Math 102 Common Final Exam Fall 2005

1. (4 pts.) For \( A = \{x \mid x \leq -3\} \) and \( B = \{x \mid x < 2\} \),
   
   a) Find \( A \cup B \). Express your answer as a number line graph.
   
   b) Find \( A \cap B \). Express your answer in interval notation.

2. (4 pts.) Factor completely. \( 5x^{-1/2} - 4x^{1/2} + x^{3/2} \)

3. (4 pts.) Solve for \( x \): \( 3x^2 + 6x - 5 = 0 \) (Simplify your answer.)

4. (5 pts.) Divide \( \frac{2 - i}{4 - 3i} \). Express your answer in the form \( a + bi \).

5. (5 pts.) Solve. Express your answer using interval notation.
   \[ \frac{3x - 5}{x - 5} \geq 4 \]

6. (4 pts.) Solve. Express your answer using interval notation.
   \[ 8 - |2x - 1| \leq -2 \]

7. Let \( A = (-2, -1) \) and \( B = (3, -5) \) be points in the \( xy \)-plane.
   
   a) (3 pts.) Find the distance from point \( A \) to point \( B \).
   
   b) (2 pts.) Find the midpoint of segment \( AB \).

8. (5 pts.) For \( f(x) = -3x^2 + 12x - 6 \),
   
   a) Express \( f(x) \) in the form \( f(x) = a(x - h)^2 + k \).
   
   b) Find the extreme value of \( f(x) \).
   
   c) Sketch \( f(x) \).

9. a) (2 pts.) Find the slope of the line given by the equation \( 3x + 4y = 12 \).
   
   b) (2 pts.) Find the equation of a line parallel to the line \( 3x + 4y = 12 \) through the point \( (3, -2) \).
   
   c) (2 pts.) Sketch both lines \( (3x + 4y = 12 \) and your answer from part b)) on one graph.

10. (5 pts.) For \( f(x) = x^2 - 4x + 3 \), find \( \frac{f(x + h) - f(x)}{h} \). (Simplify your answer completely.)

11. The graph of \( y = -\sqrt{x} + 2 \) can be sketched by applying two transformations to the graph of \( f(x) = \sqrt{x} \).
   
   a) (2 pts.) Which of the following correctly describes this process? (circle one)
      
      (i) Shift the graph of \( f(x) \) up two units and then reflect in the \( x \)-axis.
      
      (ii) Shift the graph of \( f(x) \) up two units and then reflect in the \( y \)-axis.
      
      (iii) Reflect the graph of \( f(x) \) in the \( x \)-axis and then shift two units to the left.
      
      (iv) Reflect the graph of \( f(x) \) in the \( x \)-axis and then shift two units up.

   b) (2 pts.) Sketch the graph of \( y = -\sqrt{x} + 2 \).
12. (4 pts.) For $f(x) = \frac{1}{3x - 4}$, find the formula for $f^{-1}(x)$.

13. (6 pts.) The graph of a polynomial $P(x)$ is given below. Use the graph to answer the following:

   a) What are the real zeros of $P(x)$? _____________________________
   b) Does $P(x)$ have odd or even degree? __________________________
   c) Which zero has an even multiplicity? _____________________________
   d) Is the leading coefficient of $P(x)$ positive or negative? __________
   e) How many local maximum values does $P(x)$ have? _______________
   f) What is the least possible degree of $P(x)$? _______________________

14. (4 pts.) Divide $x^4 + 3x^2 + 1$ by $x^2 - 2x + 3$.
   a) The quotient is ______________________
   b) The remainder is ______________________

15. For $P(x) = 3x^3 - x^2 - 6x + 2$,
   a) (2 pts.) List all the possible rational zeros of $P(x)$.
   b) (1 pt.) Use synthetic division to show that $\frac{1}{3}$ is a zero of $P(x)$.
   c) (2 pts.) Find the remaining zeros of $P(x)$.
   d) (2 pts.) Factor $P(x)$ completely into linear factors.

16. (5 pts.) Find a fourth degree polynomial $P(x)$ with $1 - i$ a zero, $0$ a zero of multiplicity 2, and a leading coefficient of 3. Express your answer in the form $P(x) = ax^4 + bx^3 + cx^2 (a, b, c$ are integers).

17. For $R(x) = \frac{2x - 5}{x - 5}$
   a) (2 pts.) Find the x and y intercepts.
   b) (1 pt.) Find the vertical asymptote. (write the equation)
   c) (1 pt.) Find the horizontal asymptote. (write the equation)
   d) (3 pts.) Sketch the graph.
18. (4 pts.) Sketch the graph of $f(x) = e^{-x} - 2$ (Label the $y$-intercept and the asymptote.)

19. (4 pts.) Evaluate the expression: $2\log_3 10 - \log_3 18 + \log_3 \left(\frac{1}{50}\right)$

20. (4 pts.) Solve for $x$: $4(1 + 2^x) = 9$ (Give an exact answer.)

21. (4 pts.) $3000$ is deposited in a savings account. How long does it take the money to grow to $5000$ if it earns 6% a year compounded continuously?