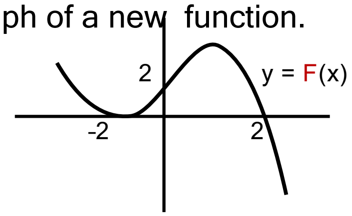
Essential Skills Chapter 3&4— additional practice

1. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for a polynomial function.

EG. for $H(x) = x^3$ find $\frac{H(x+h) - H(x)}{h}$ ...and simplify completely.

2. Sketch the graph of a function using basic shapes combined with transformations.

E.G.: Apply transformations to basic functions to obtain the graph of a new function.



a) $f(x) = 2 - \sqrt{x + 1}$ b) $g(x) = \frac{1}{2}|-x| + 2$

c) $h(x) = F(x + 2) - 3$ d) $G(x) = F(x/2)$ e) $H(x) = F(-x)$

3. Identify the extreme value & graph of a quadratic function by changing the form $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ (completing the square).

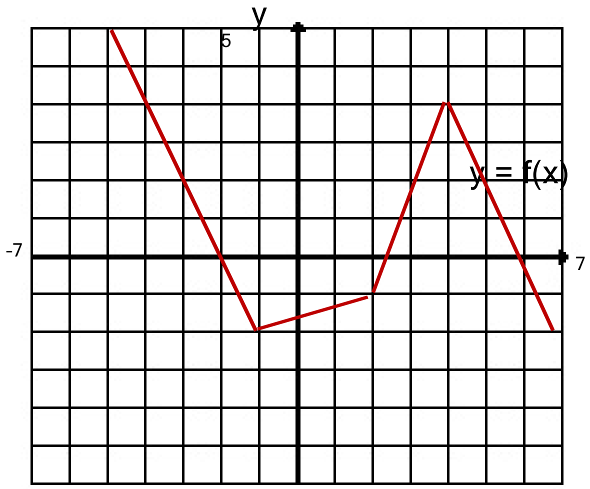
E.G.: For $f(x) = 2x^2 - 8x + 11$

- a) Express f(x) in the form $f(x) = a(x - h)^2 + k$.
- b) Find the extreme value of f(x). What type of extreme value is it?
- c) Sketch f(x), including all the important points.

4. Find the average rate of change of a function on a specified interval.

a) Find the average rate of change of $f(x) = x^3 + 2$ on the interval [0, 2].

b) Given the graph of y = f(x) below,



- find the average rate of change of f on [-3, 2] and
- identify the intervals on which f is increasing, and
- identify the intervals on which f is decreasing.

- (Does f appear to have any extreme points? Explain.)

5. Find the domain of the function.

EG For $f(x) = \frac{1}{x + 3}$ and $g(x) = \frac{x + 1}{x - 2}$ ... find:

a) the domain of $f + g$ b) the domain of $\frac{f}{g}$

c) the domain of $h(x) = \sqrt{g(x)}$

d) the domain of $k(x) = \sqrt{g(x) + 1}$

Solutions to E S Chapter 3&4– additional practice

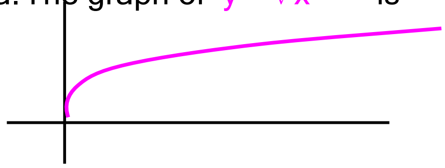
1. $$\frac{H(x+h) - H(x)}{h} = \frac{(\ (x+h)^3\ ) \ - \ (\ x^3\ )}{h}$$

$$= \frac{(\cancel{x^3} + 3x^2h + 3xh^2 + h^3\ ) \ - \ (\cancel{x^3}\ )}{h}$$

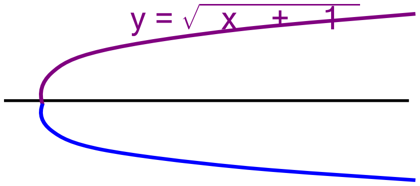
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$

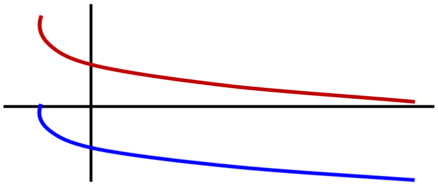
$$= \frac{h\ (3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2 \qquad \text{...provided } h \neq 0$$

2a. The graph of $y = \sqrt{x}$ is



The graph of $y = \sqrt{x + 1}$ is the graph $y = \sqrt{x}$ shifted 1 unit to the left.
PS: Consider the respective domains of $\sqrt{x}$ and $\sqrt{x + 1}$!



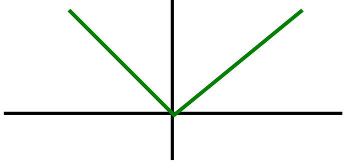


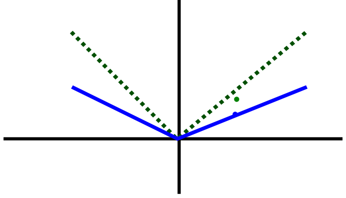
$y = -\sqrt{x + 1} + 2$

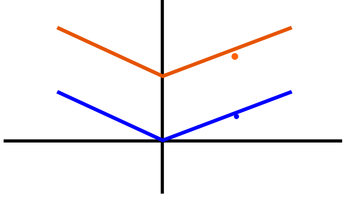
$y = -\sqrt{x + 1}$

& the graph of $y = -\surd\sqrt{x + 1}$ & finally the graph of $y = -\sqrt{x + 1} + 2$

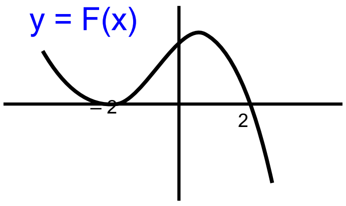
2b. The graph of $y = |-x|$ is the reflection (or "flip") of $y = |x|$ through the y-axis.

