(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\dfrac{1}{x+2}}{\dfrac{x}{x-1}} = \dfrac{1}{x+2} \cdot \dfrac{x-1}{x} = \dfrac{x-1}{x(x+2)}$

The domain of $\dfrac{f}{g}$ consists of the numbers x for which $g(x) \neq 0$ that are in the domains of both f and g. Since $g(x) = 0$ when $x = 0$, we exclude 0 as well as -2 and 1. The domain of $\dfrac{f}{g}$ is $\{x | x \neq -2, x \neq 0, x \neq 1\}$.

NOW WORK PROBLEM 61.

In calculus, it is sometimes helpful to view a complicated function as the sum, difference, product, or quotient of simpler functions. For example,

$F(x) = x^2 + \sqrt{x}$ is the sum of $f(x) = x^2$ and $g(x) = \sqrt{x}$.

$H(x) = \dfrac{x^2 - 1}{x^2 + 1}$ is the quotient of $f(x) = x^2 - 1$ and $g(x) = x^2 + 1$.

Summary

We list here some of the important vocabulary introduced in this section, with a brief description of each term.

Function
A relation between two sets of real numbers so that each number x in the first set, the domain, has corresponding to it exactly one number y in the second set.
A set of ordered pairs (x, y) or $(x, f(x))$ in which no first element is paired with two different second elements.
The range is the set of y values of the function for the x values in the domain.
A function f may be defined implicitly by an equation involving x and y or explicitly by writing $y = f(x)$.

Unspecified domain
If a function f is defined by an equation and no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number.

Function notation
$y = f(x)$
f is a symbol for the function.
x is the independent variable or argument.
y is the dependent variable.
$f(x)$ is the value of the function at x, or the image of x.

3.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The inequality $-1 < x < 3$ can be written in interval notation as ________. (pp. 125–126)

2. If $x = -2$, the value of the expression $3x^2 - 5x + \dfrac{1}{x}$ is ________. (pp. 20–21)

3. The domain of the variable in the expression $\dfrac{x-3}{x+4}$ is ________. (pp. 20–21)

4. Solve the inequality: $3 - 2x > 5$. Graph the solution set. (pp. 129–133)

Concepts and Vocabulary

5. If f is a function defined by the equation $y = f(x)$, then x is called the __________ variable and y is the __________ variable.

6. The set of all images of the elements in the domain of a function is called the __________.

7. If the domain of f is all real numbers in the interval $[0, 7]$ and the domain of g is all real numbers in the interval $[-2, 5]$, the domain of $f + g$ is all real numbers in the interval __________.

8. The domain of $\frac{f}{g}$ consists of numbers x for which $g(x)$ __________ 0 that are in the domains of both __________ and __________.

9. If $f(x) = x + 1$ and $g(x) = x^3$, then __________ $= x^3 - (x + 1)$.

10. *True or False:* Every relation is a function.

11. *True or False:* The domain of $(f \cdot g)(x)$ consists of the numbers x that are in the domains of both f and g.

12. *True or False:* The independent variable is sometimes referred to as the argument of the function.

13. *True or False:* If no domain is specified for a function f, then the domain of f is taken to be the set of real numbers.

14. *True or False:* The domain of the function $f(x) = \frac{x^2 - 4}{x}$ is $\{x \mid x \neq \pm 2\}$.

Exercises

In Problems 15–26, determine whether each relation represents a function. For each function, state the domain and range.

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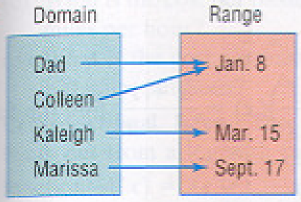

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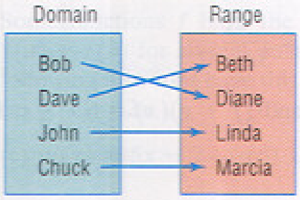

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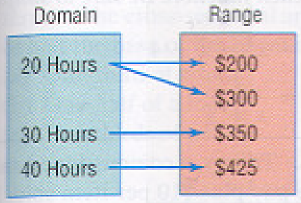

