1. For $f(x) = x^2 - 5x + 1$, find $\dfrac{**f(x+h) - f(x)}{h}$ and simplify completely.

NOTE:
**f(x+h) is NOT f(x)+h !
f(x+h) is NOT f(x) (x+h) !

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 1 - (x^2 - 5x + 1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h + 1 - (x^2 - 5x + 1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h + 1 - (x^2 - 5x + 1)}{h}$$

Many terms "cancel".
Notice, eg $-5x - (-5x) = 0$

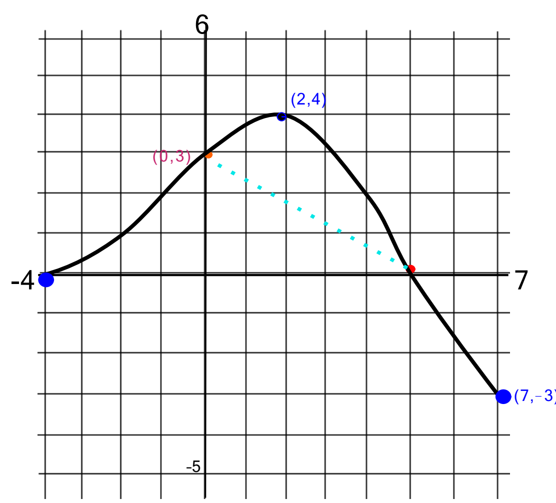
$$= \frac{2xh + h^2 - 5h}{h}$$

$$= \frac{h(2x + h - 5)}{h}$$

A factor must be a factor of the entire numerator and of the entire denominator in order to reduce (in order to "cancel").

$$= \boxed{2x + h - 5}$$

2.



a. What is f(-2) ?

Since the graph contains the point (-2,1)
f(−2) must be $\boxed{1.}$

b. On what interval is $f(x) \geq 0$?

The graph lies on or above the x-axis where $-4 \leq x \leq 5$.... On $\boxed{[-4,5]}$

c. What is the range of f ?

$\boxed{[-3,4]}$

d. On what interval(s) is f increasing/decreasing ?

The curve connecting (2,4) to (7,−3) is falling (as we read from left to right, as x increases).
The function $\boxed{\text{f is decreasing on } [2,7]}$.

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e. Does f have any local minimum?

Yes, f has endpoint minima (which can be considered local minima) at $\boxed{(-4,0) \text{ and } (7,-3)}$.

f. Does f have any local maximum?

Yes, f has a high point at approximately $\boxed{(2,4)}$.

g. Average rate of change of f on [0,5] is slope from (0,f(0)) to (5,f(5)):

$$= \frac{f(5) - f(0)}{5 - 0} = \frac{0 - 3}{5 - 0} = \boxed{\frac{-3}{5}}$$

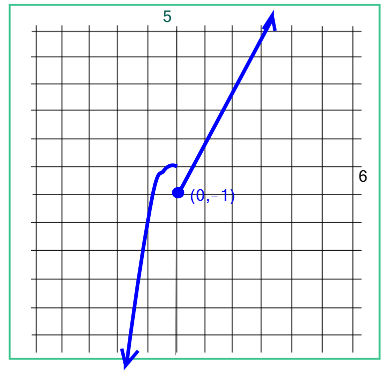
→ Note slope of dotted line in picture ↑

3. Let f be the function given by $f(x) = \begin{cases} x^3 & \text{if } x < 0 \\ 2x - 1 & \text{if } x \geq 0 \end{cases}$

a. $f(-2) = (-2)^3 = \boxed{-8}$

$f(\tfrac{1}{2}) = 2 \cdot \tfrac{1}{2} - 1 = \boxed{0}$

b. The graph of f is shown at right.

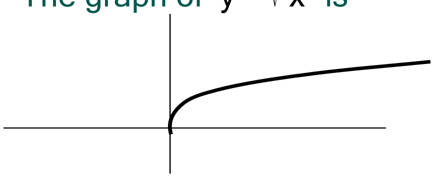


4. a. The graph of $g(x) = -\sqrt{x+4} + 1$ can be obtained from the graph of $y = \sqrt{x}$, using transformations, by:

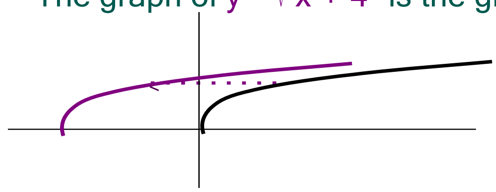
(a) shifting left 4 units,
(b) then flipping (reflecting) across the x-axis,
(c) and, finally shifting upward 1 unit.

