SELF-TEST B: Some pre-college-algebra basics & answers

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1. Solve an equation for one variable in terms of another. Solve for the variable named:

a. x: $a - 2(b - 3(c - x)) = 6$ b. a: $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$

c. i: $A = P\left(1 + \frac{i}{100}\right)^2$ d. r: $S = \frac{a}{1-r}$

e. c: $\frac{1}{s+a} + \frac{2}{s-a} = \frac{5}{s+c}$

2. Solve a quadratic equation. (Simplify your answers.)

a. $3x^2 - 12x - 1 = 0$ b. $5x = 2x^2 + 1$

c. $2x^2 + 12x + 1 = 0$ d. $2x^2 + 4x + 3 = 0$ e. $x^2 + 10 = -6x$

3. Simplify an expression involving complex numbers. (Express answers in the form a + bi .)

a. $(2 - \sqrt{-2})(\sqrt{8} - \sqrt{-4})$ b. $(1 - \sqrt{-3})(2 + \sqrt{-4})$ f. $i^{97} + i^{98} + i^{99} + i^{100}$

c. $\frac{1 - \sqrt{-1}}{1 + \sqrt{-1}}$ d. $\frac{2 + \sqrt{-8}}{1 + \sqrt{-2}}$ e. $(2 - \sqrt{-36})^{-1}$

4. Solve an equation by factoring.

a. $x^6 + 9x^4 - 4x^2 - 36 = 0$ b. $2(x-4)^{7/3} - (x-4)^{4/3} - (x-4)^{1/3} = 0$

c. $x^{1/2} - 3x^{1/3} = 3x^{1/6} - 9$ d. $x^{1/2} + 3x^{-1/2} = 10x^{-3/2}$

e. $x^2\sqrt{x+3} = (x+3)^{3/2}$ f. $x^2 - 4 = 3x$

5. Solve a rational inequality. (Express answers using interval notation.)

a. $\frac{x}{x+2} \le \frac{1}{x}$ b. $\frac{2x+5}{x+1} \le 1$ c. $\frac{9}{x} < x$

d. $\frac{3}{x-1} - \frac{x}{x+1} \ge 1$ e. $-3 \le \frac{x+1}{x-3}$

6. Solve an inequality involving absolute value.

a. $3 - |2x + 4| \le 1$ b. $4|3 - x| + 3 \ge 15$

c. $2\left|\frac{1}{2}x + 3\right| + 3 \le 51$ d. $\left|\frac{x+1}{2}\right| > 6$ e. $\left|\frac{x-2}{3}\right| < 2$

SELF-TEST B ANSWERS

1. a. x : $a - 2(b - 3(c - x)) = 6$ Simplify, clear the parentheses...

$a - 2(b - 3c + 3x) = 6$ & these...

$a - 2b + 6c - 6x = 6$ Now isolate x

$$-6x = 6 - a + 2b - 6c$$

$$x = -1 - a/3 + 2b/3 - c \quad \text{or } (1/6)(6 - a + 2b - 6c)$$

b. a : $\frac{a+1}{b} = \frac{a-1}{b} + \frac{b+1}{a}$ Clear the fractions, by multiplying through by a·b

$\cancel{a^2} + a = \cancel{a^2} - a + b^2 + b$ Simplify, and add a to both sides

$$2a = b^2 + b$$

$$a = \frac{b^2 + b}{2}$$

c. i : $A = P\left(1 + \frac{i}{100}\right)^2$ How am I going to get to that i , inside that $(\ )^2$? I'm going to have to take the square root ....

$\sqrt{A} = \sqrt{P}\left(1 + \frac{i}{100}\right)$ Now divide both sides by $\sqrt{P}$

$\frac{\sqrt{A}}{\sqrt{P}} = 1 + \frac{i}{100}$ Subtract 1 from both sides; then multiply by 100 to obtain:

$$i = 100\left(\frac{\sqrt{A}}{\sqrt{P}} - 1\right)$$

d. r : $S = \frac{a}{1 - r}$ Multiply both sides by (1 – r) ...assuming $r \neq 1$

$S(1 - r) = a$ ←Distribute the S or divide by S → $1 - r = \frac{a}{S}$

$$S - Sr = a$$

$$-Sr = a - S$$

$$r = 1 - \frac{a}{S} \quad \text{or} \quad r = \frac{S - a}{S}$$

e. c : $\frac{1}{s+a} + \frac{2}{s-a} = \frac{5}{s+c}$ Working with the given equation, OR

$\frac{3s + a}{(s+a)(s-a)} = \frac{5}{s+c}$ with the second version, MULTIPLY both sides by (s+a)(s – a)(s+c) which clears all the fractions.