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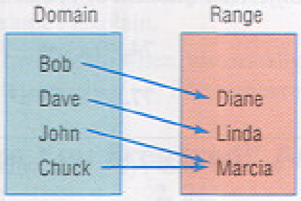


19. $\{(2, 6), (-3, 6), (4, 9), (2, 10)\}$

20. $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$

21. $\{(1, 3), (2, 3), (3, 3), (4, 3)\}$

22. $\{(0, -2), (1, 3), (2, 3), (3, 7)\}$

23. $\{(-2, 4), (-2, 6), (0, 3), (3, 7)\}$

24. $\{(-4, 4), (-3, 3), (-2, 2), (-1, 1), (-4, 0)\}$

25. $\{(-2, 4), (-1, 1), (0, 0), (1, 1)\}$

26. $\{(-2, 16), (-1, 4), (0, 3), (1, 4)\}$

In Problems 27–34, find the following values for each function:

(a) $f(0)$ (b) $f(1)$ (c) $f(-1)$ (d) $f(-x)$ (e) $-f(x)$ (f) $f(x + 1)$ (g) $f(2x)$ (h) $f(x + h)$

27. $f(x) = 3x^2 + 2x - 4$

28. $f(x) = -2x^2 + x - 1$

29. $f(x) = \frac{x}{x^2 + 1}$

30. $f(x) = \frac{x^2 - 1}{x + 4}$

31. $f(x) = |x| + 4$

32. $f(x) = \sqrt{x^2 + x}$

33. $f(x) = \frac{2x + 1}{3x - 5}$

34. $f(x) = 1 - \frac{1}{(x + 2)^2}$

In Problems 35–46, determine whether the equation is a function.

35. $y = x^2$

36. $y = x^3$

37. $y = \frac{1}{x}$

38. $y = |x|$

39. $y^2 = 4 - x^2$

40. $y = \pm\sqrt{1 - 2x}$

41. $x = y^2$

42. $x + y^2 = 1$

43. $y = 2x^2 - 3x + 4$

44. $y = \frac{3x - 1}{x + 2}$

45. $2x^2 + 3y^2 = 1$

46. $x^2 - 4y^2 = 1$

In Problems 47–60, find the domain of each function.

47. $f(x) = -5x + 4$ **48.** $f(x) = x^2 + 2$ **49.** $f(x) = \dfrac{x}{x^2 + 1}$ **50.** $f(x) = \dfrac{x^2}{x^2 + 1}$

51. $g(x) = \dfrac{x}{x^2 - 16}$ **52.** $h(x) = \dfrac{2x}{x^2 - 4}$ **53.** $F(x) = \dfrac{x - 2}{x^3 + x}$ **54.** $G(x) = \dfrac{x + 4}{x^3 - 4x}$

55. $h(x) = \sqrt{3x - 12}$ **56.** $G(x) = \sqrt{1 - x}$ **57.** $f(x) = \dfrac{4}{\sqrt{x - 9}}$ **58.** $f(x) = \dfrac{x}{\sqrt{x - 4}}$

59. $p(x) = \sqrt{\dfrac{2}{x - 1}}$ **60.** $q(x) = \sqrt{-x - 2}$

In Problems 61–70, for the given functions f and g, find the following functions and state the domain of each.

(a) $f + g$ (b) $f - g$ (c) $f \cdot g$ (d) $\dfrac{f}{g}$

61. $f(x) = 3x + 4;\ \ g(x) = 2x - 3$

62. $f(x) = 2x + 1;\ \ g(x) = 3x - 2$

63. $f(x) = x - 1;\ \ g(x) = 2x^2$

64. $f(x) = 2x^2 + 3;\ \ g(x) = 4x^3 + 1$

65. $f(x) = \sqrt{x};\ \ g(x) = 3x - 5$

66. $f(x) = |x|;\ \ g(x) = x$

67. $f(x) = 1 + \dfrac{1}{x};\ \ g(x) = \dfrac{1}{x}$

68. $f(x) = \sqrt{x - 2};\ \ g(x) = \sqrt{4 - x}$

69. $f(x) = \dfrac{2x + 3}{3x - 2};\ \ g(x) = \dfrac{4x}{3x - 2}$

70. $f(x) = \sqrt{x + 1};\ \ g(x) = \dfrac{2}{x}$

71. Given $f(x) = 3x + 1$ and $(f + g)(x) = 6 - \dfrac{1}{2}x$, find the function g.

72. Given $f(x) = \dfrac{1}{x}$ and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x + 1}{x^2 - x}$, find the function g.

