1. Find composite functions and their domains

For $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{1}{x-2}$ find $f \circ g$ and its domain.

Existence of $f \circ g(x) = f(g(x))$ is dependent on the existence of $g(x)$ first- if $g(x)$ does not exist, then $f(g(x))$ certainly cannot. Thus x may not be 2. Secondly:

$f(g(x)) = \frac{1}{g(x)+3}$, so $g(x)$ must not be -3. Solving $\frac{1}{x-2} = -3$, we see x must not be $\frac{5}{3}$.

$$f(g(x)) = \frac{1}{g(x)+3} = \frac{1}{\frac{1}{x-2}+3} = \frac{1}{\frac{1}{x-2}+3} \cdot \frac{(x-2)}{(x-2)} = \frac{x-2}{1+3(x-2)} = \frac{x-2}{3x-5} \text{ for } x \neq 2, \frac{5}{3}$$

↗Notice here
It is evident x cannot be 2

And here we see
x cannot be $^5/_3$.

2. Find inverse functions

For $f(x) = \frac{1}{3x-2}$ find f^{-1} and its domain and range.

Although we can find the inverse by reversing the steps of f, that is not always practical. Therefore we will be solving the equation for f^{-1}.

$y = \frac{1}{3x-2}$ Note that the domain of f is all reals except for ⅔. This is the range of f^{-1}.

$x = \frac{1}{3y-2}$ We interchange x and y (since that's what the inverse does). Solve for y!

$3xy - 2x = 1$ (Obtained by mutliplying both sides of the equation by (3y -2)

$3xy = 1 + 2x$ Solve for y!

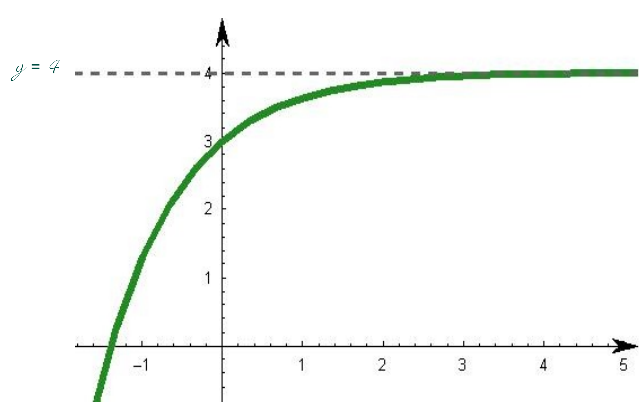
$y = \frac{1+2x}{3x}$ This gives us y as a function of x.
To write this as f^{-1} in function notation, we simply note that y IS $f^{-1}(x)$!

$$f^{-1}(x) = \frac{1+2x}{3x}$$

Note the above function is also ⅔ + 1/(3y). Finding the inverse by the "short method", we note that f triples x, subtracts 2, then takes reciprocal.
So f^{-1} must take reciprocal, add 2, then divide by 3: $f^{-1}(x) = \frac{(1/x)+2}{3} = \frac{1+2x}{3x}$

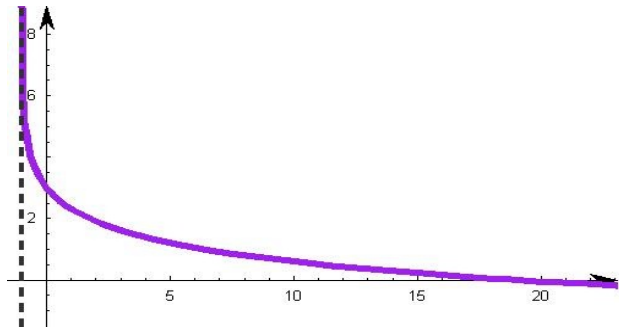
3. Sketch the graph of an exponential function. $f(x) = 4 - e^{-x}$.

(1) We should know the graph of $y = e^{x}$. (If not, plot points to determine the graphs of 2^{x} & 3^{x}.)
(2) The graph of $y = e^{-x}$ is the same as above, but reflected through the y-axis.
(3) The graph of $y = -e^{-x}$ is the same as in (2), but reflected through the x-axis.
Note that y=0 is still a horizontal asymptote.
(4) The graph of $y = 4 - e^{-x}$ is the same as in part (3), but shifted vertically +4 units.
The horizontal asymptote is then $y = 0+4$.



4. Graph logarithm function: $f(x) = 3 - \ln(x+1)$

$\ln$ is the inverse of $\exp_e(x) = e^x$.
(1) $\ln$ has domain $(0,\infty)$ and range $(-\infty,\infty)$; $y = \ln x$ has a vertical asymptote at $x=0$.
(2) $y = \ln(x+1)$ is the same as (1), but shifted 1 unit to the left. Vertical asymptote now $x = -1$.
(3) $y = -\ln(x+1)$ is the same as (2), but reflected ("flipped") about the x-axis.
(4) $y = 3 - \ln(x+1)$ is the same as (3), but shifted upward 3 units.



5. Simplify logarithm expressions: $20 \log_2 \sqrt[4]{x} + \log_2 (4x^3) - \log_2 4$

$20 \log_2 \sqrt[4]{x} + \log_2 (4x^3) - \log_2 4$

$20 \log_2 x^{1/4} + \log_2 (4x^3) - \log_2 4$

$20 \log_2 x^{1/4} + \log_2 4 + \log_2 x^3 - \log_2 4$

$20(1/4) \log_2 x + \log_2 4 + 3\log_2 x - \log_2 4$

$8 \log_2 x$

$\sqrt[4]{x} = x^{1/4}$

$\log AB = \log A + \log B$

$\log x^P = P \log x$

6. Solve equations involving logarithms. Solve $\log_{15} x + \log_{15} (x - 2) = 1$

$\log_{15} x + \log_{15} (x - 2) = 1$

$\log_{15} (x^2 - 2x) = 1$

$x^2 - 2x = 15$

$(x - 5)(x + 3) = 0$

$x = 5$ or ~~-3~~

$\log_{15} 5 + \log_{15} (5 - 2) = 1$ is true. ✓

Combine those logs using:

$\log AB = \log A + \log B$

Eliminate those logarithms, using:

If $Q = T$, then $15^Q = 15^T$

Solutions must be checked

$\log_{15} -3 + \log_{15} (-3 - 2)$ Does not compute. ✗

7. Solve exponential equations: $2^{x+3} = 5^x$.

$2^{x+3} = 5^x$

$\ln 2^{x+3} = \ln 5^x$

$(x + 3) \ln 2 = x \ln 5$

$3 \ln 2 = x \ln 5 - x \ln 2$

$x (\ln 5 - \ln 2) = 3 \ln 2$

$$x = \frac{3 \ln 2}{\ln 5 - \ln 2}$$

Here there are two different bases, so we use the natural logarithm. Take ln both sides.

$\log x^P = P \log x$