# Math 102 Common Final Exam Fall 2005

1. (4 pts.) For $A = \{x \mid x \leq -3\}$ and $B = \{x \mid x < 2\}$,

    a) Find $A \cup B$. Express your answer as a number line graph.

    b) Find $A \cap B$. Express your answer in interval notation.

2. (4 pts.) Factor completely. $5x^{-\frac{1}{3}} - 4x^{\frac{1}{3}} + x^{\frac{2}{3}}$

3. (4 pts.) Solve for x: $3x^2 + 6x - 5 = 0$ (Simplify your answer.)

4. (5 pts.) Divide $\dfrac{2-i}{4-3i}$. Express your answer in the form $a + bi$.

5. (5 pts.) Solve. Express your answer using interval notation. $\dfrac{3x-5}{x-5} \geq 4$

6. (4 pts.) Solve. Express your answer using interval notation. $8 - |2x-1| \leq -2$

7. Let A $= (-2,-1)$ and B $= (3,-5)$ be points in the xy-plane.

    a) (3 pts.) Find the distance from point A to point B.

    b) (2 pts.) Find the midpoint of segment AB.

8. (5 pts.) For $f(x) = -3x^2 + 12x - 6$,

    a)   Express $f(x)$ in the form $f(x) = a(x-h)^2 + k$.

    b) Find the extreme value of $f(x)$.

    c) Sketch $f(x)$.

9. a) (2 pts.) Find the slope of the line given by the equation $3x + 4y = 12$.

    b) (2 pts.) Find the equation of a line parallel to the line $3x + 4y = 12$ through the point $(3,-2)$.

    c) (2 pts.) Sketch both lines ($3x + 4y = 12$ and your answer from part b)) on one graph.

10. (5 pts.) For $f(x) = x^2 - 4x + 3$, find $\dfrac{f(x+h) - f(x)}{h}$. (Simplify your answer completely.)

11. The graph of $y = -\sqrt{x} + 2$ can be sketched by applying two transformations to the graph of $f(x) = \sqrt{x}$.

    a) (2 pts.) Which of the following correctly describes this process? (circle one)

        (i) Shift the graph of $f(x)$ up two units and then reflect in the x-axis.

        (ii) Shift the graph of $f(x)$ up two units and then reflect in the y-axis.

        (iii) Reflect the graph of $f(x)$ in the x-axis and then shift two units to the left.

        (iv) Reflect the graph of $f(x)$ in the x-axis and then shift two units up.

    b) (2 pts.) Sketch the graph of $y = -\sqrt{x} + 2$.

12. (4 pts.) For $f(x) = \dfrac{1}{3x-4}$, find the formula for $f^{-1}(x)$.

13. (6 pts.) The graph of a polynomial $P(x)$ is given below. Use the graph to answer the following:
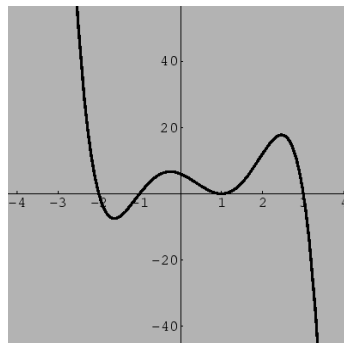
   a) What are the real zeros of $P(x)$? _____

   b) Does $P(x)$ have odd or even degree? _____

   c) Which zero has an even multiplicity? _____

   d) Is the leading coefficient of $P(x)$ positive or negative? _____

   e) How many local maximum values does $P(x)$ have? _____

   f) What is the least possible degree of $P(x)$? _____

14. (4 pts.) Divide $x^4 + 3x^2 + 1$ by $x^2 - 2x + 3$.
     a) The quotient is _____
     b) The remainder is _____

15. For $P(x) = 3x^3 - x^2 - 6x + 2$,

   a) (2 pts.) List all the possible rational zeros of $P(x)$.

   b) (1 pt.) Use synthetic division to show that $\dfrac{1}{3}$ is a zero of $P(x)$.

   c) (2 pts.) Find the remaining zeros of $P(x)$.

   d) (2 pts.) Factor $P(x)$ completely into linear factors.

16. (5 pts.) Find a fourth degree polynomial $P(x)$ with $1-i$ a zero, 0 a zero of multiplicity 2, and

   a leading coefficient of 3. Express your answer in the form $P(x) = ax^4 + bx^3 + cx^2$ (a,b,c are
   integers).

17. For $R(x) = \dfrac{2x-5}{x-5}$

     a)   (2 pts.) Find the x and y intercepts.
     b)   (1 pt.) Find the vertical asymptote. (write the equation)
     c)   (1 pt.) Find the horizontal asymptote. (write the equation)
     d)   (3 pts.) Sketch the graph.

18. (4 pts.) Sketch the graph of $f(x) = e^{-x} - 2$ (Label the y-intercept and the asymptote.)

19. (4 pts.) Evaluate the expression: $2\log_3 10 - \log_3 18 + \log_3\left(\dfrac{1}{50}\right)$

20. (4 pts.) Solve for x: $4(1 + 2^{3x}) = 9$ (Give an exact answer.)

21. (4 pts.) $3000 is deposited in a savings account. How long does it take the money to grow to $5000

if it earns 6% a year compounded continuously?