In Problems 73–78, find the difference quotient of f, that is, find $\dfrac{f(x + h) - f(x)}{h}$, *$h \neq 0$, for each function. Be sure to simplify.*

73. $f(x) = 4x + 3$ **74.** $f(x) = -3x + 1$ **75.** $f(x) = x^2 - x + 4$

76. $f(x) = x^2 + 5x - 1$ **77.** $f(x) = x^3 - 2$ **78.** $f(x) = \dfrac{1}{x + 3}$

79. If $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$, what is the value of A?

80. If $f(x) = 3x^2 - Bx + 4$ and $f(-1) = 12$, what is the value of B?

81. If $f(x) = \dfrac{3x + 8}{2x - A}$ and $f(0) = 2$, what is the value of A?

82. If $f(x) = \dfrac{2x - B}{3x + 4}$ and $f(2) = \dfrac{1}{2}$, what is the value of B?

83. If $f(x) = \dfrac{2x - A}{x - 3}$ and $f(4) = 0$, what is the value of A? Where is f not defined?

84. If $f(x) = \dfrac{x - B}{x - A}$, $f(2) = 0$, and $f(1)$ is undefined, what are the values of A and B?

85. Geometry Express the area A of a rectangle as a function of the length x if the length of the rectangle is twice its width.

86. Geometry Express the area A of an isosceles right triangle as a function of the length x of one of the two equal sides.

87. Constructing Functions Express the gross salary G of a person who earns $10 per hour as a function of the number x of hours worked.

88. Constructing Functions Tiffany, a commissioned salesperson, earns $100 base pay plus $10 per item sold. Express her gross salary G as a function of the number x of items sold.

89. Effect of Gravity on Earth If a rock falls from a height of 20 meters on Earth, the height H (in meters) after x seconds is approximately

$$H(x) = 20 - 4.9x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds? $x = 1.3$ seconds?

(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?

(c) When does the rock strike the ground?

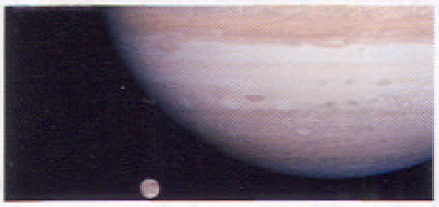
90. Effect of Gravity on Jupiter If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately

$$H(x) = 20 - 13x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds?

(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?

(c) When does the rock strike the ground?



Cost of Trans-Atlantic Travel A Boeing 747 crosses the Atlantic Ocean (3000 miles) with an airspeed of 500 miles per hour. The cost C (in dollars) per passenger is given by

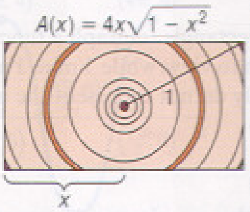
$$C(x) = 100 + \frac{x}{10} + \frac{36{,}000}{x}$$

where x is the ground speed (airspeed $\pm$ wind).

(a) What is the cost per passenger for quiescent (no wind) conditions?
(b) What is the cost per passenger with a head wind of 50 miles per hour?
(c) What is the cost per passenger with a tail wind of 100 miles per hour?
(d) What is the cost per passenger with a head wind of 100 miles per hour?

Cross-sectional Area The cross-sectional area of a beam cut from a log with radius 1 foot is given by the function $A(x) = 4x\sqrt{1 - x^2}$, where x represents the length of half the base of the beam. See the figure. Determine the cross-sectional area of the beam if the length of half the base of the beam is as follows:

(a) One-third of a foot
(b) One-half of a foot
(c) Two-thirds of a foot



93. Economics The **participation rate** is the number of people in the labor force divided by the civilian population (excludes military). Let $L(x)$ represent the size of the labor force in year x and $P(x)$ represent the civilian population in year x. Determine a function that represents the participation rate R as a function of x.

