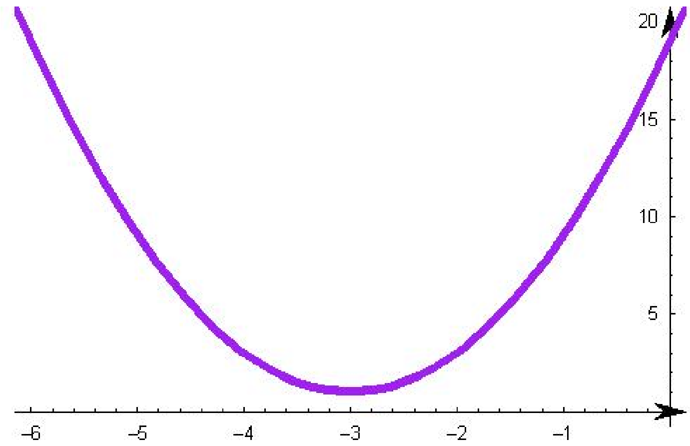
Essential Skills Chapter 3

1. (§3.1) Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for a given function.

   EG. for $f(x) = 3 - 4x - 4x^2$ find $\frac{f(x+h) - f(x)}{h}$ ...and simplify completely.

2. (§3.1) Find the domain of a function.

   EG. for $f(x) = \frac{4}{\sqrt{x-9}}$ ...find the domain



3. (§3.2, 3.3) Analyze function from graph.

   EG: for the graph at right, with vertex at (-3,1)

   a. On what interval is f increasing?

   b. On what interval is f decreasing?

   c. What is the domain of the function?

   d. What is the range of the function?

   e. What is the maximum or minimum value, if any?

   f. Which of the following could be a formula for f?

   (i) $f(x) = (x+3)^2 +1$ (ii) $f(x) = (x - 1)^2 +3$ (iii) $f(x) = 2(x - 3)^2 +1$ (iv) $f(x) = 2(x+3)^2 +1$

4. (§3.3) Find average rate of change of function.

   EG Find the average rate of change of the function $f(x) = (1 - x)^{1/3} = \sqrt[3]{1 - x}$ on [-7,9].

5. (§3.4) Sketch graphs of basic functions.

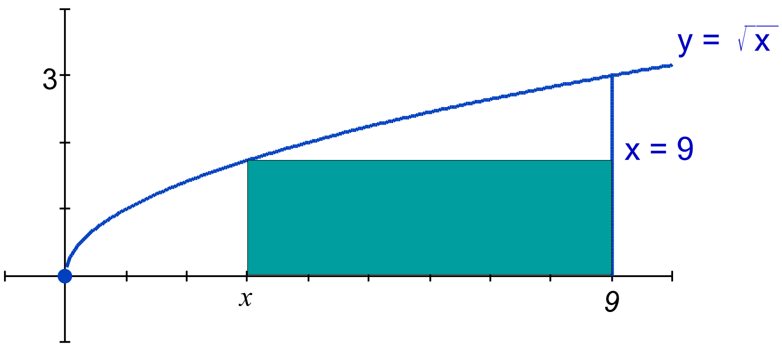
   EG: Sketch the graph of $f(x) = x^{1/3}$, the cube-root function.

6. (§3.4) Sketch graphs of basic functions using transformations.

   EG: Sketch the graph of $f(x) = (x + 2)^3 - 3$

7. (§3.6) Construct and use functions that model the relationship between two quantities.

   EG: Consider the region bounded by the graphs of $y = \sqrt{x}$ and $x = 9$, and the x-axis.
   For each value of x ($0 \le x \le 9$) there is a rectangle inscribed in this region, with its right edge on the line x = 9, and whose opposite vertices lie on the x-axis and the curve $y = \sqrt{x}$.
   Write a function that expresses the area of the inscribed rectangle as a function of x.



