❦ Essential Skills for Chapter 4 ❧

1. (§4.1) Identifying properties of linear functions

   EG Given $f(x) = -\frac{3}{2}x + 2$

   a) Determine the slope and y-intercept of f.
   b) Use the slope and y-intercept to graph f.
   c) Determine the average rate of change of f on the interval
   d) Determine whether f is increasing, decreasing, or constant.

2. (§4.1) Using linear functions as models

   EG In 2002, major league baseball signed a labor agreement with the players. In this agreement, any team whose payroll exceeds $128 million starting in 2005 must pay a luxury tax of 22.5% (for first-time offenses). The linear function describes the luxury tax T of a team whose payroll is p (in millions of dollars).

   a) What is the implied domain of this function?
   b) What is the luxury tax for a team whose payroll is is $160 million?
   c) What is the payroll of a team that pays a luxury tax of $11.7 million?

3. (§4.3) Graph a quadratic function.

   EG Sketch the graph of the quadratic function $f(x) = -2x^2 - 4x - 3$.
   Label the vertex and y-intercept.

4. (§4.3) Find optimal values using quadratic models.

   EG Paradise Travel Agency's monthly profit P (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent on advertising per month according to the rule $P(x) = 7 - 2x(x - 4)$. What is Paradise's maximum monthly profit?

5. (§4.4) Constructing and using quadratic models

   EG A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

Brief Answers:

1. a) $-{}^{3}/_{2}$, 2 b) locate (0,2), then locate the point down three and right two units. c) $-{}^{3}/_{2}$ d) dec.

2. a) $[128,\infty)$ b) $7.2 million c) $180 million

3. Vertex is at (−1,−1), opens down, and y-intercept is (0,−3).

4. $15,000

5. 500,000 square meters

❦ Solutions to Essential Skills for Chapter 4 ❧

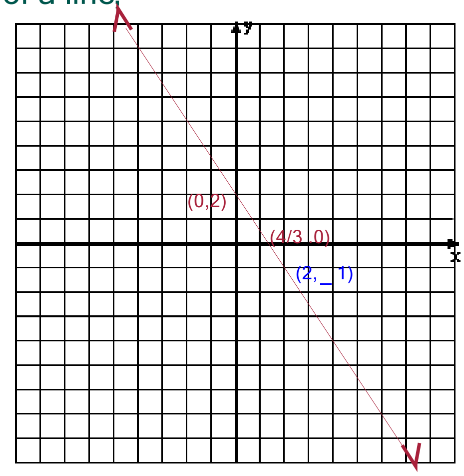
1. Identify properties of linear functions

EG Given $f(x) = -\dfrac{3}{2}x + 2$

a) By comparison with the slope-intercept form of the equation of a line,

$y = mx + b,$

m, the slope, is $-{}^3/_2$ and b, the y-intercept, is 2.



b) The y-intercept, "2" , says point: (0,2) is on the graph.
The slope, $-{}^3/_2$ (" -3 over 2"), lets us know that as x "moves over 2" (→→), y must change by -3 .
"Moving over2", "down 3", the point $(2, -1)$ belongs to the graph,

To maintain good habits, we also label the x-intercept.

c) Find the average rate of change of f on the interval $[0.3, {}^4/_9]$

Method ONE:
The line has a constant slope- i.e. constant rate of change of y with respect to x.
That slope is $-{}^3/_2$.

Method TWO:
$f(0.3) = -{}^3/_2(.3) + 2$
$f({}^4/_9) = -{}^3/_2({}^4/_9) + 2$

So

$$\frac{\Delta y}{\Delta x} = \frac{f({}^4/_9) - f(.3)}{{}^4/_9 - .3} = \frac{-{}^3/_2({}^4/_9) + 2 - \{-{}^3/_2(.3) + 2\}}{{}^4/_9 - .3}$$

$$= \frac{-{}^3/_2({}^4/_9) - -{}^3/_2(.3)}{{}^4/_9 - .3} = \frac{-{}^3/_2({}^4/_9 - .3)}{{}^4/_9 - .3} = -{}^3/_2$$

d) Determine whether f is increasing, decreasing, or constant.

The values for y = f(x) decrease as x increases. The slope is negative.
Either of these facts indicate f is decreasing.

2. In 2002, major league baseball signed a labor agreement with the players. In this agreement, any team whose payroll exceeds \$128 million starting in 2005 must pay a luxury tax of 22.5% (for first-time offenses). The linear function $T(p) = 0.225(p - 128)$ describes the luxury tax T of a team whose payroll is p (in millions of dollars).

a) What is the implied domain of this function?

