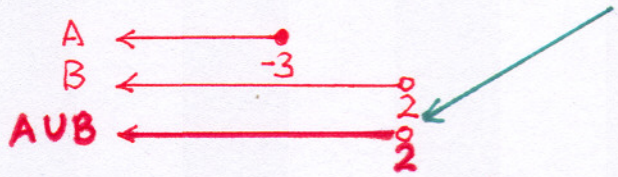
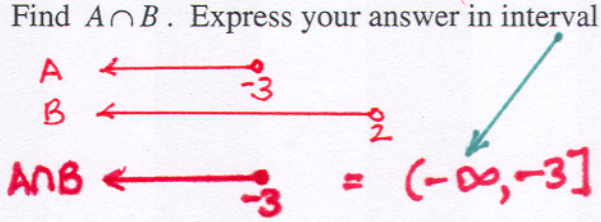
ANSWERS PAGE 1 COMMON FINAL – FALL 2005 –

1. (4 pts.) For $A=\{x \mid x \le -3\}$ and $B=\{x \mid x<2\}$,

a) Find $A \cup B$. Express your answer as a number line graph.



b) Find $A \cap B$. Express your answer in interval notation.



2. (4 pts.) Factor completely.

$3x^{-1/2} - 4x^{1/2} + x^{3/2} = \frac{3 - 4x + x^2}{x^{1/2}}$

$x^2 - 4x + 3$
$(x-3)(x-1)$

To 'kill off' that $x^{-1/2}$!!

$= \frac{(x-3)(x-1)}{x^{1/2}}$

3. (4 pts.) Solve for x: $3x^2 + 6x - 5 = 0$ (Simplify your answer.)

$x = \frac{-6 \pm \sqrt{36+60}}{6}$

$= \frac{-6 \pm \sqrt{96}}{6}$

$= \frac{-6 \pm 4\sqrt{6}}{6} = -1 \pm \frac{2\sqrt{6}}{3}$

4. (5 pts.) Divide $\frac{2-i}{4-3i}$. Express your answer in the form $a+bi$.

$\frac{2-i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{8+3-4i+6i}{16+9} = \frac{11+2i}{25} = \frac{11}{25} + \frac{2}{25}i$

ANSWERS - p. 2

5. (5 pts.) Solve. Express your answer using interval notation.

$$\frac{3x-5}{x-5} \geq 4$$

$\frac{3x-5}{x-5} - 4\frac{x-5}{x-5} \geq 0$

$\frac{-x+15}{x-5} \geq 0$

TWO CRITICAL VALUES: 15 & 5

$\frac{-x+15}{x-5}$: $\frac{+}{-}$ | $\frac{+}{+}$ | $\frac{-}{+}$ on the number line at 5 and 15; > 0 here

at $x=5$: NO SOLUTION

at $x=15$: $\frac{0}{10} \geq 0$ ✓ (solves the inequality)

the solution set is $(5, 15]$

6. (4 pts.) Solve. Express your answer using interval notation.

$$8 - |2x-1| \leq -2$$

$10 \leq |2x-1| \Rightarrow 2x-1 \leq -10$ or $2x-1 \geq 10$

$2x \leq -9$ $\quad 2x \geq 11$

$x \leq -\frac{9}{2}$ $\quad x \geq \frac{11}{2}$

the solution set is $(-\infty, -9/2] \cup [11/2, \infty)$

METHOD 2:

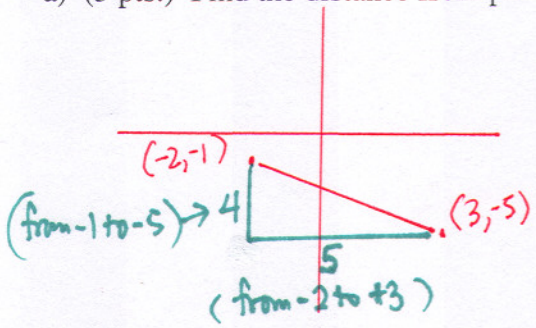
$10 \leq |2x-1|$

$5 \leq |x-1/2| \Rightarrow$ distance between x & $1/2$ must be 5 or more.

