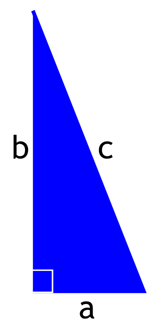
A Proof of the Pythagorean Theorem

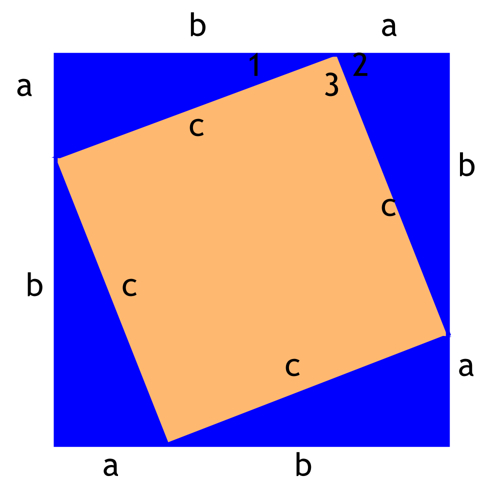
(There are hundreds of proofs of the Pythagorean Theorem. A free look at 71 of them is offered at http://www.cut-the-knot.org/pythagoras/index.shtml. Don't miss the great Java applet link in Proof #1. Proof #9 in that list is a nice variation on the following proof, with a link to an animated version.)

The following proof is attributed to the Indian astronomer Bhaskara (1114-1185).

Given any right triangle with sides a,b,c

We can construct a square
with sides of length a + b,
so four copies of the given right triangle
can be placed as shown, forming a
c-by-c square inside the a+b square.





That the c-c-c-c rhombus in the center is indeed a square can be verified by observing that the angles marked 1 and 2 are complementary (since they are the acute angles of a right triangle, copied four times), and that 1 & 2 & 3 form a straight (180°) angle. (If you don't see this, try the following: mark all the angles that are congruent to angle 1 with a "1", and all angles that are congruent to angle 2 with a "2" . Also keep in mind that the triangle is a right triangle.)

So, now that we know for sure that the square in the center truly is a square....

We argue that the area of the large square must equal the sum of the areas of the smaller square and the four right triangles:

The area of the large (a+b) square is $(a+b)(a+b) = a^2 + 2ab + b^2$.

The area of the smaller square plus the four triangular areas

$$= c \bullet c + 4 \bullet (\tfrac{1}{2})ab = c^2 + 2ab .$$

Since these must be equal:

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

Therefore:

$$a^2 + b^2 = c^2$$

— QED (Quod Erat Demonstratum.)

By the way: If the illustration of the a+b-sided square above appears "cockeyed", know that that is an optical illusion caused by the interior c-by-c square.