94. Crimes Suppose that $V(x)$ represents the number of violent crimes committed in year x and $P(x)$ represents the number of property crimes committed in year x. Determine a function T that represents the combined total of violent crimes and property crimes in year x.

95. Health Care Suppose that $P(x)$ represents the percentage of income spent on health care in year x and $I(x)$ represents income in year x. Determine a function H that represents total health care expenditures in year x.

96. Income Tax Suppose that $I(x)$ represents the income of an individual in year x before taxes and $T(x)$ represents the individual's tax bill in year x. Determine a function N that represents the individual's net income (income after taxes) in year x.

97. Some functions f have the property that $f(a + b) = f(a) + f(b)$ for all real numbers a and b. Which of the following functions have this property?

(a) $h(x) = 2x$ (b) $g(x) = x^2$
(c) $F(x) = 5x - 2$ (d) $G(x) = \frac{1}{x}$

98. Are the functions $f(x) = x - 1$ and $g(x) = \frac{x^2 - 1}{x + 1}$ the same? Explain.

99. Investigate when, historically, the use of the function notation $y = f(x)$ first appeared.

'Are You Prepared?' Answers

1. $(-1, 3)$ **2.** 21.5
3. $\{x \mid x \neq -4\}$ **4.** $\{x \mid x < -1\}$

3.2 The Graph of a Function

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Graphs of Equations (Section 2.2, pp. 165–168)
- Intercepts (Section 2.2, pp. 169–170)

Now work the 'Are You Prepared?' problems on page 236.

OBJECTIVES
1 Identify the Graph of a Function
2 Obtain Information from or about the Graph of a Function

In applications, a graph often demonstrates more clearly the relationship between two variables than, say, an equation or table would. For example, Table 1 shows the price per share of IBM stock at the end of each month manufacturing $x = 41$ computers minimizes the average cost at \$1231.74 per computer.

Table 2

X	Y_1
1	21179
2	11146
3	7781.1
4	6084
5	5054.6
6	4359.7
7	3856.4

Y_1 ■.56X²−34.39X…

Table 3

X	Y_1
38	1240.7
39	1235.9
40	1233
41	1231.7
42	1232.2
43	1234.4
44	1238.1

Y_1=1231.74487805

NOW WORK PROBLEM 29.

Summary

Graph of a function The collection of points (x, y) that satisfies the equation $y = f(x)$.
A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point (vertical-line test).

3.2 Assess Your Understanding

'Are You Prepared?'

Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. The intercepts of the equation $x^2 + 4y^2 = 16$ are ________. (pp. 169–170)

2. *True or False:* The point $(-2, 3)$ is on the graph of the equation $x = 2y - 2$. (pp. 165–168)

Concepts and Vocabulary

3. A set of points in the xy-plane is the graph of a function if and only if every ________ line intersects the graph in at most one point.

4. If the point $(5, -3)$ is a point on the graph of f, then $f(\_\_\_\_\_\_) = \_\_\_\_\_\_$.

5. Find a so that the point $(-1, 2)$ is on the graph of $f(x) = ax^2 + 4$.

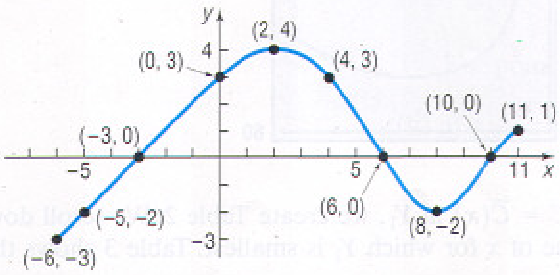
6. *True or False:* A function can have more than one y-intercept.

7. *True or False:* The graph of a function $y = f(x)$ always crosses the y-axis.

8. *True or False:* The y-intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.