$\Rightarrow$ x is below or = $1/2 - 5$ or x is above or = $\frac{1}{2} + 5$

7. Let A = $(-2, -1)$ and B = $(3, -5)$ be points in the xy-plane.

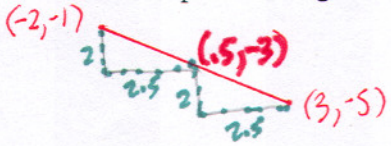
a) (3 pts.) Find the distance from point A to point B.



$D = \sqrt{4^2 + 5^2}$

$= \sqrt{41}$ units

b) (2 pts.) Find the midpoint of segment AB.



$\left(\frac{-2+3}{2}, \frac{-1+-5}{2}\right)$

$= \left(\frac{1}{2}, -3\right)$

ANSWERS – p.3

8. (5 pts.) For $f(x) = -3x^2 + 12x - 6$,

a) Express $f(x)$ in the form $f(x) = a(x-h)^2 + k$.

$f(x) = -3(x^2 - 4x\ \ ) - 6$

$= -3(x^2 - 4x + 4) - 6 + 12$

$= -3(x-2)^2 + 6$

= A PARABOLA OPENING DOWN, vertex at $(2, 6)$

b) Find the extreme value of $f(x)$. = 6

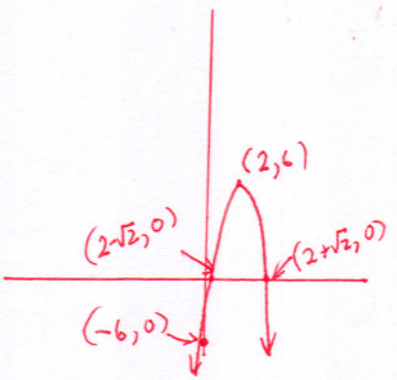
MAXIMUM at $(2, 6)$

c) Sketch $f(x)$.

$-3(x^2 - 4x + 2) = 0$

when $x = \dfrac{4 \pm \sqrt{16-8}}{2}$

$= 2 \pm \sqrt{2}$



9. a) (2 pts.) Find the slope of the line given by the equation $3x + 4y = 12$.

$y = -\frac{3}{4}x + 3$

$\Rightarrow$ slope $= -\frac{3}{4}$

b) (2 pts.) Find the equation of a line parallel to the line $3x + 4y = 12$ passing through the point $(3, -2)$.

MUST BE $3x + 4y = k$

$3 \cdot 3 + 4 \cdot (-2) = k$

$3x + 4y = 1$

$y - (-2) = -\frac{3}{4}(x - 3)$

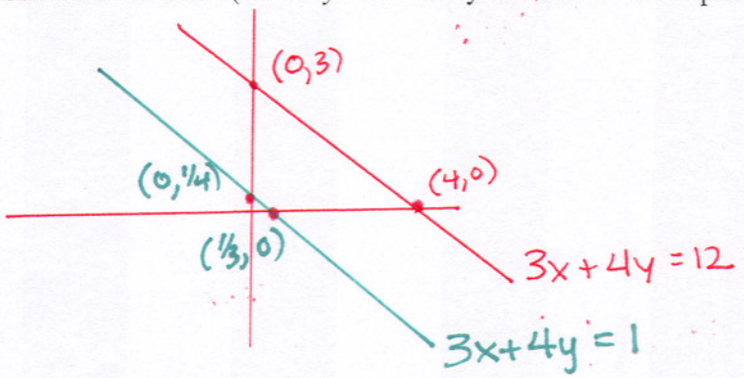
$y + 2 = -\frac{3}{4}(x - 3)$

or $y = -\frac{3}{4}x + b$

$-2 = -\frac{3}{4} \cdot 3 + b \Rightarrow b = \frac{1}{4}$

$y = -\frac{3}{4}x + \frac{1}{4}$

c) (2 pts.) Sketch both lines ($3x + 4y = 12$ and your answer from part b)) on one graph.