"p" stands for payroll, so p must be non-negative.
But, further, the description prior to the expression for T states that the luxury tax (T) is imposed only on those teams whose payrolls exceed 128 million. So p must be at least 128 million for this formula to apply. Thus the "implied domain" is $[128, \infty)$.

b) What is the luxury tax for a team whose payroll is is \$160 million?

$T(160) = .225(160 - 128) = .225\,(32) = 7.2$ The tax is \$7.2 million, or \$7,200,000.

c) What is the payroll of a team that pays a luxury tax of \$11.7 million?

If $T(p) = 11.7$, then $.225(p - 128) = 11.7$. So $p = 128 + 11.7/.225 = 180$.
That team's payroll must be 180 million.

3. Sketch the graph of the quadratic function $f(x) = -2x^2 - 4x - 3$. Label the vertex and y-intercept.

The most direct way to do this, in general, is by completing the square.
We can also use the zeroes (x-intercepts) to locate the x-coordinate of the vertex.

$f(x) = -2x^2 - 4x - 3$

$= -2(x^2 + 2x \quad) - 3$

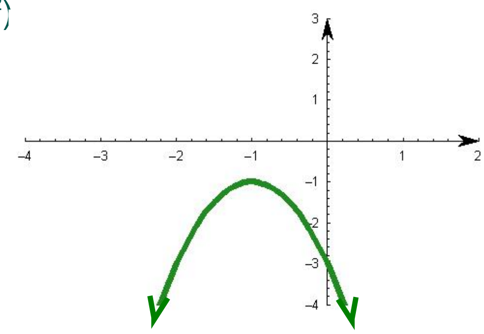
(First factor out the -2, carefully of course!)

$= -2(x^2 + 2x + 1) - 3 + 2$

$= -2(x + 1)^2 - 1$

Then, seeing $x^2 + 2x$, we know the square we need is $(x + 1)^2$ ...and we know $(x + 1)^2$ is $x^2 + 2x + 1$ so we add the needed $+1$ in the middle, & offset at end [$+2$ since we added $-2(+1) = -2$].

We now can say this is a parabola (like the graph of $y = x^2$)
...shifted left one unit ($y = (x+1)^2$)
...stretched vertically by factor 2 ($y = 2(x+1)^2$)
...flipped upside down over the x-axis ($y = -2(x+1)^2$)
...then shifted down one unit ($y = -2(x+1)^2 - 1$)



The vertex is $(-1, -1)$.
The y-intercept is $(0, -3)$
(since $f(0) = -2 \cdot 0^2 - 4 \cdot 0 - 3$)

(Borrowed graph↗)

4. Find optimal values using quadratic models.
Paradise Travel Agency's monthly profit P (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent on advertising per month according to the rule $P(x) = 7 - 2x(x - 4)$. What is Paradise's maximum monthly profit?

$P(x) = 7 - 2x(x - 4)$

$= -2x^2 + 8x + 7$

$= -2(x^2 - 4x \quad) + 7$

$= -2(x^2 - 4x + 4) + 7 + 8$

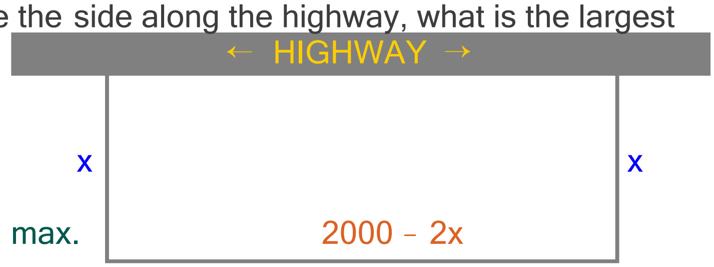
$= -2(x - 2)^2 + 15$

We complete the square

First factor out the -2
Then, seeing $x^2 - 4x$, we know the square we need is $(x - 2)^2$ ...and we know that $(x - 2)^2$ is $x^2 - 4x + 4$
...so
we add the necessary adjustments in the middle & end.

We identify this as a parabolic function (opening down) with a maximum at (2,15).
So maximum is 15 (thousand) and the advertising budget should be 2 (thousand)

5. A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?



Area = width • length

$= x(2000 - 2x)$

... a parabola opening down, so its vertex is a max.

x-intercepts are obvious: $x = 0$ and $x = 1000$, so by symmetry the vertex must be halfway between, at $x = 500$. (That is x should be 500 meters to maximize the area enclosed.)
The maximum area is 500 m (2000m – 2•500m) = 500m (1000m) = 500,000 m^2 .