(These orders will also work: b-a-c , b-c-a .)

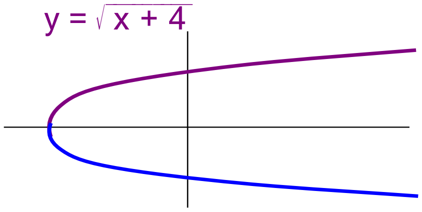
The graph of $y = \sqrt{x}$ is



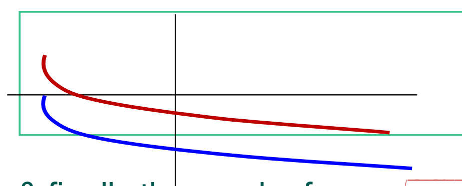
The graph of $y = \sqrt{x+4}$ is the graph of $y = \sqrt{x}$ shifted 4 units to the left. (Consider the domain of $y = \sqrt{x+4}$!)



$y = \sqrt{x+4}$



& the graph of $y = -\sqrt{x+4}$ (flipped about the x-axis)



$y = -\sqrt{x+4} + 1$ raised up 1 unit

$y = -\sqrt{x+4}$

& finally the graph of $y = -\sqrt{x+4} + 1$

5. Consider the quadratic model $h(t) = -16t^2 + 68t + 60$ for the height h (in feet), of an object t seconds after the object has been projected straight up into the air.
a. At what time does the projectile achieve its maximum height? [Find t where h(t) reaches maximum.]
b. What is its maximum height?

Method A- complete the square

$h(t) = -16(t^2 - 4.25t \quad) + 60$

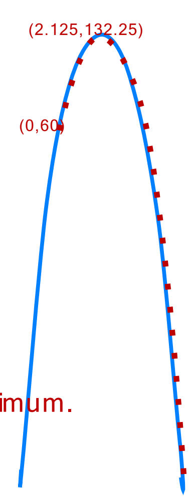
$= -16(t - 2.125)^2 + 60 + 16(2.125)^2$

$= -16(t - 2.125)^2 + 60 + 72.25$

A parabola, opening down, with vertex = maximum at (2.125, 132.25)

a. At $\boxed{t = 2.125 \text{ seconds}}$, height will be maximum.

b. This maximum will be $\boxed{132.25 \text{ feet}}$.



Method B- find the zeroes

$h(t) = -16t^2 + 68t + 60$

$= -4(4t^2 - 17t - 15)$

$= -4(4t + 3)(t - 5)$

$h(t) = 0$ when $t = -\,^3/_4$ and when $t = 5$
by the symmetry of parabolic function, the vertex must occur when $t = (\tfrac{1}{2})(-\,^3/_4 + 5) = 2.125$
Then the maximum height must be
$h(7.125) = 60 + 68(2.125) - 16(2.125)^2$

Method C- formulas...

←parabola, projectile's path is portion dotted red.

6. The monthly cost C, in dollars for usage on a certain cellular phone plan is given by the function $C(t) = .40\,t + 10$, where t is the number of minutes used.

a. What is the cost if you use just 60 minutes in one month?

$C(60) = .40(60) + 10$

$= 24 + 10$

$= 34$ The cost of the phone for 60 minutes in one month is **$34.00**.

b. Suppose you budget yourself for $60 per month for the phone.
What is the maximum number of minutes you can talk?

The Q is: For what number of minutes will the cost be $60. Well we know it isn't 60 minutes!

The Q is: Find t so that C(t) = $60.

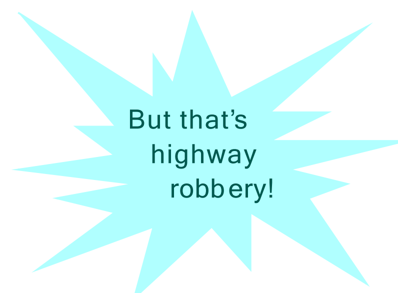
$C(t) = 60$ when $.40\,t + 10 = 60$,

$.4\,t = 50$

$t = 50/.4 = 500/4$

$t = 125$

You can gab on the phone for **125 minutes** for your $60 on this plan.



... Or shop around and get unlimited use for that money!

7. Let P = (x,y) be a point on the graph of $y = 3x - 3$.
Hint: the distance between two points is given by the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
a. Express the distance from P to the point (0,1) as a function of x.
b. What is the distance between the point on the graph where x=2 and the point (0,1)?

You'd do better to answer part b first!

