Essential Skills for Chapter 4

NOTE: There are six questions for chapter 4, four were covered on the midterm, two are new.

1. (§4.1) Graph a quadratic function.

   EG Sketch the graph of the quadratic function $f(x) = -2x^2 - 4x - 3$.
   Label the vertex and y-intercept.

2. (§4.1) Find optimal values using quadratic models .

   EG Paradise Travel Agency's monthly profit P (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent on advertising per month according to the rule $P(x) = 7 - 2x(x - 4)$. What is Paradise's maximum monthly profit?

3. (§4.2) Graph polynomial functions.

   EG Sketch the graph of $f(x) = (x-2)^2 (x-3) (x + 1)$

4. (§4.3,§4.4) Graph rational functions.

   EG Sketch the graph of $R(x) = \dfrac{x^2 - x - 12}{x + 1}$

5. (§4.5) Solve rational inequality.

   EG Solve $\dfrac{x}{x+2} < \dfrac{1}{x}$

6. (§4.6,§4.7) Find zeros of polynomials.

   EG Find all the roots/zeros of $P(x) = 2x^3 - 5x^2 + 6x - 2$

An Old Quiz on Chapter 4

a. In each of the following statements, circle T if true, F if false.
(In each statement, assume any function called "P" is a polynomial function.)

T F If P(1) = -2 and P(2) = 7, then P (r) must be 0 for some number r between 1 and 2.
T F If r is a root of P, then (x - r) is a factor of P(x).
T F If (x - 8) is a factor of P, then P(8) must be 0.
T F If $P(x) = 5(x-2)^2(x+4)$, then the only roots of P are 2 and -4.
T F If $P(x) = (x-3)Q(x) + 2$, for some polynomial Q, then 3 is a root of P.

b. Use polynomial long division to fill in the blanks with polynomials of degree < 2:

$$\frac{x^3 + x^2 + 1}{x^2 + 1} = \boxed{} + \frac{}{x^2 + 1}$$

3. List all theoretically possible* rational roots of the polynomial $4x^4 - 8x^3 + 7x^2 + 2x - 9$
   * based on rational zeroes theorem.

4. $P(x) = x^3 + 2x - 3$
   a. USE synthetic division to locate a rational root of P.
   b. Find all the roots of P(x)
   c. Sketch the graph of y = P(x)

7. Sketch the graph of $y = \dfrac{3x + 5}{x + 2}$ Label all the intercepts & asymptotes.

8. a. How can we know that $P(x) = x^4 + 3x^2 + 1$ has no real roots, without a lot of work?
   b. List all the theoretically possible rational roots of $P(x) = 2x^3 - \frac{1}{2}x^2 - 32x + 8$.
   c. Find all the roots of the polynomial given in #2. Any surprises?
   d. List all the theoretically possible rational roots of $P(x) = 2x^3 - 5x^2 - 3x$.
   e. Find all the roots of $P(x) = 2x^3 - 5x^2 - 3x$.