ANSWERS – p. 4

10. (5 pts.) For $f(x)=x^2-4x+3$, find $\dfrac{f(x+h)-f(x)}{h}$. (Simplify your answer completely.)

$$\frac{(x+h)^2-4(x+h)+3-(x^2-4x+3)}{h}$$

$$\frac{x^2+2xh+h^2-4x-4h+3-(x^2-4x+3)}{h}$$

MARKED PAIRS EACH ADD TO 0

$$\frac{2xh+h^2-4h}{h} = 2x+h-4$$

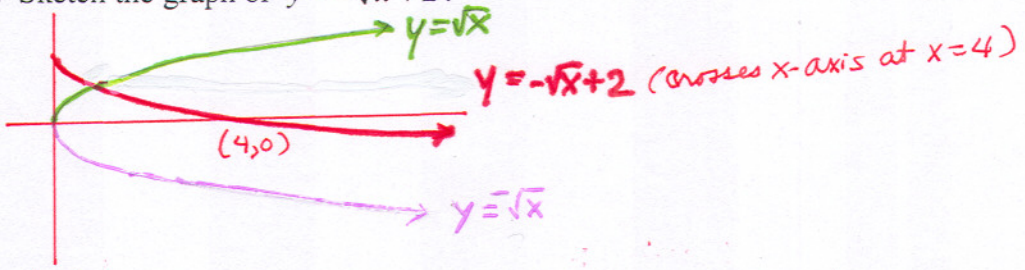
11. The graph of $y=-\sqrt{x}+2$ can be sketched by applying two transformations to the graph of $f(x)=\sqrt{x}$.

a) (2 pts.) Which of the following correctly describes this process? (circle one)

(i) Shift the graph of $f(x)$ up two units and then reflect in the x-axis.
(ii) Shift the graph of $f(x)$ up two units and then reflect in the y-axis.
(iii) Reflect the graph of $f(x)$ in the x-axis and then shift two units to the left.
(iv) Reflect the graph of $f(x)$ in the x-axis and then shift two units up.

$-\sqrt{x}$ $+2$

b) (2 pts.) Sketch the graph of $y=-\sqrt{x}+2$.



12. (4 pts.) For $f(x)=\dfrac{1}{3x-4}$, find the formula for $f^{-1}(x)$.

METHOD TWO

f multiplies x by 3,
subtracts 4,
takes the reciprocal.
THEREFORE
f^{-1} takes reciprocal,
adds 4,
divides by 3

$\Rightarrow f^{-1}(x)=\dfrac{\frac{1}{x}+4}{3}=\dfrac{1+4x}{3x}$

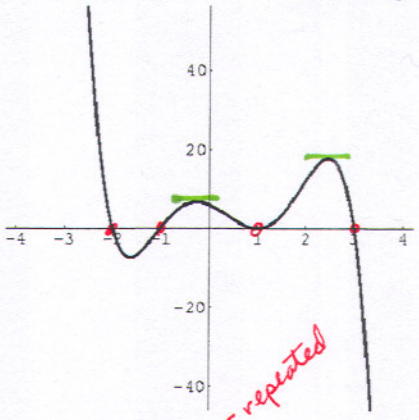
$y=\dfrac{1}{3x-4}$ (solve for x)

$3xy-4y=1$
$3xy=1+4y$
$x=\dfrac{1+4y}{3y} \Rightarrow f^{-1}(y)=\dfrac{1+4y}{3y}$

so $f^{-1}(x)=\dfrac{1+4x}{3x}$

ANSWERS- p.5

13. (6 pts.) The graph of a polynomial $P(x)$ is given below. Use the graph to answer the following:



a) What are the real zeros of $P(x)$? -2, -1, 1, 3

b) Does $P(x)$ have odd or even degree? ODD

c) Which zero has an even multiplicity? 1

d) Is the leading coefficient of $P(x)$ positive or negative? negative (as in $-x^5$...)

