1. In each of the following statements, circle T if true, F if false.
(In each statement, assume any function called "P" is a polynomial function.)

- (T) F If P(1) = -2 and P(2) = 7, then P (r) must be 0 for some number r between 1 and 2.
- (T) F If r is a root of P, then (x - r) is a factor of P(x).
- (T) F If (x - 8) is a factor of P, then P(8) must be 0.
- (T) F If $P(x) = 5(x-2)^2(x+4)$, then the only roots of P are 2 and -4.
- T (F) If P(x) = (x - 3)Q(x) + 2, for some polynomial Q, then 3 is a root of P.

2. Use polynomial long division to fill in the blanks with polynomials of degree < 2:

$$\frac{x^3 + x^2 + 1}{x^2 + 1} = \boxed{x + 1} + \frac{-x}{x^2 + 1}$$

This polynomial and fraction may be added together to check this result.

```
               x  +  1
         ________________
x^2 + 1 ) x^3 + x^2     + 1
          x^3       + x
                x^2 - x + 1
                x^2     + 1
                    - x
```

Notice it is important to maintain alignment of like terms, and thus it is necessary to leave space for the "missing" x term.

3. List all theoretically possible rational roots of the polynomial $4x^4 - 8x^3 + 7x^2 + 2x - 9$

Since this polynomial has all coefficients in the set of integers,
any rational roots must be of the form p/q
where p is a factor of the constant term (so p = ± 9, 3, or 1) and
q is a factor of the leading coefficient (so q = ± 4, 2, or 1).

Thus candidates for rational roots of this polynomial are:

± 9   ±3   ±1   ±9/2   ± 3/2   ±1/2   ± 9/4   ±3/4   ±1/4

For #4-6, $P(x) = x^3 + 2x - 3$

4. USE synthetic division to locate a rational root of P. (Synthetic division MUST be shown.)

The potential rational roots (see #3) are ±1, ±3.

The first one we are likely to try is 1, but let's pretend that we check 3 first:

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 2 & -3 \\ & & 3 & 9 & 33 \\ \hline & 1 & 3 & 11 & 30 \end{array}$$

So P(3) = 30, not 0.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -3 \\ & & 1 & 1 & 3 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

From this we know 1 is a root
and
$P(x) = (x - 1)(x^2 + x + 3)$

5. Find all the roots of P(x).

We examine $x^2 + x + 3$, finding its roots via the quadratic formula.

$x^2 + x + 3 = 0$ when $x = \dfrac{-1 \pm \sqrt{1 - 12}}{2} = \dfrac{-1 \pm \sqrt{-11}}{2} = \dfrac{-1 \pm \sqrt{11}\,i}{2}$

Roots of P are 1 and $\dfrac{-1 + \sqrt{11}\,i}{2}$ and $\dfrac{-1 - \sqrt{11}\,i}{2}$

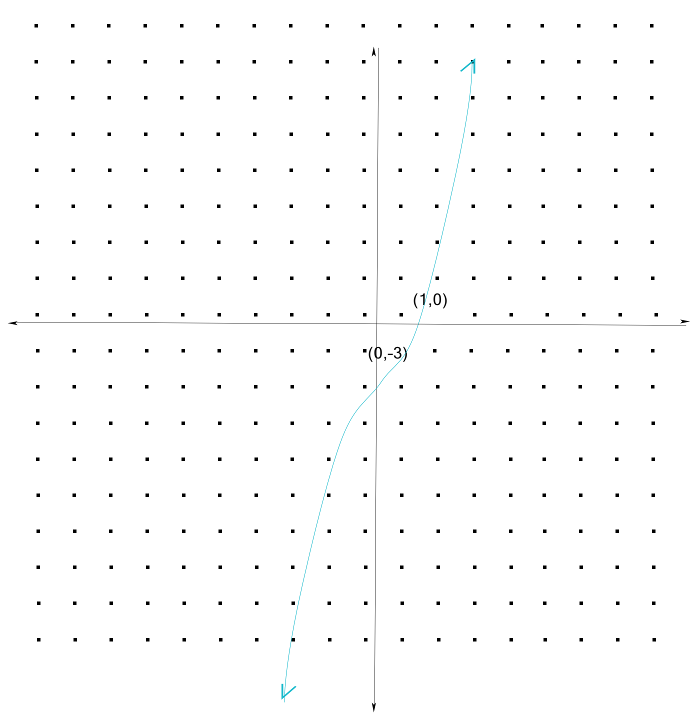
6. Sketch the graph of $y = P(x)$

x-intercept is just (1,0) [...from above]