$(3s + a)(s + c) = 5(s + a)(s - a)$ Here's a cheap finish: divide by (3s+a), subtract s

$$c = \frac{5(s+a)(s-a)}{(3s+a)} - s \qquad \text{OR} \quad c = \frac{2s^2 - as - 5a^2}{(3s+a)}$$

2. Each of these can be solved using the quadratic formula (after expressing as $ax^2 + bx^2 + c = 0$). We show other techniques here, for variety, especially where appropriate.

a. $3x^2 - 12x - 1 = 0$ $\quad x = \frac{12 \pm \sqrt{144 + 12}}{6} = 2 \pm \sqrt{39}/3$

Note 144 + 12 = 12·12 + 12 = 12·13 = 4·3·13

2 cont'd b. $5x = 2x^2 + 1$ $\quad 2x^2 - 5x + 1 = 0$ $\quad x = \dfrac{5 \pm \sqrt{25-8}}{4} = \dfrac{5 \pm \sqrt{17}}{4}$

c. $2x^2 + 12x + 1 = 0$ $\quad x = \dfrac{-12 \pm \sqrt{144-8}}{4} = \dfrac{-12 \pm \sqrt{4 \cdot 34}}{4} = \dfrac{-6 \pm \sqrt{34}}{2} =$

d. $2x^2 + 4x + 3 = 0$ $\quad x = \dfrac{-4 \pm \sqrt{16-24}}{4} = \dfrac{-4 \pm \sqrt{-4 \cdot 2}}{4} = \dfrac{-2 \pm \sqrt{2}\,i}{2}$

e. $x^2 + 10 = -6x$ $\quad x^2 + 6x + 10 = 0$ Ah ha! Finally a place to do something different

$x^2 + 6x + 9 + 1 = 0$

$(x + 3)^2 + 1 = 0$

$(x + 3)^2 = -1$

$(x + 3) = \pm i$ $\quad x = -3 \pm i$

3. Simplify an expression involving complex numbers. (Express answers in the form a + bi .)

a. $(2 - \sqrt{-2})(\sqrt{8} - \sqrt{-4})$

$= (2 - \sqrt{2}i)(\sqrt{8} - 2i)$

$= 2\sqrt{8} + \sqrt{2} \cdot 2i^2 + (-4i - 4i)$

$= 2 \cdot 2\sqrt{2} - \sqrt{2} \cdot 2 + (-4 - 4)i$

$= 2\sqrt{2} + -8i$

b. $(1 - \sqrt{-3})(2 + \sqrt{-4})$

$= (1 - \sqrt{3}i)(2 + \sqrt{4}i)$

$= 2 + \sqrt{3 \cdot 4} + (-\sqrt{3} \cdot 2 + \sqrt{4})i$

$= 2 + 2\sqrt{3} + (2 - 2\sqrt{3})i$

f. $i + -1 + -i + 1 = 0$

Whoa, could we save time by noting that $2 + \sqrt{-8}$ is just $2(1 + \sqrt{-2})$ and $2w/w = 2$

c. $\dfrac{1 - \sqrt{-1}}{1 + \sqrt{-1}}$

$= \dfrac{1 - i}{1 + i} \cdot \dfrac{1 - i}{1 - i}$

$= \dfrac{1 - 2i + i^2}{1 - i^2}$

$= \dfrac{-2i}{2} = -i$

d. $\dfrac{2 + \sqrt{-8}}{1 + \sqrt{-2}}$

$\dfrac{2 + \sqrt{8}i}{1 + \sqrt{2}i} \cdot \dfrac{1 - \sqrt{2}i}{1 - \sqrt{2}i}$