e) How many local maximum values does $P(x)$ have? two

f) What is the least possible degree of $P(x)$? 5

14. (4 pts.) Divide x^4+3x^2+1 by x^2-2x+3.

a) The quotient is x^2+2x+4

b) The remainder is $2x-11$

$$
\begin{array}{r}
x^2+2x+4 \\
x^2-2x+3 \overline{)\,x^4+3x^2+1} \\
\underline{x^4-2x^3+3x^2} \\
2x^3+1 \\
\underline{2x^3-4x^2+6x} \\
4x^2-6x+1 \\
\underline{4x^2-8x+12} \\
2x-11
\end{array}
$$

ANSWERS – p.6

15. For $P(x) = 3x^3 - x^2 - 6x + 2$,

a) (2 pts.) List all the possible rational zeros of $P(x)$. Since coefficients of P are integers...

$\pm \frac{1, 2}{1, 3} \rightarrow \pm 1, 2, 1/3, 2/3$ ... by the "rational zeroes theorem"

b) (1 pt.) Use synthetic division to show that $\frac{1}{3}$ is a zero of $P(x)$.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -6 & 2 \\ & & 1 & 0 & -2 \\ \hline & 3 & 0 & -6 & 0! \end{array} \Rightarrow P(x) = (x - 1/3)(3x^2 - 6)$$

$\hookrightarrow P(1/3) = 0$

$= (x - 1/3) \cdot 3 \cdot (x^2 - 2)$

$\pm\sqrt{2}$

c) (2 pts.) Find the remaining zeros of $P(x)$.

d) (2 pts.) Factor $P(x)$ completely into linear factors.

$P(x) = (x - 1/3) \cdot 3 \cdot (x^2 - 2)$

$= (x - 1/3) \cdot 3 \cdot (x + \sqrt{2})(x - \sqrt{2})$

OR $(3x - 1)(x + \sqrt{2})(x - \sqrt{2})$

EITHER FORM IS ACCEPTABLE.

16. (5 pts.) Find a fourth degree polynomial $P(x)$ with $1 - i$ a zero, 0 a zero of multiplicity 2, and a leading coefficient of 3. Express your answer in the form $P(x) = ax^4 + bx^3 + cx^2$ (a, b, c are integers).

If a, b & c are INTEGERS, they must be real numbers, therefore any complex root must be accompanied by a conjugate root. Since $1 - i$ is a root, $1 + i$ must be too

ROOTS $1 - i$, $1 + i$, 0, 0

$\Rightarrow P(x) = k \cdot (x - (1 - i))(x - (1 + i))(x - 0)^2$

$= k(x^2 - 2x + 2) \cdot x^2$

leading coefficient 3 $\Rightarrow$ k must be 3.

$P(x) = 3(x^2 - 2x + 2) \cdot x^2$

$= 3x^4 - 6x^3 + 6x^2$

ANSWERS – p.7

17. For $R(x)=\dfrac{2x-5}{x-5}$

a) (2 pts.) Find the x and y intercepts.

$R(0)=\frac{0-5}{0-5}=1 \Rightarrow$ (0,1) is the y-intercept

$\frac{2x-5}{x-5}=0$ when $2x-5=0$ ... when $x=\frac{5}{2} \Rightarrow$ (5/2, 0) is the x-intercept

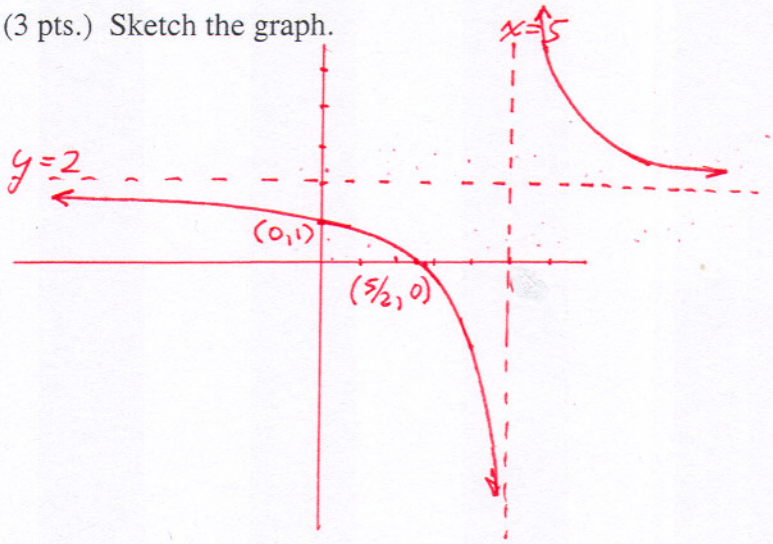
b) (1 pt.) Find the vertical asymptote. (write the *equation*)