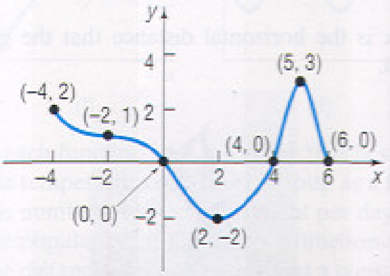
Exercises

9. Use the graph of the function f given below to answer parts (a)–(n).



(a) Find $f(0)$ and $f(-6)$.
(b) Find $f(6)$ and $f(11)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-4)$ positive or negative?
(e) For what numbers x is $f(x) = 0$?
(f) For what numbers x is $f(x) > 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = \frac{1}{2}$ intersect the graph?
(l) How often does the line $x = 5$ intersect the graph?
(m) For what values of x does $f(x) = 3$?
(n) For what values of x does $f(x) = -2$?

10. Use the graph of the function f given below to answer parts (a)–(n).



(a) Find $f(0)$ and $f(6)$.
(b) Find $f(2)$ and $f(-2)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-1)$ positive or negative?
(e) For what numbers is $f(x) = 0$?
(f) For what numbers is $f(x) < 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = -1$ intersect the graph?
(l) How often does the line $x = 1$ intersect the graph?
(m) For what value of x does $f(x) = 3$?
(n) For what value of x does $f(x) = -2$?

In Problems 11–22, determine whether the graph is that of a function by using the vertical-line test. If it is, use the graph to find:
(a) *Its domain and range*
(b) *The intercepts, if any*
(c) *Any symmetry with respect to the x-axis, the y-axis, or the origin*

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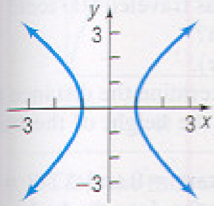

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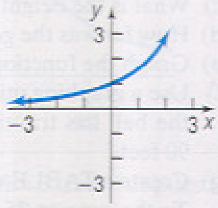

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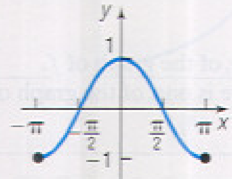

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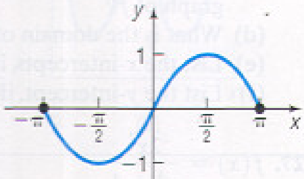

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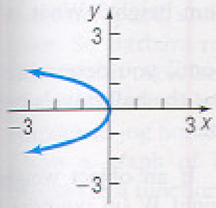

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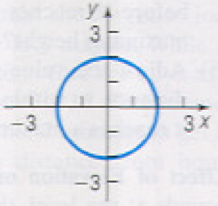

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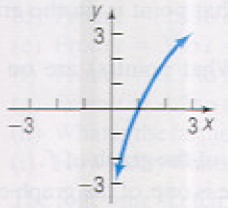

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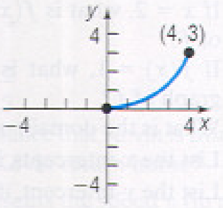

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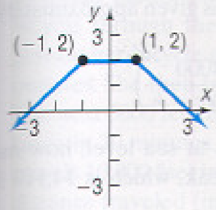

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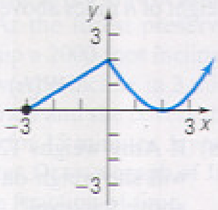

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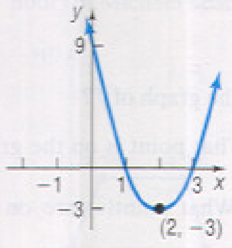

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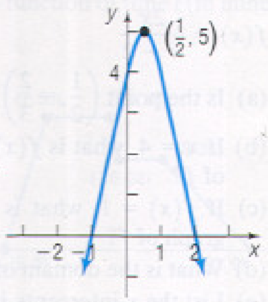


In Problems 23–28, answer the questions about the given function.

