x+3$ does not shrink toward 0 as x approaches -1. Thus there is a vertical asymptote at $x = -1$.

Horizontal asymptotes of rational functions occur when the function values approach one particular number as x approaches infinity. This cannot occur when the degree of the numerator exceeds the degree of the denominator.

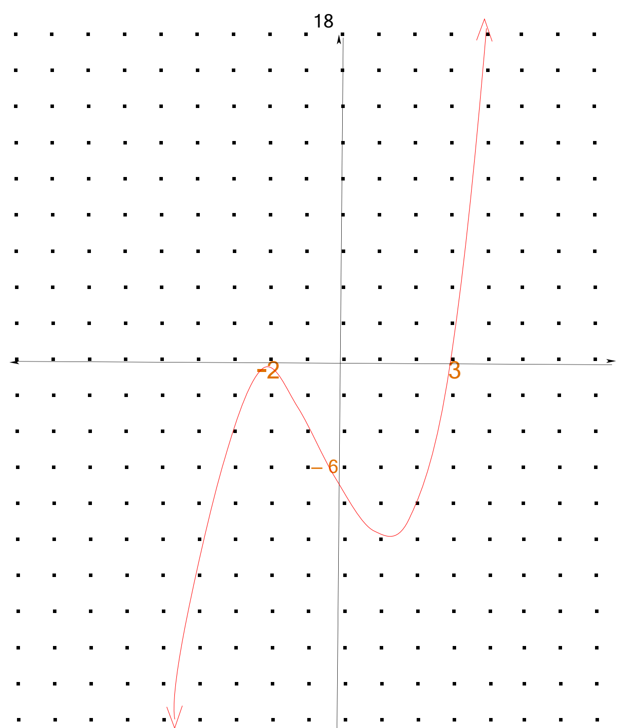
So f has no horizontal asymptote (f resembles $y = 3x^2$ when x is very large).

g(x) approaches 1 as $x \to \infty$ so g has HA $y = 1$.

h(x) approaches 0 as $x \to \infty$ so h has HA $y = 0$.

6. Sketch the graph of $y = P(x) = \dfrac{1}{2} (x + 2)^2 (x - 3)$

Polynomial, domain is all of R, continuous, etc.



y-intercept: $P(0) = ½ (0+2)^2 (0 - 3) = -6$

x-intercept(s): $P(x) = 0$
only when $(x + 2)^2 (x - 3) = 0$
. . . when $x = -2$ (twice), $x = 3$

What happens to y=P(x) when $x \to \infty$ (increases towards ∞) ? Consider that $(x + 2)^2$ and $(x - 3)$ both grow unboundedly large, both positive, as $x \to \infty$. So
$(½)(x + 2)^2 (x - 3) \to \infty$ as $x \to \infty$.
(i.e. grows unboundedly large)

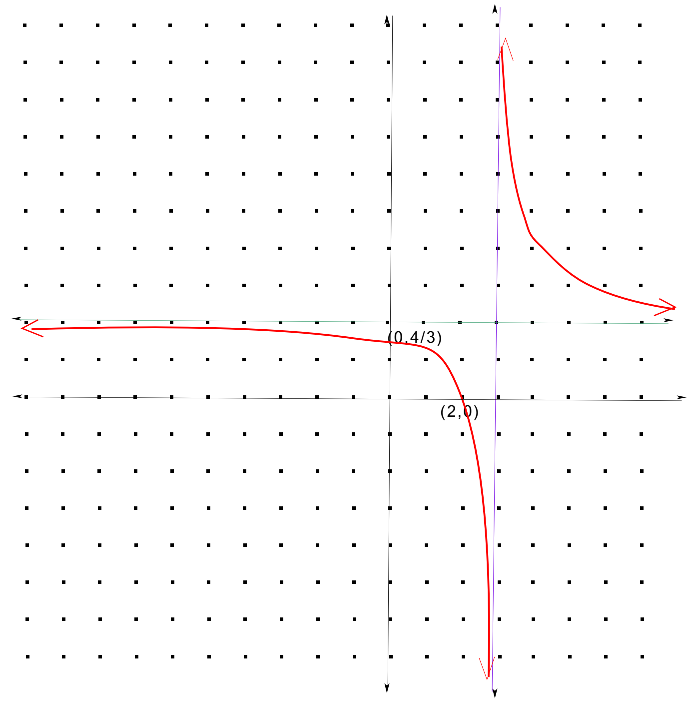
... And when $x \to -\infty$, $(x + 2)^2$ & $(x - 3)$ both grow unboundedly large, but $(x-3)$ is large negative, so the product is large & negative.
$(½)(x + 2)^2 (x - 3) \to -\infty$ as $x \to -\infty$.

Plotting a few additional points, such as (-5,-36) & (-4,-14) & (1, -9) & (2, -8) & (4,18) adds accuracy & serves as a check on the analysis.

7. Sketch the graph of $y = \frac{4 - 2x}{3 - x}$ Label all the intercepts & asymptotes.

Rational function, therefore:
Domain: y is defined for all values except where denominator = 0: x = 3

As x approaches 3, $4 - 2x \to -2$,
while $3 - x \to 0$,
so the quotient $\to \pm \infty$
Thus there is a vertical asymptote at x = 3.**



As $x \to \infty$ $y \to 2$
As $x \to -\infty$ $y \to 2$
Thus the function has a horizontal asymptote✳ at y = 3.

y-intercept: when x = 0
y = 4/3

x-intercept: when x = 2, y = 0

** More on the Vertical Asymptote:

As x approaches 3 from below e.g. take x = 2.5 , 2.8 , 2.9 , 2.99 , 2.9999 . . . → 3
y grows unboundedly large, negative: $y = \frac{-1}{.5}, \frac{-1.6}{-.2}, \frac{-1.8}{-.1}, \frac{-1.98}{-.01}, \frac{-1.9998}{-.0001} \to \infty$

Similarly, computing function values for x that are just above 3, demonstrates that the function values are positive and grow unboundedly large $(\to\infty)$ as x-values approach 3 from above.

✳Another way to discover the horizontal asymptote:

...divide & conquer:

$$\frac{-2x + 4}{-x + 3} = \frac{2x - 4}{x - 3} = 2 + \frac{2}{x - 3}$$ As $x \to \infty$, the latter portion $\to 0$

& $f(x) \to 2 + 0$ as $x \to \infty$