$\dfrac{2 + \sqrt{16} + (\sqrt{8} - 2\sqrt{2})i}{1 + 2}$

$\dfrac{6 + 0i}{3} = 2$

e. $(2 - \sqrt{-36})^{-1}$

$= \dfrac{1}{2 - 6i} \cdot \dfrac{2 + 6i}{2 + 6i}$

$= \dfrac{2 + 6i}{4 + 36}$

$= \dfrac{1 + 3i}{20}$

4. Solve an equation by factoring.

a. $x^6 + 9x^4 - 4x^2 - 36 = 0$ Do you see the

$x^4(x^2 + 9) - 4x^2 - 36 = 0$ hidden common factor?

$(x^2 + 9)(x^4 - 4) = 0$

$(x^2 + 9)(x^2 + 2)(x^2 - 2) = 0$

$x^2 = -9$ or $x^2 = -2$ or $x^2 = 2$

$x = \pm 3i$ $\quad x = \pm\sqrt{2}i$ $\quad x = \pm\sqrt{2}$ $\quad$ Solutions: $x = \pm 3i, \pm\sqrt{2}i, \pm\sqrt{2}$

ANSWERS p 3

4 cont'd b. $2(x-4)^{7/3} - (x-4)^{4/3} - (x-4)^{1/3} = 0$

$(x-4)^{1/3}\,(2(x-4)^2 - (x-4) - 1) = 0$

$(Y)^{1/3}\,(2(Y)^2 - (Y) - 1) = 0$

$(Y)^{1/3}\,(2Y+1)(Y-1) = 0$

So $Y = 0$ or $Y = -\frac{1}{2}$ or $Y = 1$

$x - 4 = 0$ $\quad x - 4 = -\frac{1}{2}$ $\quad x - 4 = 1$

$x = 4$ or $x = 3\frac{1}{2}$ or $x = 5$

Here is a common factor, not hidden at all!

You can substitute $Y = x - 4$, if you wish

(which does make factoring appear easier...)

Solutions: $x = 4,\ 3\frac{1}{2},\ 5$

c. $x^{1/2} - 3x^{1/3} = 3x^{1/6} - 9$

$x^{1/2} - 3x^{1/3} - 3x^{1/6} + 9 = 0$

$Y^3 - 3Y^2 - 3Y + 9 = 0$

$Y^2(Y-3) - 3(Y-3) = 0$

$(Y-3)(Y^2 - 3)$

$Y = 3$ or $Y = \pm\sqrt{3}$

$x^{1/6} = 3$ $\qquad x^{1/6} = \pm\sqrt{3}$

So $x = 3^6 = 729$ $\qquad x = (\pm\sqrt{3})^6 = 27$

Do they really expect me to solve these? Yes.

Put in standard form: descending powers, = 0

Try to factor. $Y = x^{1/6}$ substitution may help.

Think pairs!

Solutions: $x = 729,\ 27$

d. $x^{1/2} + 3x^{-1/2} = 10x^{-3/2}$

$x^{3/2}(x^{1/2} + 3x^{-1/2} - 10x^{-3/2}) = 0\cdot x^{3/2}$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5$ $\quad x = 2$

FIRST: No negatives! Put in standard form & ...

multiply by $x^{3/2}$. ...Don't forget: Provided $x \neq 0$

The $x = -5$ is interesting; does it solve the equation? Substitute & see...

e. $x^2\sqrt{x+3} = (x+3)^{3/2}$

$x^2(x+3)^{1/2} - (x+3)^{3/2} = 0$

$(x+3)^{1/2}(x^2 - (x+3)) = 0$

$(x+3)^{1/2}(x^2 - (x+3)) = 0$

$x = -3$ OR $x = \dfrac{1 \pm \sqrt{-11}}{2}$

e. $x^2\sqrt{x+3} = (x+3)^{3/2}$

$x^2 = (x+3)$

Et cetera....

Alternatively, divide both sides by $(x+3)^{1/2}$,
Being careful to state this works IF $x \neq -3$
But test -3 (& see it is a solution).

(Check YOUR solution!)