23. $f(x) = 2x^2 - x - 1$
(a) Is the point $(-1, 2)$ on the graph of f?
(b) If $x = -2$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = -1$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

24. $f(x) = -3x^2 + 5x$
(a) Is the point $(-1, 2)$ on the graph of f?
(b) If $x = -2$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = -2$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

25. $f(x) = \dfrac{x+2}{x-6}$
   (a) Is the point $(3, 14)$ on the graph of f?
   (b) If $x = 4$, what is $f(x)$? What point is on the graph of f?
   (c) If $f(x) = 2$, what is x? What point(s) are on the graph of f?
   (d) What is the domain of f?
   (e) List the x-intercepts, if any, of the graph of f.
   (f) List the y-intercept, if there is one, of the graph of f.

26. $f(x) = \dfrac{x^2+2}{x+4}$
   (a) Is the point $\left(1, \frac{3}{5}\right)$ on the graph of f?
   (b) If $x = 0$, what is $f(x)$? What point is on the graph of f?
   (c) If $f(x) = \frac{1}{2}$, what is x? What point(s) are on the graph of f?
   (d) What is the domain of f?
   (e) List the x-intercepts, if any, of the graph of f.
   (f) List the y-intercept, if there is one, of the graph of f.

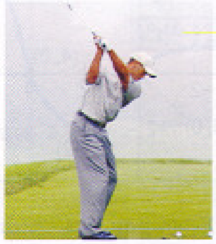
27. $f(x) = \dfrac{2x^2}{x^4+1}$
   (a) Is the point $(-1, 1)$ on the graph of f?
   (b) If $x = 2$, what is $f(x)$? What point is on the graph of f?
   (c) If $f(x) = 1$, what is x? What point(s) are on the graph of f?
   (d) What is the domain of f?
   (e) List the x-intercepts, if any, of the graph of f.
   (f) List the y-intercept, if there is one, of the graph of f.

28. $f(x) = \dfrac{2x}{x-2}$
   (a) Is the point $\left(\frac{1}{2}, -\frac{2}{3}\right)$ on the graph of f?
   (b) If $x = 4$, what is $f(x)$? What point is on the graph of f?
   (c) If $f(x) = 1$, what is x? What point(s) are on the graph of f?
   (d) What is the domain of f?
   (e) List the x-intercepts, if any, of the graph of f.
   (f) List the y-intercept, if there is one, of the graph of f.

29. **Motion of a Golf Ball** A golf ball is hit with an initial velocity of 130 feet per second at an inclination of 45° to the horizontal. In physics, it is established that the height h of the golf ball is given by the function

$$h(x) = \frac{-32x^2}{130^2} + x$$

where x is the horizontal distance that the golf ball has traveled.



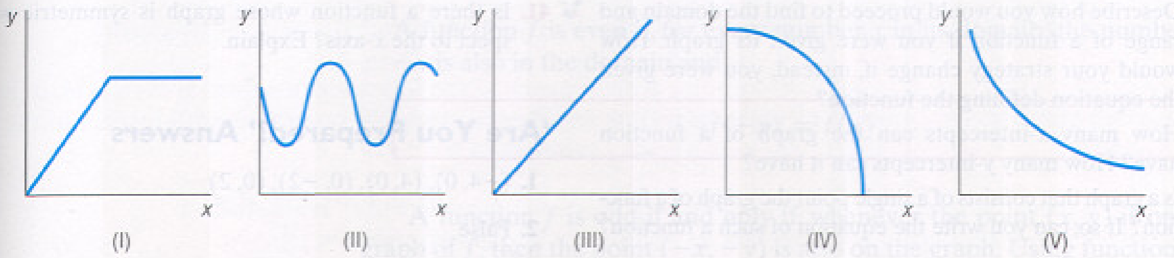
   (a) Determine the height of the golf ball after it has traveled 100 feet.
   (b) What is the height after it has traveled 300 feet?
   (c) What is the height after it has traveled 500 feet?
   (d) How far was the golf ball hit?
   (e) Graph the function $h = h(x)$.
   (f) Use a graphing utility to determine the distance that the ball has traveled when the height of the ball is 90 feet.
   (g) Create a TABLE with TblStart = 0 and ΔTbl = 25. To the nearest 25 feet, how far does the ball travel before it reaches a maximum height? What is the maximum height?
   (h) Adjust the value of ΔTbl until you determine the distance, to within 1 foot, that the ball travels before it reaches a maximum height.