y-intercept is f(0) = -3 ...(0,-3)

as $x \to \infty$ $y \to \infty$

as $x \to -\infty$ $y \to -\infty$



7. Sketch the graph of $y = \dfrac{3x + 5}{x + 2}$ Label all the intercepts & asymptotes.

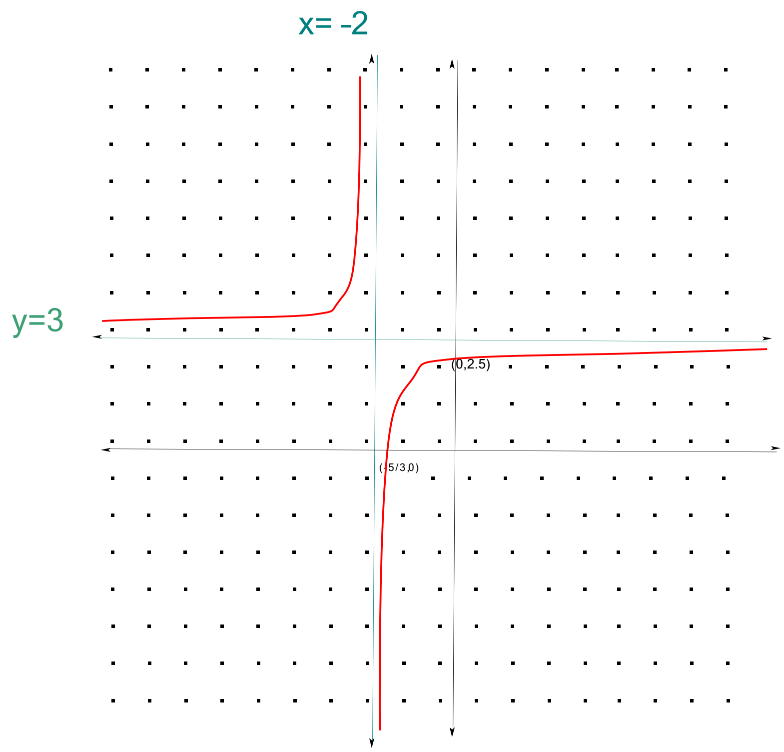
Domain: y is defined for all values except x = -2

As x approaches -2, 3x+5 → -1, while x+2 → 0, so the quotient → ± ∞
Thus there is a vertical asymptote at x = -2.

As x →∞ y →3
As x →∞ y →3
Thus the function has a horizontal asymptote at y = 3.

y-intercept: when x = 0
y = 5/2

x-intercept: when x = -5/3, y = 0



8. Some tricky questions here.

a. How can we know that $P(x) = x^4 + 3x^2 + 1$ has no real roots, without a lot of work?
DesCartes' Law of Signs tells us: P(x) has 0 sign changes; P(-x) has no sign changes.
Therefore P has no real roots, positive or negative.

b. List all the theoretically possible rational roots of $P(x) = 2x^3 - ½ x^2 - 32x + 8$.
Your knees might say p/q where p is a factor of 8, and q is a factor of 2.

c. Find all the roots of the polynomial given in #2. Any surprises?
The roots are 4, -4, and 1/4. If 1/4 did not show up in your list in #2, it is because you did not work with a polynomial with INTEGER coefficients, which is a requirement of the rational roots theorem. (The moral of the story is: Don't trust your knees; use your brain.)
Work with 2P(x), which is $4x^3 - x^2 - 64x + 16$.

d. List all the theoretically possible rational roots of $P(x) = 2x^3 - 5x^2 - 3x$.
0. (We warned you about those knees!)

e. Find all the roots of $P(x) = 2x^3 - 5x^2 - 3x$.
$P(x) = 2x^3 - 5x^2 - 3x = x(2x^2 - 5x - 3) = x(2x+1)(x-3)$
So the roots are 0, -½, and 3. These, being rational, may all be found using the rational roots theorem, but in order to apply it, you must first factor out the x, then use the theorem on the new polynomial, $2x^2 - 5x - 3$, whose constant term is NOT 0.