1. $$\frac{f(x+h) - f(x)}{h} = \frac{(3 - 4(x+h) - 4(x+h)^2) - (3 - 4x - 4x^2)}{h}$$

$$= \frac{3 - 4x - 4h - 4(x^2 + 2xh + h^2) - (3 - 4x - 4x^2)}{h}$$

$$= \frac{\cancel{3} - \cancel{4x} - 4h - \cancel{4x^2} - 8xh - 4h^2 - (\cancel{3} - \cancel{4x} - \cancel{4x^2})}{h}$$

$$= \frac{-4\cancel{h} - 8x\cancel{h} - 4h^{\cancel{2}}}{\cancel{h}} = -4 - 8x - 4h$$

provided $h \neq 0$

2. To find the domain of $f(x) = \dfrac{4}{\sqrt{x-9}}$ ...we ask: for what x can we compute f(x)?

You may look at this and immediately realize that $x - 9$ must be positive.
If not, then try the step-by-step consideration below:

The approach: Compute f(10) ... or any other value.

To find f(10), what is the first computation you must make? (Subtract 9.)
This was no problem if x = 10. Can this ever be an obstacle? (No, we can always subtract 9.)

What is the next computation? (Take square root.)
Can this ever be an obstacle? (Yes, square root of negative is not real; so the quantity $x - 9$ must be non-negative.) $x - 9 \geq 0$ ... requires that $x \geq 9$.

Finally, we must divide 4 by the result of the square root. Division cannot be by 0, so additionally, $x - 9$ must not be 0, so x cannot be 9. This leaves us with $x > 9$.... $x \in (9,\infty)$

3. In the analysis of this graph, we assumed the graph continues as shown (in parabolic shape).

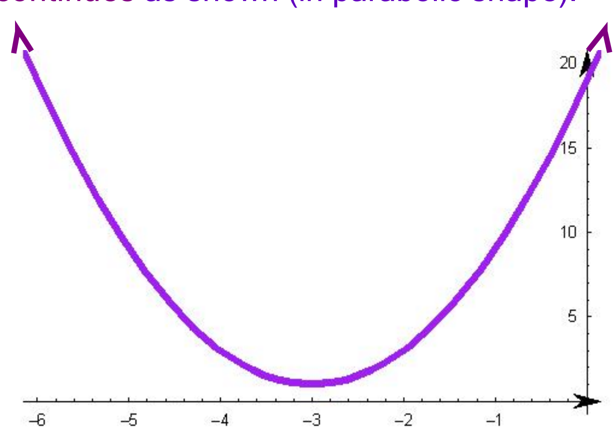
a. f is increasing when $x \geq -3$. So
f is increasing on the interval $[-3, \infty)$.

b. f is decreasing when $x \leq -3$. So
f is decreasing on the interval $(-\infty, -3]$.

c. If the graph indeed continues, domain is $(-\infty, \infty)$.

d. If the graph indeed continues, range is $[1,\infty)$

e. The minimum value is 1, and this occurs at the extreme point (-3, 1).



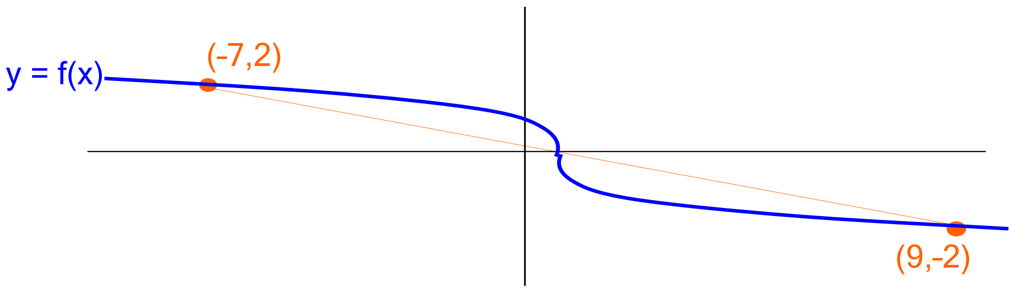
f decreases... on $(-\infty,-3]$ ↑ hits bottom at (-3,1) ...then increases on $[-3,\infty)$

f. Vertex of parabola at (-3,1) says function could be (i) or (iv), but the location of the y-intercept tells us it's got to be (iv).

4. The average rate of change of a function f, on an interval [a,b] is the slope of the line joining the points (a,f(a)) and (b,f(b)). Let's call this "*m*"

$$m = \frac{\Delta y}{\Delta x} = \frac{f(9) - f(-7)}{9 - -7} = \frac{\sqrt[3]{1-9} - \sqrt[3]{1--7}}{9 - -7} = \frac{\sqrt[3]{-8} - \sqrt[3]{8}}{16} = \frac{-4}{16} = -\tfrac{1}{4}$$

On the interval from -7 to 9, the function drops from 2 to -2.



