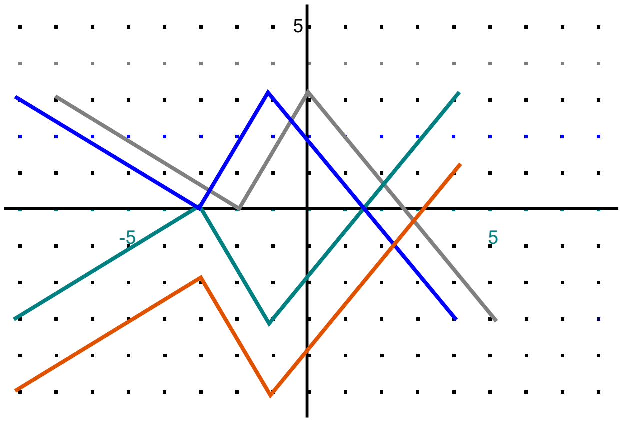
Math 102 ◇ 50-pt-quiz # 1 ♡ Spring ♥2008 ♡ NAME: *Les Réponses ! !*

() 1. Below is a sketch of the graph of the function g: y = g(x) .
On the same coordinate system, graph the function f given by f(x) = − g(x+1) − 2 .
We are showing intermediate graphs.



y = g(x) is shown in grey.

y = g(x +1) shifts the graph of y=g(x) 1 units left.

y = − g(x +1) flips the graph of y = g(x +1) about the x-axis.

y = − g(x +1) − 2 shifts the graph of y = − g(x +1) down 2 units

() 2. Find the domain of the function given by $f(x) = \dfrac{2x-6}{x^2-4}$. Write the domain using interval notation.

The value of f(x) can be computed, and is a real number, any time $x^2 - 4 \neq 0$.

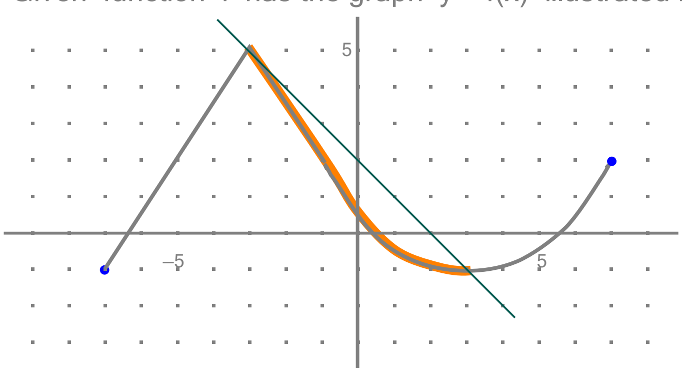
So x may not be 2 or −2.... We could say the domain is $\{ x \mid x \neq \pm 2 \}$

Thinking in terms of intervals, x may be below −2, above 2, or between −2 & 2.

On the number line, this looks like ➜

...and in interval notation, the domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

() 3. Given function f has the graph y = f(x) illustrated below, Give best estimates for:



a. The graph shows f descends from (–3,5) to (3,–1); so f is decreasing on [–3,3].

b. What x-intercept(s) does f appear to have, if any ?
The graph appears to cross the x-axis at approximately: x = -6, x = ½, and x = 5¾

c. What y-intercept(s) does f appear to have, if any ?
A function may have only one y-intercept, and this one appears to be about ½ .

d. What appears to be the range of f ? Range of f is $-1 \leq y \leq 5$ also written $[-1,5]$

e. The average rate of change of f on the interval [−3, 3]
is the slope of the line through (–3,5) and (3,-1).

This is: $\text{Slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{f(3) - f(-3)}{3 - (-3)} = \dfrac{-1 - 5}{3 + 3} = \dfrac{-6}{6} = -1$

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(10) 4. The function f is given by $f(x) = -2x^2 + 8x + 10$. Find and simplify:

$$\frac{f(x+h) - f(x)}{h} = \frac{-2(x+h)^2 + 8(x+h) + 10 - (-2x^2 + 8x + 10)}{h}$$

← Notice f(x+h) is NOT f(x) + h !!!

$$= \frac{(-2x^2 - 4xh - 2h^2 + 8x + 8h + 10 - (-2x^2 + 8x + 10)}{h}$$

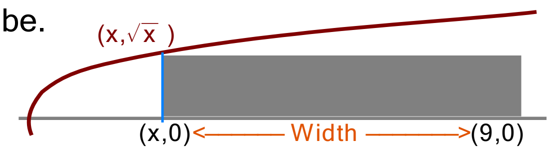
$$= \frac{-4xh - 2h^2 + 8h}{h}$$

$$= \frac{h(-4x - 2h + 8)}{h}$$

$$= -4x - 2h + 8$$

() 5. Consider the region bounded by the graphs of $y = \sqrt{x}$ and $x = 9$, and the x-axis.
For each x $(0 \le x \le 9)$ there is a rectangle inscribed in this region, with its right edge on the line x = 9, and opposite vertices on the x-axis [at x] and the curve $y = \sqrt{x}$.
Write a function that expresses the area of the inscribed rectangle as a function of x.

Area = width • height Identifying the coordinates of three vertices shows what the width and height must be.



Area = $(9 - x)(\sqrt{x} - 0)$

We might write this: $A(x) = \sqrt{x}\,(9 - x)$

() 6. Suppose the height (in feet), h, of an object t seconds after it has been projected straight up into the air is given by $h(t) = -16t^2 + 40t + 50$.
a. At what height was the object initially (when is was projected upward)?
b. How much time elapses before the object hits the ground?

a. Initial height of object is height at time 0.

$h(0) = -16 \cdot 0^2 + 40 \cdot 0t + 50 = 50$ So height was initially 50 ft.

b. To find how much time elapses before object hits the ground, we need the time (t) at which the height of the object becomes 0 ft. That is, we solve to find t where h(t) = 0.

$$h(t) = 0$$

$$-16 \cdot t^2 + 40 \cdot t + 50 = 0$$

$$8 \cdot t^2 - 20 \cdot t - 25 = 0$$ At which point we realize this does not factor !

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{20 \pm \sqrt{400 + 4 \cdot 8 \cdot 25}}{16}$$ quadratic formula, always works .

$$t = \frac{20 \pm \sqrt{1200}}{16} = \frac{20 \pm 4\sqrt{75}}{16}$$ One of these “solutions” does not work because in our problem t must be ≥ 0.

So the one and only solution is $t = \dfrac{5 + \sqrt{75}}{4}$ seconds (≈ 3.4 s)