5. Solve a rational inequality SOP: compare to 0 and simplify:

a. $$\frac{x}{x+2} \le \frac{1}{x}$$

Subtract 1/x from both sides.

$$\frac{x}{x+2} - \frac{1}{x} \le 0$$

Simplify: add the fractions

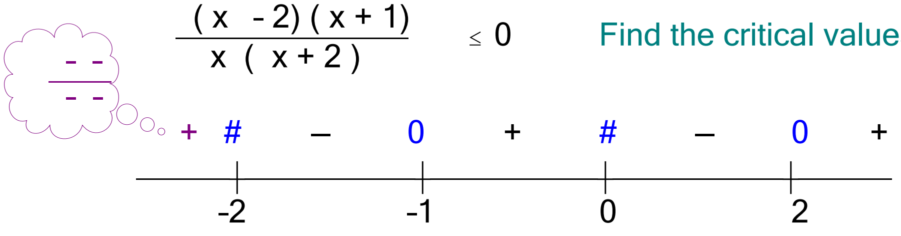
$$\frac{x}{x}\,\frac{x}{x+2} - \frac{1}{x}\,\frac{x+2}{x+2} \le 0$$

$$\frac{x^2 - x - 2}{x(x+2)} \le 0$$

Simplify: factor it !

$$\frac{(x-2)(x+1)}{x(x+2)} \le 0$$

Find the critical values : 2 , -1 , 0 , -2



Determine the sign of the quotient on each subinterval demarcated by the cvs , and the value AT each cv.

The inequality is true for x in $(-2, -1] \cup (0, 2]$.

State solutions in interval notation.

b. $$\frac{2x+5}{x+1} \le 1$$

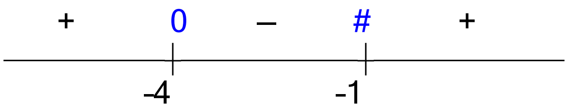
Compare to 0, not 1.

$$\frac{2x+5}{x+1} - 1 \le 0$$

Simplify.

$$\frac{2x+5-(x+1)}{x+1} \le 0$$

$$\frac{x+4}{x+1} \le 0$$



x must be in $[-4, -1)$

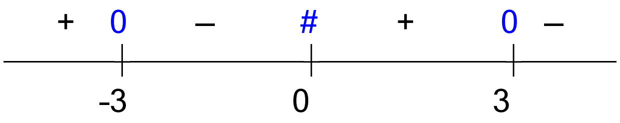
c. $$\frac{9}{x} < x$$

Stick to the plan !

$$\frac{9}{x} - x < 0$$

$$\frac{9 - x^2}{x} < 0$$

$$\frac{(3-x)(3+x)}{x} < 0$$



The solution set is $[-3, 0) \cup [3, \infty)$

d. $\frac{3}{x-1} - \frac{x}{x+1} \geq 1$

$\frac{3}{x-1} - \frac{x}{x+1} - 1 \geq 0$

$\frac{x+1}{x+1}\,\frac{3}{x-1} - \frac{x-1}{x-1}\,\frac{x}{x+1} - 1\,\frac{x^2-1}{x^2-1} \geq 0$

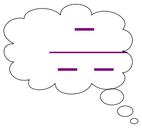
Watch those signs !

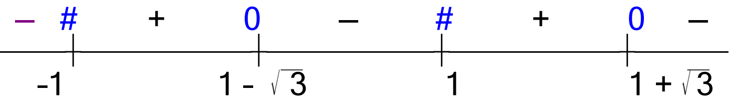
$\frac{3x+3 - x^2 + x - x^2 + 1}{(x-1)(x+1)} \geq 0$

$\frac{-2x^2 + 4x + 4}{(x-1)(x+1)} \geq 0$

We need to know where $-2x^2 + 4x + 4$ changes sign. We use the quadratic formula to locate the zeroes of $-2x^2 + 4x + 4$.