5. You should recognize AND be able to sketch:

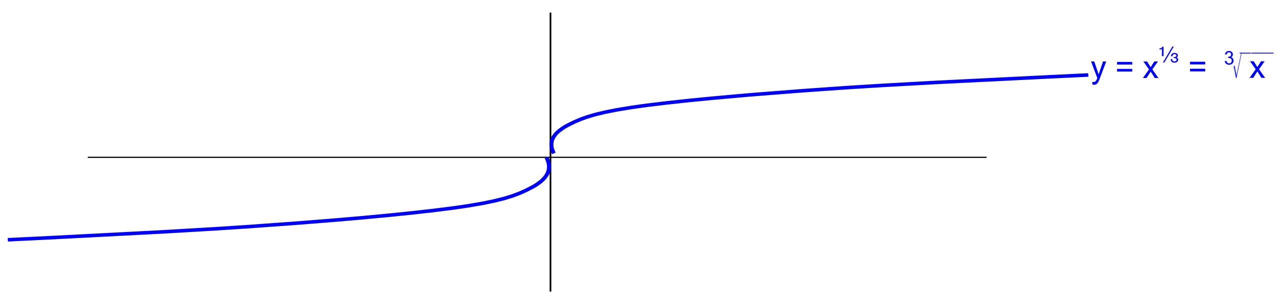
$y = x^2$ $y = x^3$ $y = x^4$ $y = x^5$ $y = x^6$ $y = \frac{1}{x}$ $y = [\,x\,]$ $y = |x|$

$y = \sqrt{x}$ $y = \sqrt[3]{x}$

The old-fashioned way to understand these graphs is to plot points until you see the patterns and trends on each function, and later to look for relationships between the different functions.

If you do this, even if you later "forget" what a particular graph looks like, plotting a few points will help you recall the understanding that you gained from discovering the graph the first time. This does not work if you just watch someone else do it.

EG: Sketch the graph of $f(x) = x^{1/3}$, the cube-root function.



Whereas the graph of $y = x^3$ contains the points:

(-2, -8)
(-1, -1)
(0, 0)
(½, $^1/_8$)
(1, 1)
(2, 8)
(3, 27)

The graph of $y = x^{1/3}$ (equivalent to saying $y^3 = x$) exactly reverses the relationship between x and y... and, so, must contain the points:

(-8,-2)
(-1, -1)
(0, 0)
($^1/_8$, ½)
(1, 1)
(8, 2)
(27,3)

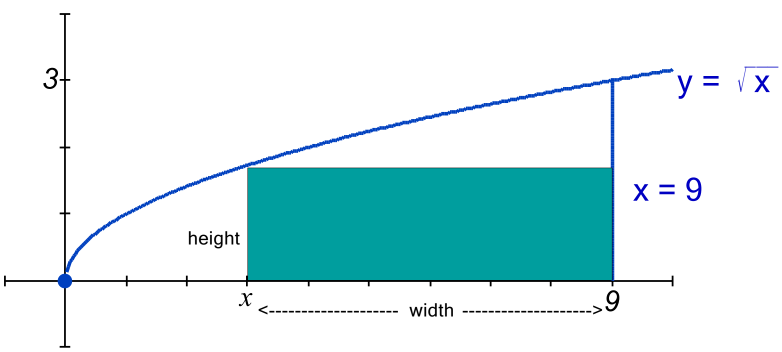
P.S. So what would the graph of $y = (1 - x)^{1/3}$ look like?

6. Sketch graphs using transformations. (The assumption here is that you know the graph of the basic function– in this case, the cubing function.)

Start with $y = x^3$. Then shift LEFT 2 for $y = (x + 2)^3$, then DOWN 3 for $y = (x + 2)^3 - 3$.

7. Write function modeling....

Area of a rectangle, inscribed as shown:



The area of this rectangle is the product of its width and height:

A = width • height

where the width (shown in the sketch)
is the (horizontal) distance between the chosen x and x=9 , and thus
is (9 - x)
and the height, the vertical distance labelled in the sketch,
is the distance between the x-axis (y=0) and the curve where $y = \sqrt{x}$, and so
is the difference between y=0 and $y = \sqrt{x}$: $\sqrt{x} - 0$

A = width • height

$A = (9 - x) \cdot (\sqrt{x} - 0)$