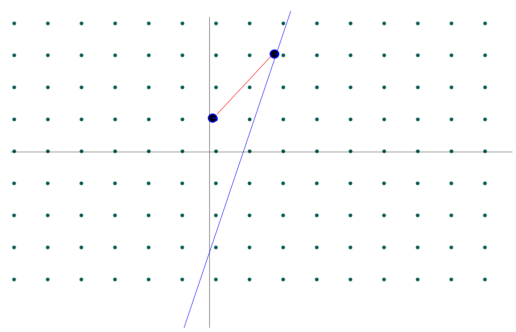
b. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(0 - 2)^2 + (1 - 3)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{8} = \boxed{2\sqrt{2}}$

When x=2, on the graph: $y = 3 \cdot 2 - 3$ so the points are (2,3) and (0,1)



a. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ Has been given to us – a free gift.

We must decide what (x_1, y_1) & (x_2, y_2) are. But one point is (0,1) and the other is (x, y= 3x – 3)
(x_1, y_1) & (x_2, y_2)

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(0 - x)^2 + (1 - (3x - 3))^2}$

$d = \sqrt{x^2 + (4 - 3x)^2}$

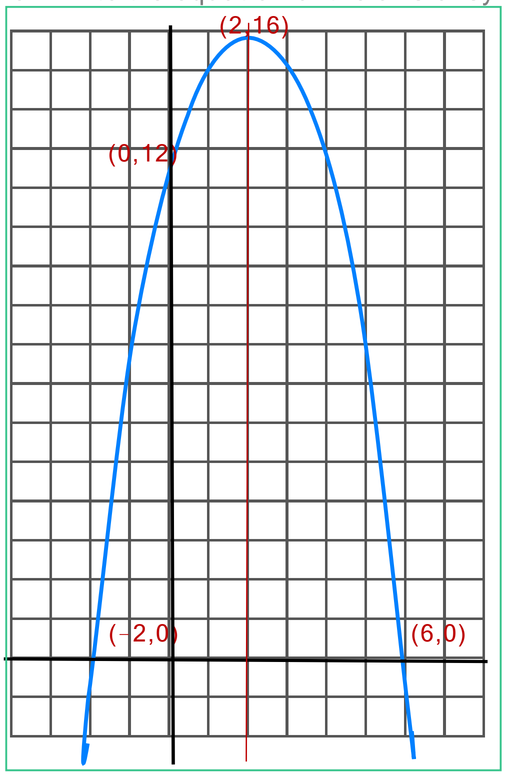
(You may waste time calculating the polynomial under the radical $= 10x^2 - 24x + 16$,
but I would not do that for this problem.)

8. Let f be the function given by: $f(x) = -x^2 + 4x + 12$

a. Find the vertex of this function **(2,16)**

b. Sketch the graph, and label all the important points.

c. Write the equation of the axis of symmetry: **x=2**



$= -(x^2 - 4x \quad) + 12$

$= -(x^2 - 4x + 4 \quad) + 12 + 4$

$= -(x-2)^2 + 16$

A parabola, opening down, with vertex (2,16)

y-intercept $f(0) = -0^2 + 4 \cdot 0 + 12 = 12$

x-intercepts: $-x^2 + 4x + 12 = 0$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

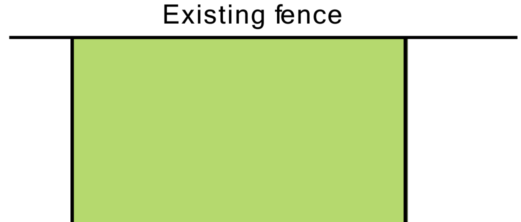
$x = 6$ or $x = -2$

The method of completing the square was used here because it was particularly easy. (But it is probably even easier to find the vertex after the x-intercepts... since all the numbers turn out to be so nice.)

9. A rectangular field is to be enclosed with 250 yards of fencing. One side of the field abuts an existing straight fence (and does not need fencing).

a. Express the area of the field, A, as a function of its width x.

b. For what value of x will the area be the greatest?



a. Area = Length • Width — Suppose we call the width "x"...

= Length • x — We must express Length in terms of x.

$A(x) = (250_{yd} - 2x) \cdot x$ — IF, for instance, x is 100, then Length would be 250 – 200.

As a pure function, A(x) is a parabola, opening down (again!), with x-intercepts easily seen to be at x=0 and x= 125 . By the symmetry of this function about its axis, the axis and vertex must occur when x = 62.5.

b. Area will be greatest when x is **62.5 yd.**

(The location of the maximum can also be found by completing the square, or by using a formula. The method used above was the easiest to use for this problem.)