$x = 1 \pm \sqrt{3}$ (about - .7 & 2.7)





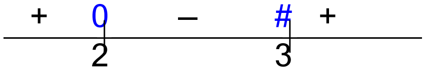
$(-1, 1 - \sqrt{3}\,] \cup (1, 1 + \sqrt{3}\,]$

e. $-3 \leq \frac{x+1}{x-3}$

$0 \leq \frac{x+1}{x-3} + 3$

$0 \leq \frac{x + 1 + 3(x-3)}{x-3}$

$0 \leq \frac{4x - 8}{x-3}$ c.v. 2 & 3



x must be in $(-\infty, 2] \cup (3, \infty)$

6. Absolute value basics: $\boxed{|x| < P \Leftrightarrow -P < x < P}$ ...and $\boxed{|x| > P \Leftrightarrow x < -P \text{ or } P < x}$

a. $3 - |2x + 4| \leq 1$

$2 \leq |2x + 4|$

$1 \leq |x + 2|$

$x + 2 \leq -1$ or $1 \leq x + 2$

$x \leq -3$ or $-1 \leq x$

Add | 2x + 4 | to both sides, subtract 1

Divide through by 2.

Solve this basic absolute value inequality.

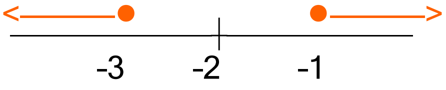
Simplify the results.

or: x must be in $(-\infty, -3\,] \cup [\,-1, \infty)$

Alternatively, we can solve the basic inequality “by inspection”, noting that $|x - c|$ is just the distance between x & c . So $|x - c| \geq r$ says the distance between x & c must be at least r.

$1 \leq |x + 2|$ says

$1 \leq |x - -2|$ which says the distance between x & -2 must be at least 1.



b. $4|3 - x| + 3 \geq 15$

Subtract 3, divide by 4.

$|3 - x| \geq 3$

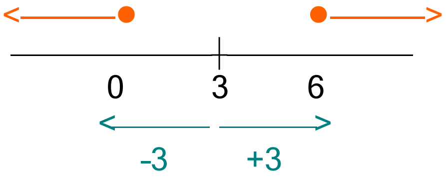
Since $|3 - x|$ is the same as $|x - 3|$, this says the distance between x and 3 must be at least 3.

$3 - x \leq -3$ or $3 \leq 3 - x$

$-x \leq -6$ or $0 \leq -x$

$6 \leq x$ or $x \leq 0$

...for those who must feel the pain



$(-\infty, 0] \cup [6, \infty)$

c. $2\left|\frac{1}{2}x + 3\right| + 3 \leq 51$

Multiply the $2|Q| = |2Q|$, then subtract 3 on both sides.

$|x + 6| \leq 48$

says the distance between x and -6 must not exceed 48.

$-48 \leq x + 6 \leq 48$

so x must be between -6 - 48 and -6 + 48

$-54 \leq x \leq 42$

an interval centered on -6, with “radius” 48: [-54, -42]

d. $\left|\frac{x+1}{2}\right| > 6$ ⇒

Multiply both sides by 2.

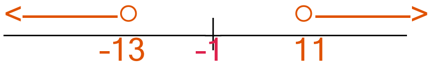
$|x + 1| > 12$

⇓

$|x - -1| > 12$

The distance between x & -1 must exceed 12:

x must be in $(-\infty, -13) \cup (11, \infty)$



The hard way:

$x + 1 < -12$ OR $x + 1 > 12$

$x < -13$ OR $x > 11$

(Actually there is nothing wrong with doing this algebraically, as shown at left. However, the above thought process leads to an immediate recognition and visualization of the meaning of $|x + 1| > 12$.)

e. $\left|\frac{x - 2}{3}\right| < 2$ ⇒

Multiply both sides by 3.

$|x - 2| < 6$

⇓

The distance between x & 2 must be less than 6.

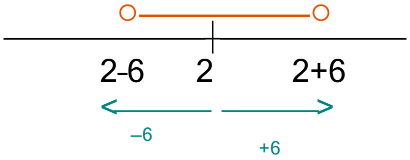
x must stay between 2 - 6 and 2 + 6.

x must be in $(-4, 8)$

Or, if you prefer to write a lot:

$-6 < x - 2 < 6$

$-4 < x < 8$



(For best results, you should be comfortable with both approaches.)

Phinally!!