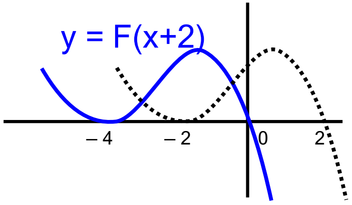


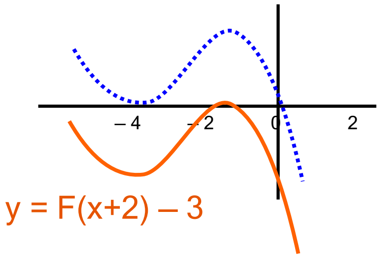


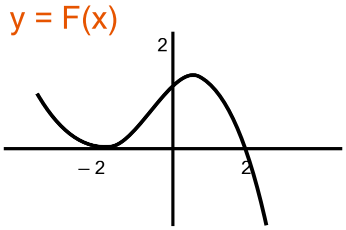


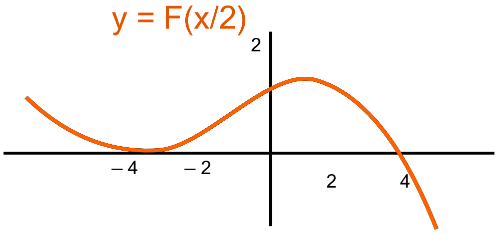
At each x, the graph of $y = (\ ½\ )\ |-x|$ is half as high as that of $y = |-x|$
At each x, the graph of $y = (\ ½\ )\ |-x| + 2$ is 2 units higher than that of $y = (\ ½\ )\ |-x|$
(so we shift upwards by 2) .

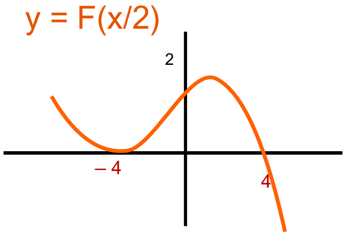












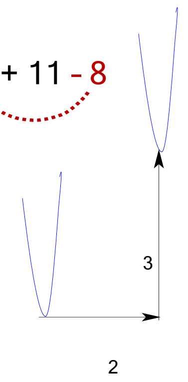
Another way to show the graph of F(x/2)– Change the x-scale.

3. Given: $f(x) = 2x^2 - 8x + 11$ we complete the square $\quad f(x) = 2x^2 - 8x + 11$

$f(x) = 2(x^2 - 4x \;+\; ?\;) + 11$ ...?? $\qquad f(x) = 2\,(x^2 - 4x + 4) + 11 - 8$

$f(x) = 2\,(x - \text{what})^2 \;+ 11$ $\qquad f(x) = 2(x - 2)^2 + 3$

We identify this as a parabola whose <u>minimum</u> occurs at (2,3)–
a parabola that is "stretched twice as tall" as the basic $y = x^2$,
shifted 2 units to the right, and 3 units upward.



4. At x = -3 the value of the function (y = f(-3)) appears to be 2.
At x = 2 the value of the function appears to be -1.
Therefore the average rate of change of f on the interval [-3, 2] is

$$\frac{\Delta f}{\Delta x} \quad \text{or} \quad \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{2--3} = \frac{-3}{5}$$

f appears to decrease on (- ∞ , -1] and on [4, ∞). f appears to increase on [-1, 4] and has a local minimum at (-1,-2) , and a local maximum at (4,4).

5. a. In order for f+g to be defined, both f & g must be defined. This is the case as long as x is not -3 or 2. So the domain of f+g is $(-\infty, -3) \cup (-3,2) \cup (2,\infty)$.

b. For f/g to be defined, not only must the conditions in part a be true, but we must also note that g(x) may not be 0, so the domain must be further restricted to avoid -1. So the domain of f/g is $(-\infty, -3) \cup (-3,-1) \cup (-1,2) \cup (2,\infty)$.

c. To find $\sqrt{g(x)}$ we must first be able to find g(x) ... so x must not be 2.
But, in addition, to take the $\sqrt{g(x)}$, g(x) must not be negative... so:
$g(x) \geq 0$ we solve: $\quad E = \frac{x+1}{x-2} \geq 0$

Critical values (where the factors x+1 and x - 2 can change sign) are -1 & 2.

E: + 0 – ! +
-1 2

So we see the required condition holds for $-\infty < x \leq -1$ and for $2 < x < \infty$

The domain of $h(x) = \sqrt{g(x)}$ is $(-\infty, -1] \cup (2, \infty)$

d) For $k(x) = \sqrt{g(x) + 1}$, we must have $g(x) + 1 \geq 0$ ... for which we solve:

$$\frac{x+1}{x-2} + 1 \geq 0 \qquad \frac{x+1+x-2}{x-2} \geq 0 \qquad \frac{2x-1}{x-2} \geq 0$$

Critical values (where the factors 2x - 1 and x - 2 can change sign) are ½ & 2.

+ 0 – ! +
½ 2

So we see the required condition holds for $-\infty < x \leq \frac{1}{2}$ and for $2 < x < \infty$

The domain of $k(x) = \sqrt{g(x) + 1}$ is $(-\infty, \frac{1}{2}] \cup (2, \infty)$