$\frac{2x-5}{x-5}$ runs to $\pm\infty$ when $\boxed{x=5}$

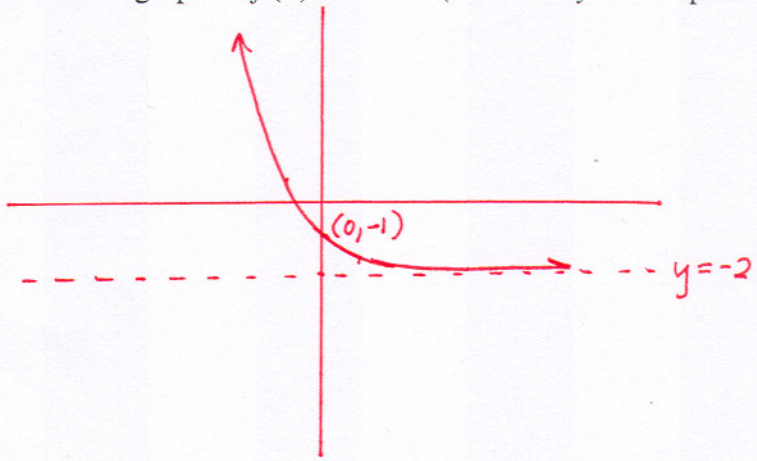
c) (1 pt.) Find the horizontal asymptote. (write the *equation*)

$\lim_{x\to\infty} \frac{2x-5}{x-5} = 2$ $\boxed{y=2}$

d) (3 pts.) Sketch the graph.



18. (4 pts.) Sketch the graph of $f(x)=e^{-x}-2$ (Label the y-intercept and the asymptote.)



ANSWERS – p. 8

19. (4 pts.) Evaluate the expression: $2\log_3 10 - \log_3 18 + \log_3\left(\frac{1}{50}\right)$

$= \log_3 \frac{10^2}{18} \cdot \frac{1}{50}$

$= \log_3 \frac{10^2}{9 \cdot 2 \cdot 50}$

$= \log_3 \frac{1}{9}$

$= -2$

20. (4 pts.) Solve for x: $4(1+2^{3x}) = 9$ (Give an exact answer.)

$1 + 2^{3x} = \frac{9}{4}$

$2^{3x} = \frac{5}{4}$

$\ln 2^{3x} = \ln \frac{5}{4}$

$3x \ln 2 = \ln \frac{5}{4}$

$x = \frac{\ln \frac{5}{4}}{3 \ln 2}$

$= \frac{1}{3} \log_2 \frac{5}{4}$

OR, EQUIVALENTLY (ALL FORMS O.K.):

$\frac{\ln 5 - \ln 4}{3 \ln 2}$ $\quad \frac{\ln 5 - \ln 4}{\ln 8}$ $\quad \frac{\ln 5}{\ln 8} - \frac{2}{3}$

21. (4 pts.) $3000 is deposited in a savings account. How long does it take the money to grow to $5000 if it earns 6% a year compounded continuously?

Find t when $\$3000 e^{.06t} = \$5000$

$e^{.06t} = \frac{5}{3}$

$\ln e^{.06t} = \ln \frac{5}{3}$

$.06t \ln e = \ln \frac{5}{3}$

$.06t = \ln \frac{5}{3}$

$t = \frac{\ln 5/3}{.06}$