30. **Effect of Elevation on Weight** If an object weighs m pounds at sea level, then its weight W (in pounds) at a height of h miles above sea level is given approximately by

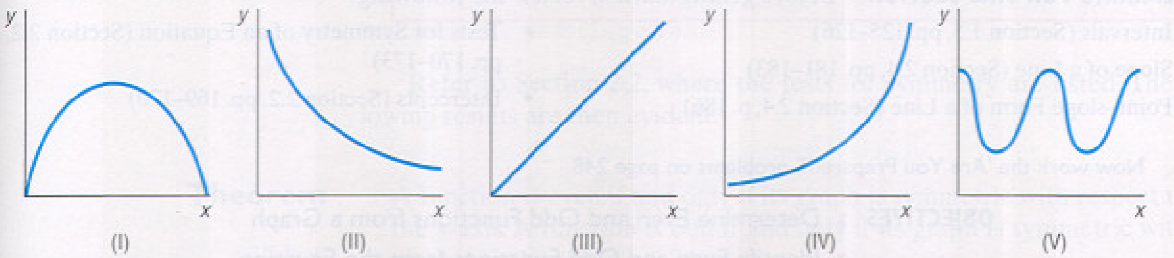
$$W(h) = m\left(\frac{4000}{4000 + h}\right)^2$$

   (a) If Amy weighs 120 pounds at sea level, how much will she weigh on Pike's Peak, which is 14,110 feet above sea level?
   (b) Use a graphing utility to graph the function $W = W(h)$. Use $m = 120$ pounds.
   (c) Create a Table with TblStart = 0 and ΔTbl = 0.5 to see how the weight W varies as h changes from 0 to 5 miles.
   (d) At what height will Amy weigh 119.95 pounds?
   (e) Does your answer to part (d) seem reasonable?

31. Match each function with the graph that best describes the situation. Discuss the reason for your choice.
   (a) The cost of building a house as a function of its square footage
   (b) The height of an egg dropped from a 300-foot building as a function of time
   (c) The height of a human as a function of time
   (d) The demand for Big Macs as a function of price
   (e) The height of a child on a swing as a function of time



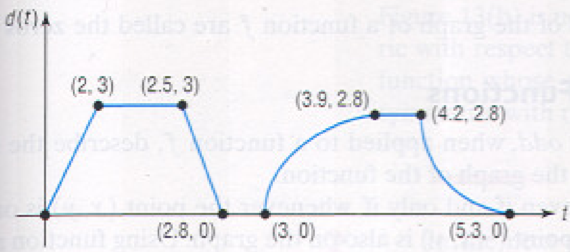
32. Match each function with the graph that best describes the situation. Discuss the reason for your choice.
(a) The temperature of a bowl of soup as a function of time
(b) The number of hours of daylight per day over a 2-year period
(c) The population of Florida as a function of time
(d) The distance of a car traveling at a constant velocity as a function of time
(e) The height of a golf ball hit with a 7-iron as a function of time



33. Consider the following scenario: Barbara decides to take a walk. She leaves home, walks 2 blocks in 5 minutes at a constant speed, and realizes that she forgot to lock the door. So Barbara runs home in 1 minute. While at her doorstep, it takes her 1 minute to find her keys and lock the door. Barbara walks 5 blocks in 15 minutes and then decides to jog home. It takes her 7 minutes to get home. Draw a graph of Barbara's distance from home (in blocks) as a function of time.

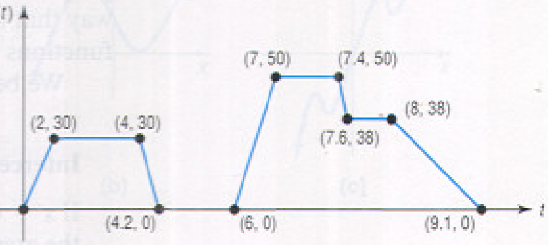
34. Consider the following scenario: Jayne enjoys riding her bicycle through the woods. At the forest preserve, she gets on her bicycle and rides up a 2000-foot incline in 10 minutes. She then travels down the incline in 3 minutes. The next 5000 feet is level terrain and she covers the distance in 20 minutes. She rests for 15 minutes. Jayne then travels 10,000 feet in 30 minutes. Draw a graph of Jayne's distance traveled (in feet) as a function of time.

35. The following sketch represents the distance d (in miles) that Kevin is from home as a function of time t (in hours). Answer the questions based on the graph. In parts (a)–(g), how many hours elapsed and how far was Kevin from home during this time?



(a) From $t = 0$ to $t = 2$
(b) From $t = 2$ to $t = 2.5$
(c) From $t = 2.5$ to $t = 2.8$
(d) From $t = 2.8$ to $t = 3$
(e) From $t = 3$ to $t = 3.9$
(f) From $t = 3.9$ to $t = 4.2$
(g) From $t = 4.2$ to $t = 5.3$
(h) What is the farthest distance that Kevin is from home?
(i) How many times did Kevin return home?

36. The following sketch represents the speed v (in miles per hour) of Michael's car as a function of time t (in minutes).



(a) Over what interval of time is Michael traveling fastest?
(b) Over what interval(s) of time is Michael's speed zero?
(c) What is Michael's speed between 0 and 2 minutes?
(d) What is Michael's speed between 4.2 and 6 minutes?
(e) What is Michael's speed between 7 and 7.4 minutes?
(f) When is Michael's speed constant?

37. Draw the graph of a function whose domain is $\{x \mid -3 \le x \le 8,\ x \ne 5\}$ and whose range is $\{y \mid -1 \le y \le 2,\ y \ne 0\}$. What point(s) in the rectangle $-3 \le x \le 8$, $-1 \le y \le 2$ cannot be on the graph? Compare your graph with those of other students. What differences do you see?

38. Describe how you would proceed to find the domain and range of a function if you were given its graph. How would your strategy change if, instead, you were given the equation defining the function?

39. How many x-intercepts can the graph of a function have? How many y-intercepts can it have?

40. Is a graph that consists of a single point the graph of a function? If so, can you write the equation of such a function?

41. Is there a function whose graph is symmetric with respect to the x-axis? Explain.

'Are You Prepared?' Answers

1. $(-4, 0), (4, 0), (0, -2), (0, 2)$
2. False

3.3 Properties of Functions

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Intervals (Section 1.5, pp. 125–126)
- Slope of a Line (Section 2.4, pp. 181–183)
- Point–slope Form of a Line (Section 2.4, p. 186)
- Tests for Symmetry of an Equation (Section 2.2, pp. 170–173)
- Intercepts (Section 2.2, pp. 169–170)

Now work the 'Are You Prepared?' problems on page 248.

OBJECTIVES
1. Determine Even and Odd Functions from a Graph
2. Identify Even and Odd Functions from the Equation
3. Use a Graph to Determine Where a Function Is Increasing, Is Decreasing, or Is Constant
4. Use a Graph to Locate Local Maxima and Minima
5. Use a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing
6. Find the Average Rate of Change of a Function

It is easiest to obtain the graph of a function $y = f(x)$ by knowing certain properties that the function has and the impact of these properties on the way that the graph will look. In this section we describe some properties of functions that we will use in subsequent chapters.

We begin with the familiar notions of intercepts and symmetry.

Intercepts

If $x = 0$ is in the domain of a function $y = f(x)$, then the y-intercept of the graph of f is the value of f at 0, which is $f(0)$. The x-intercepts of the graph of f, if there are any, are the solutions of the equation $f(x) = 0$.

The x-intercepts of the graph of a function f are called the **zeros of** f.

Even and Odd Functions

1 The words *even* and *odd*, when applied to a function f, describe the symmetry that exists for the graph of the function.

A function f is even if and only if, whenever the point (x, y) is on the graph of f, then the point $(-x, y)$ is also on the graph. Using function notation, we define an even function as follows: